$$
\begin{aligned}
u_1 &= -b(\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4); \\
u_2 &= lb(\Omega_4 - \Omega_2); \\
u_3 &= lb(\Omega_1 - \Omega_3) \\
u_4 &= db(-\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4)
\end{aligned} \tag{3}
$$

Using Eqs.(1), (2) and (3), the equation of motion of the Quadrotor can be represented as

$$
\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \frac{1}{m}\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} + g\begin{bmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{bmatrix} - \begin{bmatrix} qw - rv \\ ru - pw \\ pv - qu \end{bmatrix} \tag{4}
$$

where u,v,w are body-axis velocities; p,q,r are angular rates; ϕ,θ,ψ are attitude angles; g is the gravity and m is the total mass. F_x, F_y, F_z are total three-axis forces

$$
\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix} + \begin{bmatrix} C_{fx} \\ C_{fy} \\ C_{fz} \end{bmatrix} \tag{5}
$$

where C_{fx}, C_{fy}, C_{fz} are three-axis aerodynamic forces. They can be neglected for low speed operations. In this work, they are neglected. Derivatives of angular rates(p,q,r) are given below:

$$
\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} u_2 / I_{xx} + qr\dfrac{I_{yy} - I_{zz}}{I_{xx}} \\ u_3 / I_{yy} + pr\dfrac{I_{zz} - I_{xx}}{I_{yy}} \\ u_4 / I_{zz} + pq\dfrac{I_{xx} - I_{yy}}{I_{zz}} \end{bmatrix} \tag{6}
$$

where I_{xx}, I_{yy}, I_{zz} are moment inertia. Eqs.(4) and (5) are equations of six degree of freedom(6DOF). Derivations of attitude(ϕ,θ,ψ) and position(X,Y,Z) are given as in the form of

$$
\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sec\theta\sin\phi & \sec\theta\cos\phi \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{7}
$$

$$
\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{8}
$$

Fig.2 shows the simulating block diagram[17-19] of the Quadrotor will be used in this work.

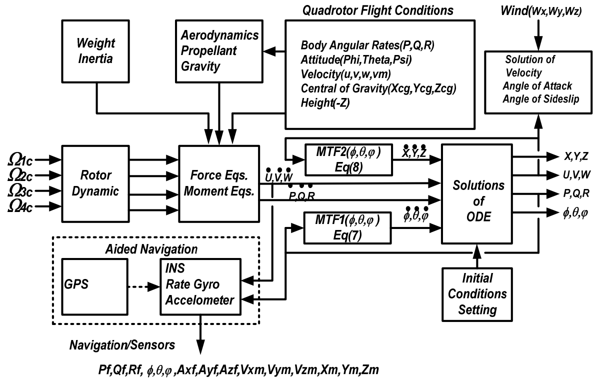

Fig.2. Simulation block diagram of the Quadrotor.

3. Guidance and Control Laws

3.1 Basic Control System Designs

The Basic Control System is discussed and designed first. It will be used for guidance laws development. The relationship between thrust(T_i) and total thrust and angular momentum can be evaluated from Eq.(2) and represented as

$$
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} -1/4 & 0 & +1/2 & -1/4 \\ -1/4 & -1/2 & 0 & +1/4 \\ -1/4 & 0 & -1/2 & -1/4 \\ -1/4 & +1/2 & 0 & +1/4 \end{bmatrix}\begin{bmatrix} -(T_1 + T_2 + T_3 + T_4) \\ T_4 - T_2 \\ T_1 - T_3 \\ -T_1 + T_2 - T_3 + T_4 \end{bmatrix} \tag{9}
$$

Eq.(9) is called as a mixer. Using Eqs.(1) and (2), the Basic Control Configuration is designed and shown in Fig.3 in which the Quadrotor dynamic is shown in Fig.2. Eq.(9) gives the maneuverability of the Quadrotor is limited by maximal thrust of the propeller(T_i). Therefore, limitation for command ($u_{1c}, u_{2c}, u_{3c}, u_{4c}$) shown in Fig.3 must be added for preventing uncontrollable nonlinear dynamics.

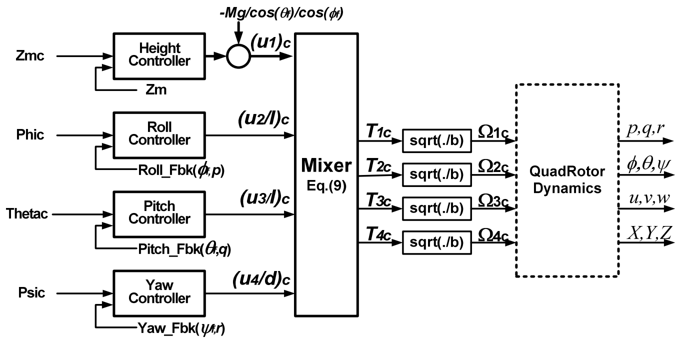

Fig.3. The Basic Control Configuration.

Fig.3 shows that there are four input command and output controls. They are altitude control command (Z_{mc}) and three attitude control command (*Phic, Thetac,Psic*). Feedback datum for command tracking and stabilization are altitude measurement Z_{mf} , vertical speed w_f , angular rates(p_f, q_f, r_f) and attitudes(ϕ_f, θ_f, ψ_f). Outputs of the basic control ($\Omega_{1c}, \Omega_{2c}, \Omega_{3c}, \Omega_{4c}$) are rotating speed of propellers. Proportion plus Integration(PI) control laws are used for command tracking and disturbance rejection. Control laws are given below:

$$u_{1c} = K_{iz}[(K_{oz} + K_{ozi}/s)(Z_{mc} - Z_{mf}) - w_f] - mg/\cos\theta_f \cos\phi_f \quad (10)$$

$$u_{2c} = K_{ip}[(K_{op} + K_{opi}/s)(Phic - \phi_f) - p_f] \quad (11)$$

$$u_{3c} = K_{iq}[(K_{oq} + K_{oqi}/s)(Thetac - \theta_f) - q_f] \quad (12)$$

$$u_{4c} = K_{ir}[(K_{or} + K_{ori}/s)(Psic - \psi_f) - r_f] \quad (13)$$

where *K(*)* are loop gains will be selected. They are determined by bandwidth of command tracking and inner loop gain crossover frequencies of each channel. In this work, 1Hz for gain crossover frequency and 0.2Hz for bandwidth are used.

3.2 Verifications for the Basic Control System

In this subsection, the Basic Control System will be designed and verified by vast digital simulations. System parameters used is given in Table 1[20].

Table 1: Parameters of the quadrotor[20].

Parameters	*Value*	*unit*
m	*4.34*	*Kg*
l	*0.315*	*m*
b	1.2953×10^{-5}	
d	*0.008*	
I_{xx}	*0.0820*	$Kg \cdot m^2$
I_{yy}	*0.0845*	$Kg \cdot m^2$
I_{zz}	*0.1377*	$Kg \cdot m^2$

Fig.4 shows performance of command trackings of the controlled system. Fig.5 shows corresponding speeds of the Quadrotor. Fig.4(a) shows that the Quadrotor flies to 100m height by vertical motion mode first. Fig.4(b) shows time responses of the rolling control; Fig.4(c) shows time responses of the pitching control; and Fig.4(d) shows time responses of the yawing control; Fig.4 shows that the controlled system gives good performance and the couplings between attitude and altitude are limited. This is the major merit of the Quadrotor.

Fig.5(a) shows X_B-axis speed(u); Fig.5(b) shows the Y_B-axis speed(v); Fig.5(c) shows the Z_B-axis speed(w); and Fig.5(d) shows the total speed *Vm*. Fig.4(b) and Fig.5(b) give that positive rolling angle increases positive lateral speed v; and negative rolling angle increasing negative lateral speed v. It implies that if there is no negative rolling angle actuation applied; the quadrotor gets positive lateral speed, zero longitudinal speed and zero rolling angle. Therefore, the quadrotor can approach the target using lateral motion by keeping heading. Fig.4(c) and Fig.5(a) show longitudinal motion by changing the pitching angle. Fig.4(d) and Fig.5(d) show speed u and v are not disturbed by changing the yawing angle. It will be shown that the lateral and longitudinal motions on inertial axis are almost not disturbed also by changing the yawing angle. Fig.6 shows the flight locus.

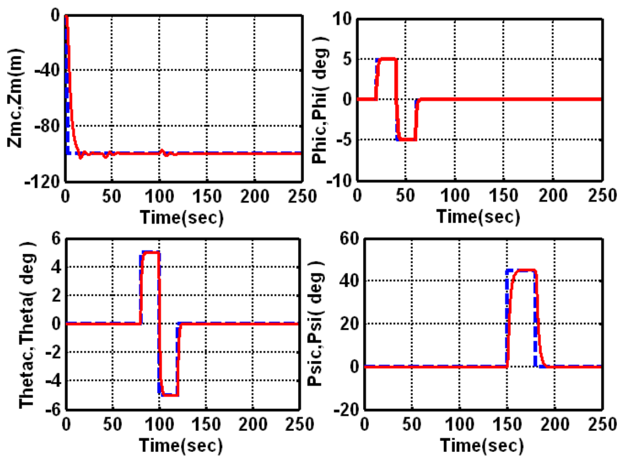

Fig.4. Command tracking performance of the Basic Control System;(a)Heigh control(*Zmc, Zm*);(b)Rolling control(*Phic, Phi*); (c)Pitching control(*Thetac, Theta*); and (d) Yawing control(*Psic,Psi*).

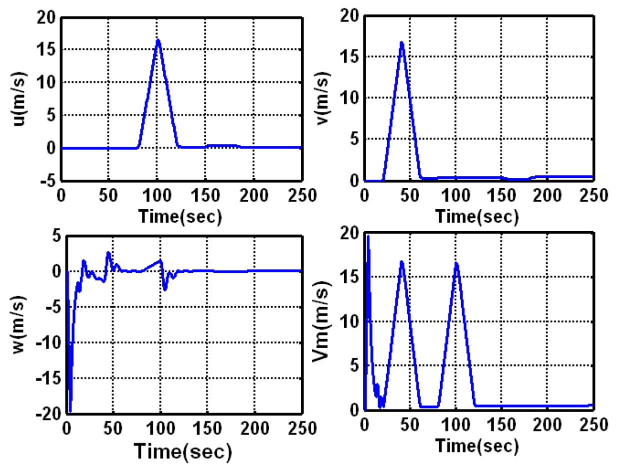

Fig.5. Corresponding vehicle speeds of the Basic Control System;(a)Speed u;(b)speed v; (c)speed w; and (d) speed Vm。

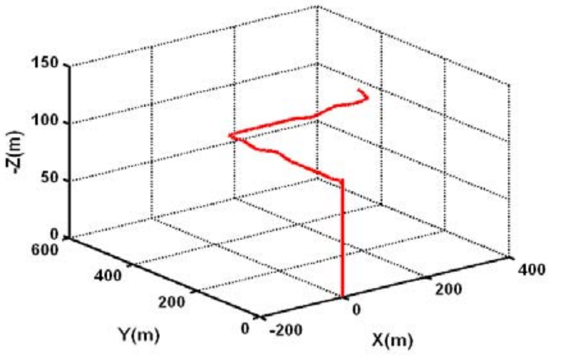

Fig.6. The flight locus

Figs.(7), (8) and (9) show another simulation verifications. The Quadrotor is first flight to 100m hight vertically before 20”, and then forward speed along X_B axis is created by a constant pitching angle (5degrees) between 20” and 60”, and the third changing the yawing angle to 45degrees between 80” and 180”. Fig.8 and Fig.9 show that the moving direction and speed of the vehicle are not disturbed by the changing the heading of the Quadrotor. The behavior is similar to pitch a rotating baseball horizontally. Therefore, one can create constant longitudinal or lateral motion and yawing for omni-directional surveillance. It is not really need the platform for the imaging seeker for large angle surveillance.

Simulating results shown in Figs. 4 to 9 give the Quadrotor can provide new operating behaviors. One can develop new guidance law for special application.

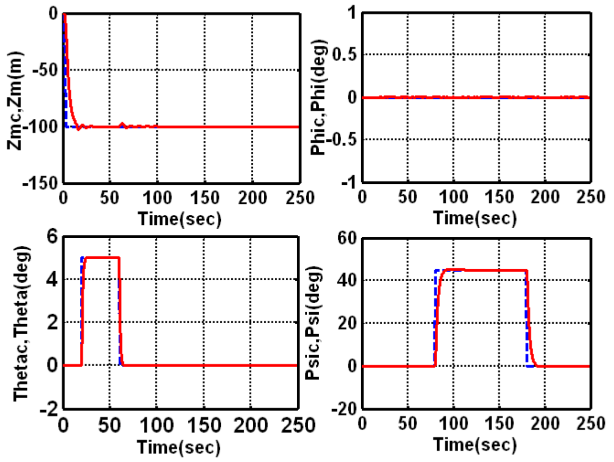

Fig.7 Time responses of Command Tracking.

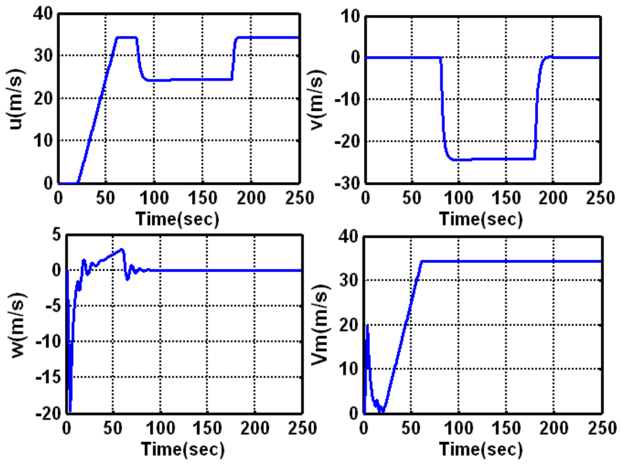

Fig.8 Time responses of Speeds.

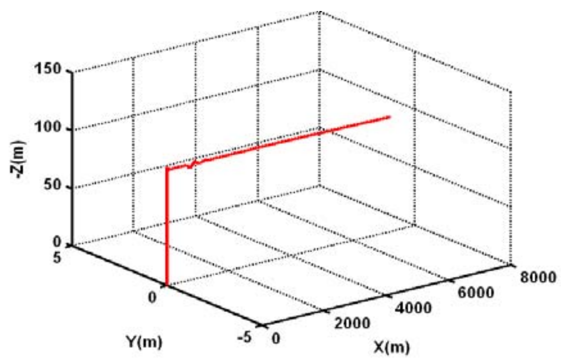

Fig.9. Time responses of flight locus.

3.3 Application of the basic control system

In this subsection, a guidance laws is proposed first for the Quadrotor to simulate the behavior of the conventional fix-fin flight vehicle. It can be done easily. The Conceptual design is give below:

(1)Speed Control for X_B-Axis: it is corresponding to forward propellant controls of the conventional fix-fin flight vehicle. The output control is the pitching angle command Thetac;

(2)Zero Speed Control for Y_B-Axis: it is corresponding to small lateral maneuverability of the conventional fix-fin flight vehicle. The output control is the rolling angle command *Phic*;

(3)Yawing angle control: it is corresponding to the yawing angle control by the rudder of the conventional fix-fin flight vehicle. The output control is yawing angle command *Psic*;

(4)Altitude control: it is corresponding to the height control by elevators of the conventional fix-fin flight vehicle. The output control is the high command Zmc.

Certainly, iterm (2) can be changed to non-zero speed control. It will give extra freedom for control than that of the conventional fix-fin flight vehicle. The speed control laws for X_B and Y_B axes are

$$Thetac = K_{ou}(u_c - u_f) \quad (14)$$

$$Phic = K_{ov}(v_c - v_f) \quad (15)$$

where u_c and v_c are speed commands, u_f and v_f are speed measurements. Naturally, one can get all speed control by disconnecting height control. Then relationship between body and inertia coordinated systems are given as below:

$$\begin{bmatrix} u_c \\ v_c \\ w_c \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix}^{-1} \begin{bmatrix} V_{xc} \\ V_{yc} \\ V_{zc} \end{bmatrix} \quad (16)$$

where (V_{xc}, V_{yc}, V_{zc}) are speeds on the inertial coordinated system; (u_c, v_c, w_c) are speeds on the body coordinated system. All speed control con-figuretion can be used for E-guidance to approach target.

Fig.10, 11 and 12 show simulation verification for simulating the conventional fix-fin UAV. The speed controls u_c=10m/s, v_c=0m/s,*Thetac*=0.0deg, and programmed altitude control. Simulation results shows they are similar to those of Bank-to-Turn (BTT) flight vehicle.

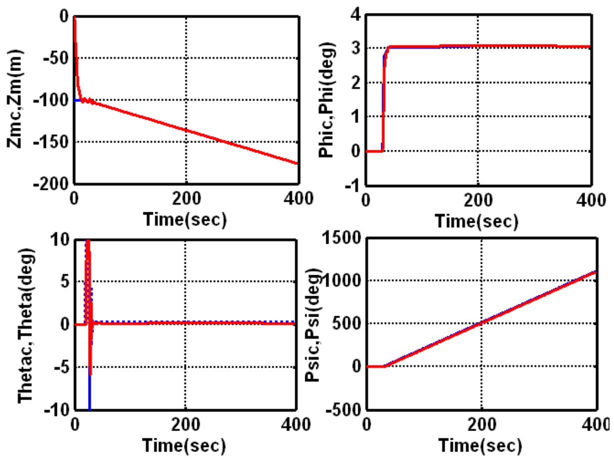

Fig.10. Time responses of Command Tracking.

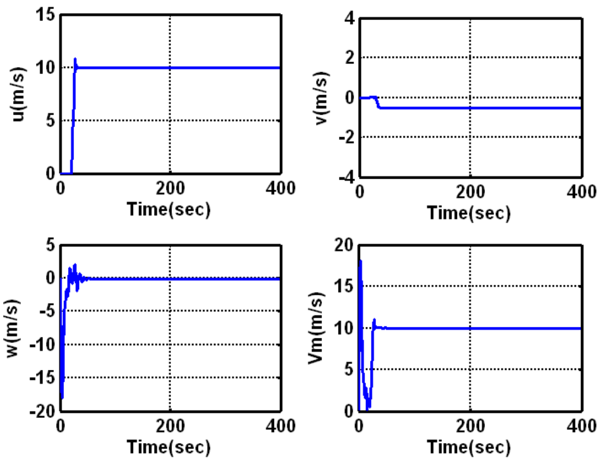

Fig.11. Time responses of Speeds.

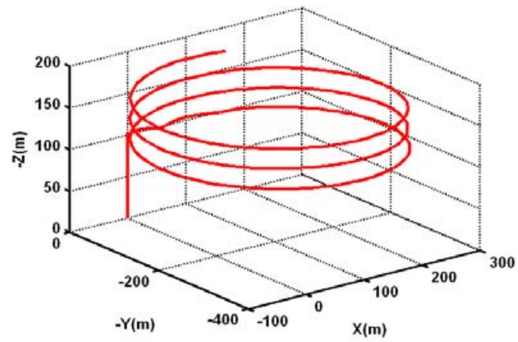

Fig.12. Time responses of flight locus.

3.4 Locus Tracking Laws

In this subsection, the new locus tracking law is developed. Locus tracking law is usually used in fix-fin flight vehicle. The conceptual design of the locus tracking laws is given below:

(1) Speed Control for X_B-Axis: it is corresponding to forward propellant controls of the conventional fix-fin flight vehicle. The output control is the pitching angle command *Thetac*;

(2) Yawing angle tracking the flight path angle(ψ_L): the output control is command *Psic*

(3) Speed Control for Y_B-axis: approaching the reference-tracking locus. The output control is speed command v_c.

The operating concept is shown in Fig.13. *HL* represents the distance between the Quadrotor and the tracking locus. The tracking laws are given below:

$$Psic = a\tan 2(Y_{i+1}\text{-}Y_i, X_{i+1}, X_i) \quad (17)$$

$$v_c = K_v(H_{LC} - H_L) \quad (18)$$

HLc=0 represents the flight locus on the tracking locus; *HLc*=50 represents the flight locus on the right of the tracking line with distance 50m. It can change the tracking locus sequentially for way point navigation(flight).

The conventional locus tracking laws are given below for comparison. They are

$$\Delta Psic = K_{lh}(H_{LC} - H_L) \quad (19)$$

$$Lim(\Delta Psic, \pm 90^\circ) \quad (20)$$

$$Psic = a\tan 2(Y_{i+1}\text{-}Y_i, X_{i+1}, X_i) + \Delta Psic \quad (21)$$

$$v_c = 0.00 \quad (22)$$

The major difference between the tracking law for the Quadrotor and the conventional locus tracking laws[17,18] is the new tracking law described by Eqs.(17) and (18) can keep heading angle along the flight path angle; i.e., lateral approaching the tracking locus without changing heading. It will get better imaging properties. The proposed new tracking law will be verified by digital simulations.

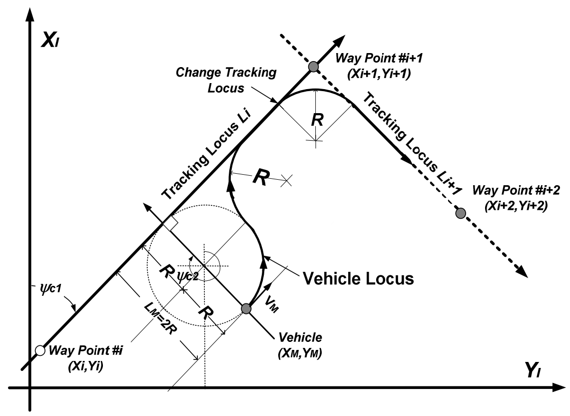

Fig.13 Concepts for locus Tracking.

4. Simulation Verifications

Tracking locus law described by Eqs.(17) and (18), basic control laws described by Eqs.(10)-(13), and speed control laws described by Eqs.(14) and (15) in the above section are used and evaluated for, digital simulation verifications. Way points in simulation are given in Table 2.

Table 2: Way points used for simulation verification

Waypoint	*X(m)*	*Y(m)*	*Z(m)*
1	*0*	*0*	*-50*
2	*500*	*500*	*-50*
3	*1000*	*0*	*-50*
4	*500*	*-500*	*-50*
5	*0*	*0*	*-50*
6	*-500*	*500*	*-50*
7	*-1000*	*0*	*-50*
8	*-500*	*-500*	*-50*
9	*0*	*0*	*-50*

The simulation results for locus tracking are given in Fig.14. Fig.14 shows good performance for locus tracking. The tracking performance of the basic control system is shown in Fig.15. Fig.15(d) shows the controlled system give the yawing angle almost tracks the flight path angle. Fig.16 shows time responses of body-axis speeds and total speed. It shows that speed of the Quadrotor was decreased to very low; i.e., near suspending.

Comparisons with conventional locus tracking law are made and shown in Figs. 17 and 18. Fig.17 shows that the proposed method can track the locus faster. Fig.18 shows that the proposed method can keep the heading angle along the flight path angle.

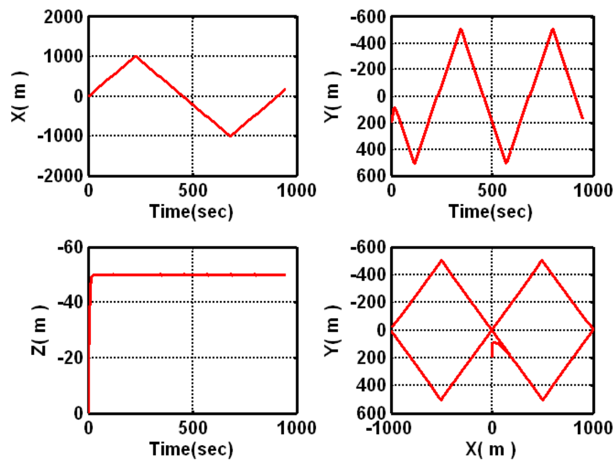

Fig.14. Flight Loci.

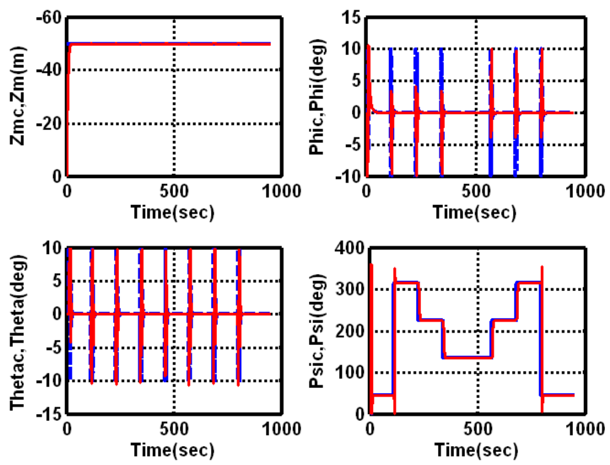

Fig.15. Time responses of basic control system.

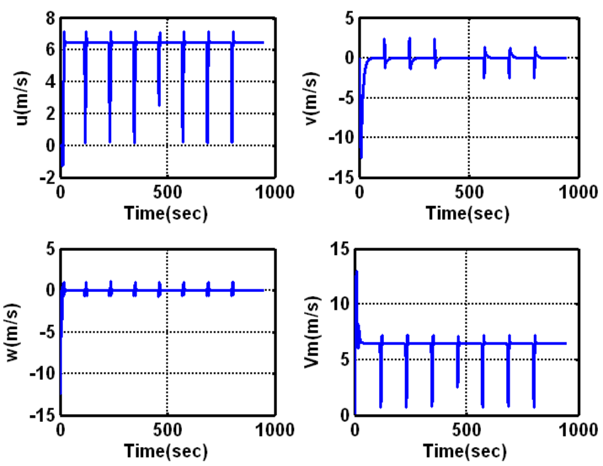

Fig.16. Time responses of Speeds.

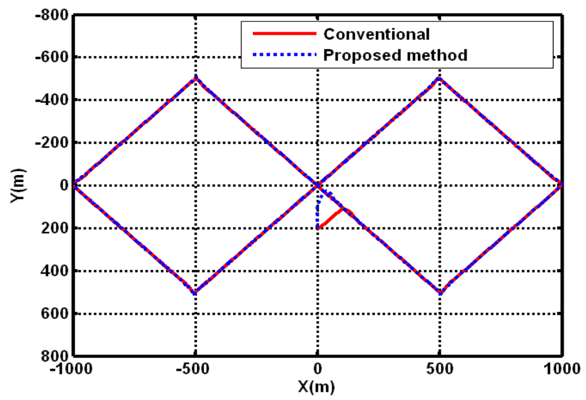

Fig.17. Comparisons of flight loci.

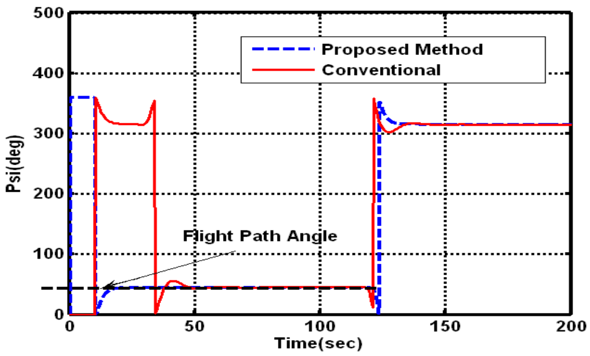

Fig.18. Comparison of yawing angles.

5. Conclusions

In this paper, new guidance and control laws have been proposed for a Quadrotor UAV. They were developed on the basic control system includes height control and attitude controls. The proposed method was verified by digital simulations. They give better performance than that of conventional guidance and control laws.

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