Multi-Agent System Rendezvous via Refined Social System and Individual Roles*

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Abstract: - In order to explore the rendezvous problems of leader-follower multi-agent system further, three different kinds of strategies, namely the democracy strategy, the autarchy strategy as well as the mixed strategy, which were inspired by the mankind system, were put forward based on the heterogeneous peculiarities of the social status and the individual roles of the swarm of the multi-agent system, whereas the last strategy was composed of the democracy one and the autarchy one. It was assumed that only the leader agents were capable of knowing the details of the global task, which was to search for and come to the same targeted object, for the follower agents, if they were kept connected with the corresponding leader ones, they could also be empowered the privilege of the location of the rendezvous task in indirect ways. It was concluded that among this arena, agents were finally divided into four different categories, that were the agent where it represented the targeted object, the leader agents where they played the leading roles and the common follower agents where they had some connections with the leader ones and the isolated follower agents where they were failed in connection with the leader ones. Results demonstrated that, the democracy strategy could fulfill the rendezvous task in an easiest way in expenses with a long convergent time and the autarchy democracy could come to the targeted object quickly with the cost of the risk of failing in completing the rendezvous task of the isolated follower agents, while the mixed strategy cared about both of the convergent speed and the utmost task, would take autarchy strategy when the connections were available for the follower agents to the leader ones, and the democracy rules had to be used when the connection to the leader agents was not possible. Both the models and the topological structures were provided to elucidate the specifications of the three strategies, and moreover, they were explained even further in a set of case studies with simulation work in the leader-follower multi-agent networks, which consisted of 4 leader agents and 5 follower agents.

Key-Words: - Leader-follower multi-agent network; Rendezvous problems; The democracy strategy; The autarchy strategy; The mixed strategy; Refined Social System.

1 Introduction

In recent years, the distributed control of multi-agent network has been paid more attentions from different disciplines. For instance, Rahmani [1] analyzed the issue of how the ratio of leaders to followers affect connectivity. Couzin [2] presented the extended numerical results on this subject. Notarstefano [3] put forward some results regarding maintaining connectivity in networks of homogeneous agents directly through the control law. Olfati-Saber [4] discussed the consensus problems in undirected networks of agents with fixed topology and switching topology, and the consensus problems in directed networks with time delays was also addressed. In Zavlanos [5], the direct control laws were also used in a heterogeneous network. Tove Gustavi [6] provided sufficient conditions for rendezvous in first order leader-follower network；Hu Jiangping [7] studied the distributed tracking control of the leader-follower directed network with noisy measurement. Based on the fixed topology which consisted of the dynamic of linear nodes, the analysis of consensus problem of leader-follower network appeared in Li Zhongkui [8]. Ni Wei [9] investigated the consensus problems in leader-follower network with fixed topology and switching topology. The other researches on consensus issue of leader-follower network include Peng Ke [10], Wang Jinzhi [11] and Sun Yongzheng [12], where Peng Ke studied the situation of variable speed leader and time-varying delays, and Wang Jinzhi analyzed robustness of system, while Sun Yongzheng considered

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the conditions of noise perturbation and time delays. The controllability of leader-follower multi-agent network was discussed in Liu Bo [13]. Fabio Pasqualetti [14] found how to control autonomous motion multi-agent. The second order multi-agent system was also investigated. For instance, the controllability and the robust control were addressed in Darina Goldin [15] and Liu Shicai [16] respectively. Chen Yangyang [17] designed the controller to make the second order multi-agent system move along the given path and further analyzed the stability. Zheng Jun [18] considered the formation control in the second order network and a type of discrete time distributed cooperative control algorithm was also given. Considering the static leaders, Liu Dejin [19] studied the consensus problem of the discrete time second order multi-agent system with communication delay. Mei Jie [20] discussed adaptive coordinated tracking control of Euler-Lagrange system in the directed graph. Song Yunzhong [21] investigated the consensus in the mixed order multi-agent network.

Although the consensus research of multi-agent networks and leader-follower multi-agent networks provided valuable results, there still have been some obvious limitations. For instance, the realization mechanism of the swarm has been considered rarely. But in actual society, the swarm members can interact with each other in various styles, for example, the followers may be kept in the group or be left alone according to the different utmost objectives; and as the same way, the leaders may also play with the followers in the unselfish ways or in the selfish ways, that can be boiled down to the costs and the capabilities of the leaders; to be in detail, we will investigate the three different situations, i.e. (1) When guiding the swarm, the leaders absorb the suggestion of the members and then change the guiding, which is known as the democracy strategy. The autarchy strategy and mixed strategy can be defined in the same way. (2) In the autarchy strategy, leaders only guide the swarm but not absorb the suggestion of the others. (3) In the mixed strategy, leaders sometimes take the democracy strategy, sometimes obey the autarchy strategy. In this paper, the three strategies namely the democracy strategy, the autarchy strategy and the mixed strategy would be used in rendezvous problem of leader-follower network. Specific to leader-follower multi-agent network, the democracy strategy means that the motion of agent is affected by the other ones located within the sensing zone of the agent. And the autarchy strategy is such a strategy, in which the motion of leader is not affected by any agents, while follower only gets the influence of the initial nearest leader within its sensing zone. The mixed strategy is composed of the autarchy one and the democracy one. In the mixed strategy, all the leaders and some followers which are affected by leader initially take the autarchy strategy, while the other followers which are called isolated followers take the democracy strategy.

The paper is organized as follows: in introduction section, we give the research situations and present our work contents and significance. In problem formation section, the descriptions of the related issues as well as the model are addressed, and the relevant conditions are also given. In results and analysis section, the three strategies are discussed in detail, which are also explained with simulation work. In the conclusions section, the results and the research prospect of the paper are summarized.

2 Problem Formulation

Consider N agents evolving in R^2. We use single integrator agents whose motions obey the model:

$$\dot{x}_i = u_i, i \in \mathbb{N} = \{1, \ldots, N\}. \quad (1)$$

Let g={V, E} describe the group topology, that consists of a set of vertices, V={1,...,N}, representing the team members, and a set of edges, $E=\{(i, j)\in V\times V \mid i\in \mathbb{N}\}$ representing the active inter-agent communication links. If g is an undirected graph, there is a line between node i and j, representing the impact between them is mutual. If g is a directed graph, the line (i, j) is directed, which consists of starting point i and finishing point j, representing node i makes an impact on node j.

According to the different functions of the agents in leader-follower network, the agents can be divided into two categories, i.e. leaders and followers. The leaders were capable of knowing the details of the global task that is to come to a given target, while the followers just need to keep in touch with leaders to reach the target. The agents belong either to the subset of leaders, N_l, or to the subset of followers, N_f , where $N_l \cup N_f = N$, $N_l \cap N_f = \phi$ and the number of agents in each set is given by $|N_l| = N_l$ and $|N_f| = N_f$ respectively. Due to limited sensing capability of the sensors, each agent has a limited sensing zone of radius $\Delta > 0$. At any given time, the set of agents located within the sensing zone of agent $i \in N$ are referred to as the neighbours of agent

i, $N_i=\{j\in N: |x_i - x_j|\leq\Delta\}$. Each agent has knowledge of the relative coordinates to its neighbouring agents, but cannot detect or communicate with agents outside its sensing zone. Note that both g and N_i are time-varying. The difference between leaders and followers is that leaders are aware of the location of the target and can control group but the followers cannot.

We start by introducing a notation for the distance between two arbitrary agents i and j. Let

$$\delta_{ij} = \delta_{ji} = \left|x_i - x_j\right| = \sqrt{\left(x_i - x_j\right)^T \left(x_i - x_j\right)} \geq 0$$

Since we are considering a physical system we can assume that $x_k(t)$ is a continuous function for any agent $k\in N$. The time derivative $\dot{\delta}_{ij}$ is not directly defined when $\delta_{ij}=0$ so here we shall instead consider the time derivative of δ_{ij}^2

$$\frac{d\delta_{ij}^2}{dt} = 2\delta_{ij}\dot{\delta}_{ij} = 2\left(x_i - x_j\right)^{\mathrm{T}}\left(\dot{x}_i - \dot{x}_j\right) \quad (2)$$

To describe the relationship between leaders and target, the target attraction function is defined as

$$F\left(x_j, d\right) = \begin{cases} f\left(\delta_j\right)\dfrac{d - x_j}{\delta_j}, & \delta_j > 0 \\ 0, & \delta_j = 0. \end{cases} \quad (3)$$

where d is the location of the target and define $\delta_j=|x_j - d|$ representing the agent j's distance to the target. At any given position $x_j\neq d$, the direction of $F(x_j, d)$ is towards the target and the magnitude is decided by the continuous scalar function $f(\delta_j)\geq 0$.

In the following analysis, the leader-follower multi-agent network that consists of four leaders and five followers is illustrated. The agents are tagged with numbers, starting at 1 first, followed with 2, 3 and the other numbers, where agent 1 to 5 belong to followers，agent 6 to 9 belong to leaders and node 10 is the target. Without loss of generality, we assume that the target is the origin of coordinates and $f(\delta_j)= \beta\delta_j$, $\beta=0.5$. The initial configuration of each agent can be seen in Tab.1.

TABLE I INITIAL VALUES OF THE SIMULATION WORK

	Followers					Leaders				Target
	1	2	3	4	5	6	7	8	9	10
x	27.5	28	28	28	32	17	21.5	22	21	0
y	0	-0.2	-4	4	-1	-2	-1	0.5	1	0

Remark：agent 1 to 5 belong to followers，agent 6 to 9 belong to leaders，node 10 is the target，x and y indicate abscissa and ordinate respectively.

3 Results and Analysis

3.1 The democracy strategy

In the democracy strategy, the motion of agent is affected by the other ones located within the sensing zone of the agent. If agent i has an impact on j, agent i is also affected by j. In other words, interactions among each agent are undirected. The topology of group is thus described by undirected graph $g_1=\{V_1, E_1\}$.

The dynamics for an arbitrary follower agent $i\in N_f$ are given by

$$\dot{x}_i = -\sum_{k\in N_i}\left(x_i - x_k\right) = -N_i x_i + \sum_{k\in N_i} x_k \quad (4)$$

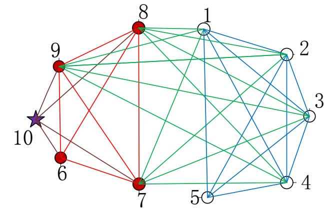
We can conclude that the motions of followers are decided by their neighbours. Considering the situation of two dimensions, the dynamics can be looked as the resultant vectors of themselves and their neighbours, where the directions of all the agents are the same corresponding to the resultant vectors, and the speeds of the swarm motion members can also be decided by the norm of the resultant vectors.

For an arbitrary leader agent $j\in N_l$, the dynamics are described by

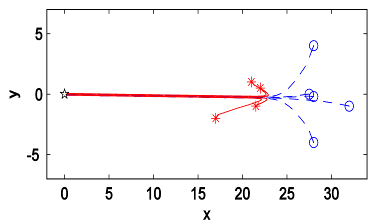
$$\dot{x}_j = -N_j x_j + \sum_{k\in N_j} x_k + F\left(x_j, d\right) \quad (5)$$

By contrast, the dynamics for leaders are the same as the followers in some way, except for the impact from the target, which is the leaders' feature.

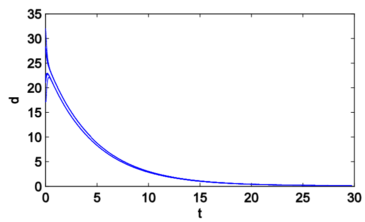
The simulation results in the democracy strategy can be seen in Fig.1.



(a) Topological structure



(b) Position rendezvous



(c) Rendezvous time

Fig. 1 Simulation results in the democracy strategy.

In the process of simulation, the distances between the agents from the leader group and the follower group and the target agent are used to measure the rendezvous time indirectly, where all the distances are turned into be zero is regarded as the network rendezvous.

3.2 The autarchy strategy

In the autarchy strategy, the motion of each of the leaders is not affected by the other ones, while the follower only gets the inormation from the initial nearest leader within its sensing zone. The impacts among the agents are directed. Therefore, the directed graph $g_2=\{V_2, E_2\}$ is used to describe the topology of group. For follower i, define x_{im} as the initial nearest leader within i's neighbors.

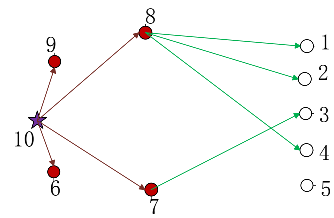
For an arbitrary follower agent $i\in N_f$, we have

$$\dot{x}_i = -\left(x_i - x_{im}\right) \quad (6)$$

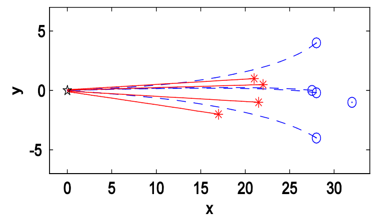
For an arbitrary leader agent $j\in N_l$, its dynamics is governed by

$$\dot{x}_j = F\left(x_j, d\right) \quad (7)$$

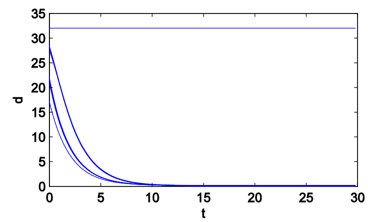
The simulation results of the autarchy strategy are shown in Fig.2.



(a) Topological structure



(b) Position rendezvous



(c) Rendezvous time

Fig. 2 Simulation results in the autarchy strategy.

Apparently, the isolated follower 5, which is alone at first, will stand alone forever, i.e. it will fail to come to the target. In other words, the autarchy strategy can't realize the utmost task. But this strategy can come to the target more quickly than the democracy one.

3.3 The mixed strategy

Based on the autarchy strategy, the mixed strategy is presented to prevent the isolated followers from not coming to the target. In the mixed strategy, all the leaders and some followers which are kept connected with the corresponding leader ones take the autarchy strategy. Therefore the mixed strategy is the mixture of the democracy one and the autarchy one. Let $g_3=\{V_3, E_3\}$ describes the topology in the mixed strategy. Due to the particularity of the location, the followers are divided into two cases, i.e. the common followers, N_{cf} and the isolated followers, N_{sf}, where $N_{cf}\cup N_{sf} = N_f$, $N_{cf}\cap N_{sf} = \phi$. In order to describe this scenario we introduce some additional notation. Let N_{fi} and N_{si} be the subsets of agent i's neighbours that belong to the group of followers and the group of isolated followers respectively, $|N_{fi}|= N_{fi}$, $|N_{si}|= N_{si}$.

For an arbitrary common follower agent $i\in N_{cf}$, we have

$$\begin{aligned}\dot{x}_i &= -\left(x_i - x_{im}\right) - \sum_{k\in N_{si}} \left(x_i - x_k\right) \\ &= -\left(N_{si}+1\right)x_i + x_{im} + \sum_{k\in N_{si}} x_k\end{aligned} \qquad (8)$$

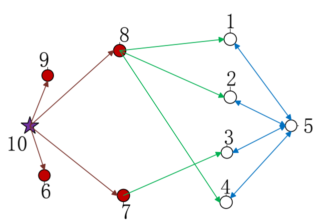
From the equation above it is easy to see that there are two kinds of impacts on the common followers, which come from both the initial nearest leaders and the isolated followers of the neighbours. While an arbitrary isolated follower is just affected by the followers of its neighbours, the dynamics for an arbitrary isolated follower agent $i\in N_{sf}$ are given by

$$\dot{x}_i = -\sum_{k\in N_{fi}} \left(x_i - x_k\right) = -N_{fi}x_i + \sum_{k\in N_{fi}} x_k \qquad (9)$$

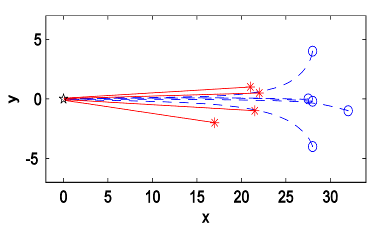
For an arbitrary leader agent $j\in N_l$, the dynamics are considered as

$$\dot{x}_j = F\left(x_j, d\right) \qquad (10)$$

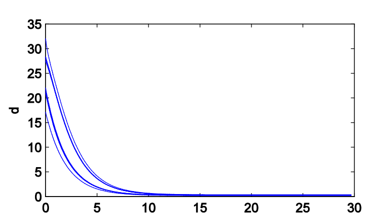
The simulation results in the mixed strategy can be seen in Fig.3.



(a) Topological structure



(b) Position rendezvous



(c) Rendezvous time

Fig. 3 Simulation results in the mixed strategy.

3.4 The analysis of the strategies

From the definitions and the topological structures, it is found that all the agents in the democracy strategy are equal, no level constraints and interactive, and the leaders are just the pointers. In the autarchy strategy, the leaders, which are not affected by the other ones, play leading roles absolutely, whereas the followers, which can only follow the leaders passively, play subordinate roles. The mixed strategy is based on the autarchy strategy and partly obeys the democracy strategy, where if considering the whole system, there are some level constraints among the agents, i.e. the leaders playing leading roles absolutely, while the followers playing subordinate roles. By contrast, both the isolated followers and other followers are equal on the partial view.

From the simulations, we can derive that the democracy strategy is so sequential that the agents first converge together and then rendezvous. The autarchy strategy is much freer than the democracy strategy, where the agents don't need to be rearranged before rendezvous, but just to come to the target. The mixed strategy is in between. On the view of rendezvous time, it is easy to find that the democracy strategy can care about the whole system to ensure all the agents to reach the target but in expenses with a long convergent time, moreover, the autarchy strategy can come to the target quickly with the cost of the risk of failing in completing the rendezvous task of the isolated follower agents, while the mixed strategy, which can care about both of the convergent speed and the utmost task, can reach the target more quickly than the democracy one but slowly than the autarchy one.

All of the above demonstrates that the democracy strategy is equal, sequential, slow and to be holy priority, and the autarchy strategy is unequal, free, quick, and to be prior in time savings, while the mixed strategy is composed by the democracy one and the autarchy one, which can save the rendezvous time in some scale and can avoid fail coming to the target of the initial isolated agent. The mixed strategy is much more perfect than the other ones if we pay the same attention to the time cost and rendezvous target.

4 Conclusions

In this paper, we have examined and discussed the convergent characteristics of the three strategies, namely the democracy strategy, the autarchy strategy as well as the mixed strategy. The simulation of leader-follower multi-agent network which consisted of four leaders and five followers demonstrated correctness of the analysis.

However, our study was in the ideal condition. For example, influences from the surroundings are omitted, sometimes it is not permitted. Besides that, vision constraints are not considered, we assume that the agents can draw information from each of the direction, and that is impossible in some time, for instance, when a person observes the surroundings in the real situation, there must be some visual angle which is not a circle. So, the further works will be needed to improve the already results in the near coming future.

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