Fault detection and isolation of fuzzy system with uncertain parameters using the bounded approach

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Abstract -- This paper deals with the problem of state estimation and fault detection and localization of nonlinear systems with uncertain parameters. The multiple model approach is used to model the nonlinear system. This approach is based on the use of fuzzy systems principle. Uncertain parameters are modeled using the bounded approach. The technique proposed in this work is used for linear case but never in the case of nonlinear systems, which presents the main contribution in this paper. The principle of the proposed method is to consider that the system parameters are uncertain and the distribution value is unknown at the time of the court. Only the extreme bounds are known in advance. Using this method it is possible to generate fault indicators (named also residuals). These residuals are not sensitive to the parameter uncertainties and allow detecting and locating the faults affecting the system. A numerical example is showing the effectiveness of the presented method.

Keywords: state estimation; multiple model; fault detection; bounded approach; uncertain parameters; bounded error; interval analysis

1. Introduction

Most of the techniques used for diagnosis are based on the development of a mathematical model. The problem that may arise is that the developed model from the system redundancy may not reflect the physical system real behavior in view of the modeling uncertainties which may affect the system.

Few studies have been developed in this context, however, can include the works of Adrot [1], Letellier [2, 3] and Bedoui [4, 5] who are interested mainly in the regular linear systems, which is the least real case.

In general case, physical processes are so complex, and it is impossible to model those using linear models. Linearity is a very conservative hypothesis. This work is interested in nonlinear systems. To model nonlinear systems, the multiple model approach is used. This approach is based on the fuzzy systems principle. Using this approach it is possible to model any class of nonlinear systems. It is a question of general kind of modeling. By this virtue, Fuzzy systems are the subject of many works [6, 7]. They can reproduce exactly a nonlinear model behavior model [6-8]. They are constructed by a set of linear models blended together with nonlinear functions holding the convex-sum property.

Approaches using fuzzy models, named also multiple models [9] are the object of many works in different contexts including the taking into account of unknown inputs or parameter uncertainties [6, 7, 29, 31].

The modeling approaches of model uncertainties have been developed using stochastic variables [10] or using the bounded approach [11, 12]. These approaches assume that only the extreme limits of the uncertainties are known.

The methods of fault detection and localization or of state estimation based on mathematical model suppose that the mathematical model of the system is known. This hypothesis leads to start any diagnosis by a step of modeling. In often cases, the obtained model in uncertain due to many problems. It is necessary in these cases to make a diagnosis or a fault

detection and localization independently of the parameter uncertainties. Usually, these uncertainties are unknown. In these work it is supposed that only, the extreme boundaries of these uncertainties are known. The proposed method is based on the interval analysis.
The main contribution in this paper is to present a technique able to generate residuals, in the case of nonlinear systems, in spite of the model uncertainties. These residuals are used to detect and localize the faults affecting the system. The advantages of this method is that it allows detecting and localizing faults even in the presence of the model uncertainties and that it can be applied for all the nonlinear systems because the principle of modeling is general and can be used for all the nonlinear systems classes.
The paper is organized as follow: after a small introduction, sections 2 and 3 recalls the bounded approach principle and the multiple models approach respectively. Section 4 presents the problem formulation and the technique of residual generation and the method that allows the fault detection and localization. A numerical example illustrating the effectiveness of the presented method is the subject of the section 5. The paper is finished by a conclusion.

2. Bounded approach

The bounded approach or interval analysis is a technique that has been used for 50 years in the Information technology (IT) sector [13, 14] whose purpose is to represent numerical errors in computer systems. For example, IT cannot represent the solution of the quotient 1/3 of an accurate, because the mathematical solution of this operation is infinite 0.33…33<1/3<0.33...34, where bounded approach is developed to solve this problem by representing the exact value of the quotient between two bounds. Basing on this approach, it can write that $1/3 \in [0{,}333 \;\; 0{,}334]$. This approach is used in various fields of research.
Indeed, in industry, no material can be modeled perfectly. The manufacturers require a tolerance interval on the values of the electrical components (resistors, inductors, capacitors, etc.) Many of these components are sensitive to external factors (temperature, trigger, etc.), so it is required to model their variation using this approach. Which make the bounded approach a rigorous tool to model these bound variations.
The interval arithmetic seeks to ensure the calculation results by finding the interval containing the real result. The solution will be in the form of an interval with sufficient accuracy. This technique is proved in automatic in various works such as those of [15] in control, [16] in state estimation, [17] in parametric estimation, [18, 19] for the fault detection, and [20] for the fault localization.
The state space representation of a LTI discrete-time system can be described as follows:

$$\begin{cases} x(k+1) = A(\eta_A)x(k) + B(\eta_B)u(k) + v(k) \\ y(k) = C(\eta_C)x(k) + w(k) \end{cases} \quad (1)$$

Where $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^q$ its command, $y \in \mathbb{R}^m$ is the measured output. $A(\eta_A), B(\eta_B)$ *and* $C(\eta_C)$ are respectively the state matrix, control matrix and measures matrix with η is the uncertain parameters vector. *n*, *q* and *m* are respectively the size of the vectors *x*, *u* and *y*.

Definition [3]
The solution domain of a linear dynamic system, on a time interval [*0* n] is:

$$S(0,n) = \{x(k,\eta,u): \;\; k \in [0,n], \eta \in \theta\} \quad (2)$$

Where x(k,η,u) is the solution of the system represented by the equation (1) at time k for an uncertain parameter vector $\eta \in \theta$.
The solution domain for a time interval [0 k] is represented as follow

$$S(k) = \{x(k,\eta,u): \eta \in \theta\} \quad (3)$$

For this, it will be assumed that the system is stable. This assumption is necessary in order to limit S(k) at each time k.
In practice, to introduce the uncertain model concept, a simple example known by all is presented: Ohm's law.

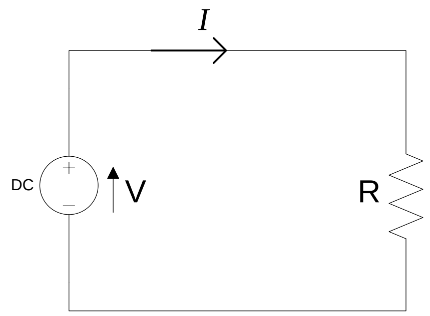

Fig. 1: R, I and V, the parameters of Ohm's law

By definition, Ohm's law provides that the current *I* through a resistor with nominal value R_0 and the voltage *V*, are proportional according to the equation $V = R_0 I$.
Assuming that the instrumentation system is perfect, then the real values *V* and *I* of the current and the

voltage, called true values, are measured without error. Denoting *Vm* and *Im* the respective measurements of *I* and *V*, the above proportional relationship becomes: $Vm = R_0 Im$. In this case, the model is deterministic in measurement.
Practically, it is impossible to find identical and perfect electronic components. For this the manufacturers always indicate a tolerance interval on the values of components quantities. In our case, the resistance value R is given with some technological precision Δ (Δ = 5%, 10%, …), where the value of R can be written in the interval form containing the nominal value R_0 $[R_0(1-\Delta),\ R_0(1+\Delta)]$. Furthermore, the value of R depends on the electronic component temperature. This may be taken into account by changing the model by adding an additional relationship reflecting the resistor resistivity.
In conclusion, due to various disturbances which can affect the circuit, the resistance has a low chance to have its nominal value R_0, by cons it certainly included in the interval $[R_0(1-\Delta), R_0(1+\Delta)]$.
It becomes clear that uncertain model allows us to grasp the disturbances affecting the system, unlike the deterministic model
$V = R.I = (1+\tau)R_0.I$ with $\tau \in [-\Delta, +\Delta]$
To better describe the problem, it should be noted that even the instrumentation chain is not perfect, whence measurement errors that will appear.
Indeed, as the resistance, the sensors also have a limited technological precision, resulting additive and multiplicative errors in addition to measured values *Im* and *Vm*
No one model perfectly reflects the exact behavior of a system under all constraints, conclusion validated by the previous example where it was shown at its modeling that several models may arise, everything depends on the assumptions introduced.
The proper solution to overcome this problem modeling is to introduce all the uncertainties in the model.

3. multiple model approach

The aim of the multiple model approach idea is to apprehend the global nonlinear system behavior by local models set. Each local model (named also sub-model) can be a linear time-invariant (LTI) system valid around an operating point. The local models are aggregated using an interpolation function. The total nonlinear system behavior is the sum of the local models balanced by weighting functions associated to each of them. These weighting functions are used to gradually quantify the membership of the system current operation point at a zone of operation. Such models can approximate a wide class of nonlinear systems [7]. They can even describe exactly some nonlinear systems[21]. Each nonlinear dynamic system can be simply, described by a fuzzy model.
The multiple model approach simplifies the study of nonlinear systems. It is able to reproduce the behavior of complex system with an exact manner [22].
A fuzzy model is the fuzzy fusion of many linear models. Each of these linear models represents the local system behavior around an operating point. A multimodel is described by fuzzy If-Then rules which represent local linear Input/output relations of the nonlinear system. It has a rule base of *M* rules, each having p antecedents, where i^{th} rule is expressed as :
R^i: IF ξ_1 is F_i^1 and ... and ξ_p is F_i^p

$$\text{Then : } \begin{cases} \dot{x}(t) = A_i x_i(t) + B_i u(t) \\ y(t) = C_i x_i(t) \end{cases} \tag{4}$$

In which i = 1, ...,M, F_i^j (j = 1, ..., p) are fuzzy sets and $\xi = \xi_1, \xi_2, \dots, \xi_p$ is a known vector of premise variables which may be the state, the input or the output.
In this modeling approach, two main structures can be considered according to nature of the coupling between local models.
In the first one, the sub-models have the same state-space and consequently the multiple models is composed of homogeneous sub-models [23].
In the second structure, decoupled multiple structure, the sub-models do not share the same state-space and the multiple model uses heterogeneous sub-models.

3.1. Coupled structure

A multiple model can be written in the coupled form:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x_i(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) C_i x_i(t) \end{cases} \tag{5}$$

Where x_i are the state vectors of the local models, y is the system output vectors and u is the system input. A_i are the state evolution matrices, they represent its dynamic behavior, B_i are the matrices of control and C_i are the matrices of observation. The matrices A_i, B_i, and C_i are matrices with known coefficients and appropriate dimensions.

$\mu_i(\xi(t)),\quad i\in\{1,\dots,M\}$ are the activation functions (named also weighting functions) and $\xi(t)$ is the decision variable vector.
The weighting functions satisfies the sum convex property expressed in the following equations:

$$0\le \mu_i(\xi(t))\le 1 \quad and \quad \sum_{i=1}^{M}\mu_i(\xi(t))=1 \tag{6}$$

If the measurement system is considered linear $C_1 = C_2 = \dots = C_M = C$, the system (5) is rewritten as follows :

$$\begin{cases}\dot{x}(t)=\sum_{i=1}^{M}\mu_i(\xi(t))(A_i x_i(t)+B_i u(t))\\ y(t)=Cx(t)\end{cases} \tag{7}$$

3.2. Decoupled structure

Similarly, a complex system can be represented by a multiple model with decoupled structure.

$$\begin{cases}\dot{x}_i(t)=\ (A_i)x_i(t)+\ (B_i)\ u(t)\\ \qquad y_i(t)=\ (C_i)x_i(t)\\ y\ (t)=\ \sum_{i=1}^{M}\ \mu_i\ (\xi(t)\ )\ \ y_i(t)\end{cases} \tag{8}$$
$$i\in\{1,\dots,M\}$$

where x_i are the state vectors of the local models, y_i are the output vectors of the local models, y is the system output vector and u is the system input. A_i are the state evolution matrices, they represent its dynamic behavior, B_i are the matrices of control and C_i are the matrices of observation. The matrices A_i, B_i, and C_i are matrices with known coefficients and appropriate dimensions.
The same representation (8) can be written in the discreet case :

$$\begin{cases}x_i(k+1)=\ (A_i)x_i(k)+\ (B_i)\ u(k)\\ \qquad y_i(k)=\ (C_i)x_i(k)\\ y\ (k)=\ \sum_{i=1}^{M}\ \mu_i\ (\xi(k)\ )\ \ y_i(k)\end{cases} \tag{9}$$
$$i\in\{1,\dots,M\}$$

where x_i, y_i, y, A_i , B_i C_i are defined as above.

4. Problem formulation

In the rest of this paper, the problem of inaccuracy modeling of a nonlinear system will be solved. The technological equipment imprecision brings us to consider uncertainty on the matrix A.
Let consider the fuzzy system described by a decoupled multiple model structure based on measurable decision variables $\xi(k)=u(k)$. The considered structure presents an uncertainty on the matrix A. This structure is given by the following equations :

$$x_i(k+1)=A_i(\eta_{A_i}(k))x_i(k)+B_i u(k) \tag{10}$$

$$y_i(k)=C_i x_i(k) \tag{11}$$

$$y_m(k)=\sum_{i=1}^{M}\mu_i(u(k))y_i(k) \tag{12}$$

With $x_i\in\mathbb{R}^n, u\in\mathbb{R}^q\ et\ y_i\in\mathbb{R}^m$ are respectively the state vector, the input and output of the i^{th} sub-model. $\mu_i(u(k))\in[0,1]$ represent the activation functions depending on the input. $A_i(\eta_{A_i}(k))\in\mathbb{R}^{n\times n}$ is the uncertain matrix of the i^{th} sub-model defined by:

$$\begin{gathered}A_i(\eta_{A_i}(k))=A_i^0\pm\Delta_{A_i}\otimes\eta_{A_i}\\ A_i^0\in\mathbb{R}^{n\times n},\Delta_{A_i}\in\mathbb{R}^{n\times n},\eta_{A_i}\in\mathbb{R}^{n\times n}\end{gathered} \tag{13}$$

Where $\otimes$ is the operator of multiplication of two matrices element by element. A_i^0 is the nominal matrix of $A_i(\eta_{A_i}(k))$. Δ_{A_i} is the matrix of amplitude of uncertainties in the elements of the matrix $A_i(\eta_{A_i}(k))$. η_{A_i} is the matrix consisting of bounded and standard variables of i^{th} sub-model. $i\in 1,\dots,M$

4.1. State estimation

From equation (11), an estimated can be provided:

$$x_i^{(1)}(k)=C_i^{-1}y_i(k) \tag{14}$$

Consider that the state variables are estimated from a range of measures initially valid. Similarly, all the C_i matrices are considered observable and square.
By returning to the equation (10) and replacing $[x_i(k)]$ by their estimated calculated in (14) $x_i^{(1)}(k)$, it is possible to estimate the state variables at time $k+1$

$$x_i^{(2)}(k+1)=A_i(\eta_{A_i}(k))x_i^{(1)}(k)+B_i u(k) \tag{15}$$

The uncertain matrix $A_i(\eta_{A_i}(k))$ can be represented by their extreme limits, where it has the following form:

$$A_i(\eta_{A_i}(k))\in[A_i]=\left[\underline{A}_i,\overline{A}_i\right],\quad \left|\eta_{A_i}(k)\right|\le 1 \tag{16}$$

Hence the expression (14) and (15) become :

$$[x_i^{(2)}(k+1)]=[A_i][x_i^{(1)}(k)]+B_i u(k) \tag{17}$$

$$[x_i^{(1)}(k)]=C_i^{-1}y_i(k) \tag{18}$$

The estimated state variable at time $(k+1)$ is calculated twice. The first $[x_i^{(1)}(k+1)]$ is calculated from measurements and the second $[x_i^{(2)}(k+1)]$ is calculated from the system model. The final estimates $[x_i(k+1)]$ are obtained by taking the intersection between the two estimated $[x_i^{(1)}(k+1)]$ and $[x_i^{(2)}(k+1)]$.

$$[x_i(k+1)] = [x_i^{(2)}(k+1)] \cap [x_i^{(1)}(k+1)] \quad i = \{1 \dots M\} \tag{19}$$

4.2. Fault detection and location

An uncertain fuzzy system is considered faulty if a fault can be detected on one of the sub-models.

For fault detection and localization, locale residuals must be generated for each of the sub models. For this, the local coherency between estimated $[x_i^{(2)}(k+1)]$ and $[x_i^{(1)}(k+1)]$ at every moment for all $i \in \{1 \dots M\}$ will be tested.

By considering that the measures aren't affected by fault at the time *k*, it is possible to deduce that the estimated obtained using the system model and the model of measures at the same time $[x_i^{(2)}(k+1)]$ are coherent with the system model. Then, this estimated can be considered as a reference to compare it with the estimated obtained from the measures at the time *k+1* $[x_i^{(1)}(k+1)]$

From the foregoing, it is possible to judge the state of the local estimated, in the case of the safe functioning and faulty functioning

For the safe functioning :

$$[x_i(k+1)] = [x_i^{(2)}(k+1)] \cap [x_i^{(1)}(k+1)] \tag{20}$$

For the faulty functioning :

$$[x_i(k+1)] = [x_i^{(2)}(k+1)] \tag{21}$$

By referring to $[x_i^{(2)}(k+1)]$, the interval estimate $[\hat{x}_i(k+1)]$ calculated from the equation (19) is analyzed. Two cases are possible:

- If $[x_i(k+1)] = [x_i^{(2)}(k+1)] \cap [x_i^{(1)}(k+1)] \neq \varnothing$ it is possible to confirm that the interval estimates $[x_i^{(2)}(k+1)]$ and $[x_i^{(1)}(k+1)]$ are coherent, which means that the sub-model measures, at the same time *k +1*, are safe.
- If $[x_i(k+1)] = [x_i^{(2)}(k+1)] \cap [x_i^{(1)}(k+1)] = \varnothing$; the two interval estimates $[x_i^{(2)}(k+1)] et [x_i^{(1)}(k+1)]$ are not coherent where the measures of the hole system at the same time *k +1*, contain at minus one fault. The i^{th} sub-model is then used to locate the fault.

In this case, the safe states variables values are considered belonging only at the reference $[x_i^{(2)}(k+1)]$, and it is not possible to consider the estimated interval, obtained from the system model and measurements at time *k +2* $[x_i^{(2)}(k+2)]$, as a reference to compare with $[x_i^{(1)}(k+2)]$. Fault must be corrected first.

For this, and by referring to the system model (10), the real state variables are replaced by the state variables estimated considered a reference at the same time $[x_i^{(2)}(k+1)]$ hence:

$$[x_i^{(2)}(k+2)] = A_i[x_i^{(2)}(k+1)] + B_i u(k+1) \tag{22}$$

To identify the measures affected by a fault, intervals residuals $[r_i(k+1)]$ are generated for each sub-model. From the equation (11), the state variables values at (*k*+1) is replace by their estimated considered as a reference at same time $[x_i^{(2)}(k+1)]$ whence:

$$[r_i(k+1)] = C_i[x_i^{(2)}(k+1)] - y_i(k+1) \tag{23}$$

A measurement is affected if the correspondent residual is abnormal i.e. the zero don't belong to the correspondent interval residuals.

5. Results

Consider an numerical example for a fuzzy system composed of three sub-models:

$$\underline{A}_1 = \begin{bmatrix} 0.450 & -0.20 & 0.182 \\ -0.087 & 0.660 & -0.08 \\ 0.120 & 0.109 & 0.350 \end{bmatrix}$$

$$\overline{A}_1 = \begin{bmatrix} 0.500 & -0.29 & 0.218 \\ -0.073 & 0.660 & -0.08 \\ 0.120 & 0.131 & 0.390 \end{bmatrix}$$

$$\underline{A}_2 = \begin{bmatrix} -0.80 & 0.260 & 0.180 \\ 0.250 & 0.660 & -0.18 \\ 0.120 & -0.109 & 0.350 \end{bmatrix}$$

$$\overline{A}_2 = \begin{bmatrix} -0.50 & 0.290 & 0.216 \\ 0.325 & 0.670 & -0.08 \\ 0.120 & -0.131 & 0.390 \end{bmatrix}$$

$$\underline{A}_3 = \begin{bmatrix} 0.600 & -0.39 & -0.282 \\ -0.067 & 0.313 & -0.180 \\ 0.320 & 0.309 & 0.700 \end{bmatrix}$$

$$\overline{A}_3 = \begin{bmatrix} 0.800 & -0.39 & -0.118 \\ -0.033 & 0.330 & -0.180 \\ 0.320 & 0.331 & 0.700 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1.00 \\ -0.80 \\ 0.91 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0.800 \\ -0.50 \\ 1.419 \end{bmatrix} \quad B_3 = \begin{bmatrix} 0.500 \\ -0.90 \\ 1.110 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -5.25 & -1.635 & 1.120 \\ 0.920 & 0.200 & -5.00 \\ -2.00 & -1.30 & 1.495 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} -4.50 & -1.635 & 1.12 \\ 1.90 & 0.150 & -5.0 \\ -2.00 & -1.30 & 1.81 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} -3.25 & 1.350 & 2.000 \\ 1.920 & 0.550 & -1.00 \\ -3.80 & -0.60 & 0.750 \end{bmatrix}$$

Consider on Figure 2 the control signal with uniform distribution between -1 and 2.

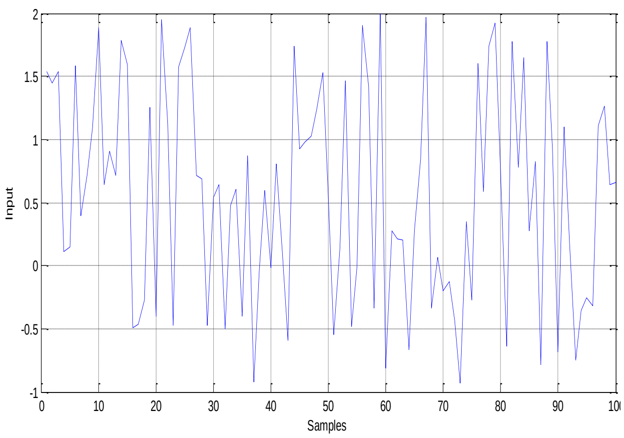

Fig. 2: The control *U(k)* applied to the system.

The transition between the sub-models is done through weighting functions that need to be normalized to meet the requirement of convexity. Figure 3 illustrates these functions $\mu(.)$ depending on the control U(k).
The system has three outputs. Three faults are injected respectively at times [20, 30], [50, 60] and [70, 80] on measurements $y_1(k)$, $y_2(k)$ et $y_3(k)$. Figure 4 shows the different affected measurements with their corrections.

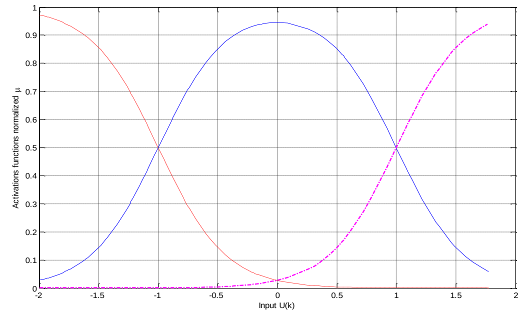

Fig. 3: Normalized weighting functions depending on the control U(k).

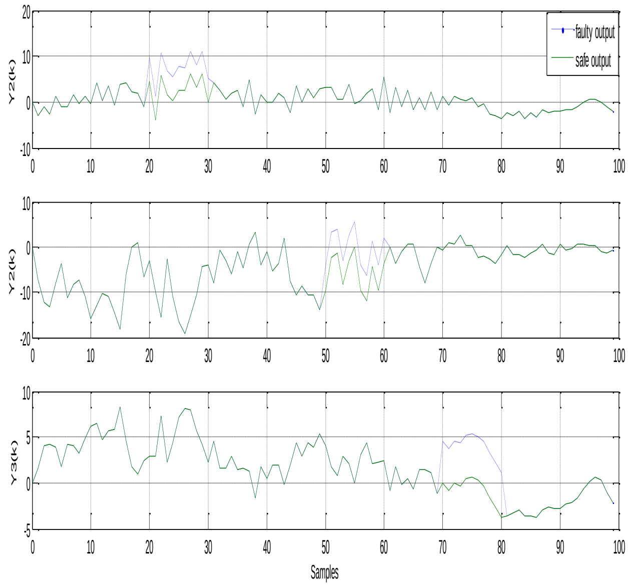

Fig. 4 : Affected outputs with their corrections.

By returning to the system equation and the relations (17), (18) and (19) it is possible to determine one state estimate.
From this state estimate it is possible to find an estimate of the system outputs.

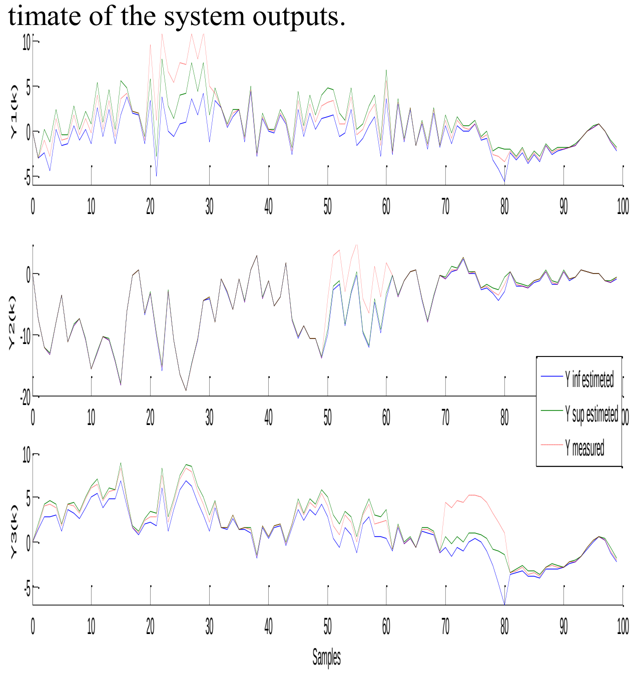

Fig. 5: Real and estimated outputs.

Figure 5 shows the lower and upper bounds of the three outputs $y_1(k)$, $y_2(k)$, and $y_3(k)$ together with the measured outputs. The real system measurements values must be included between these two bounds, else it will be declared faulty.
At moments [20, 30], [50, 60] and [70, 80], it is possible to remark that the output is out of bounds. From these results, residuals are easily generated and they will be used as fault indicator.
Figure 6 represent the three residuals calculated from the real and estimated measurements. The residue is a domain limited by two bounds. The zero must be included into this domain to assert that the system is safe; else it will be declared faulty.

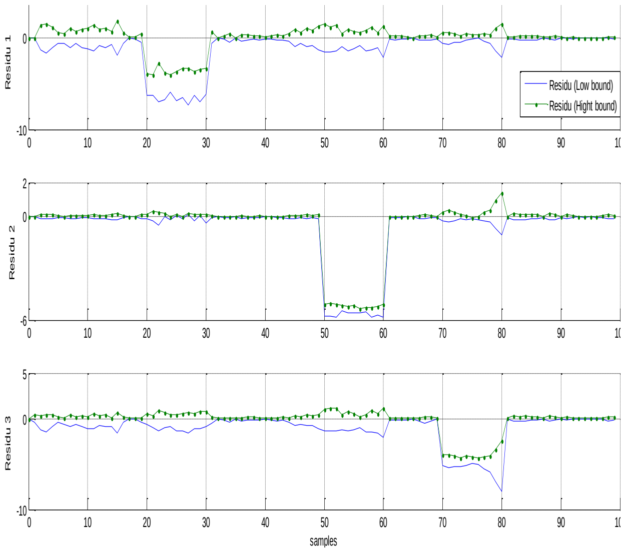

Fig. 6 :Residuals.

It is clear that residuals perfectly detect faults at the times of their interventions; as well they locate each fault on the affected output.
The bounded approach proves its effectiveness in term of fault detection and localization in cases of fuzzy systems with a parametric uncertainty.

6. Discussion

The advantage of the proposed method in this work is to offer the possibility to detect and locate faults affecting the system. Using the multiple model approach, the process can be modeled by several local models aggregated by weighting function to form the global model. It is possible that some local model are affected by faults, the proposed method can detect them.
This approach is very important since it make the generalization of the proposed method to the class of nonlinear system.
In general, works deals with nonlinear systems choose a particular class to apply their theories, it is impossible to use an approach for all the nonlinear systems classes. Using the multiple model approach, this problem is solved because all the nonlinear systems classes can be modeled using this approach.
By exploiting the numerical application, it appears that the proposed approach is interesting because it could deal with cases of uncertain fuzzy systems.
Based on interval analysis, a state estimation is made for a nonlinear system with uncertain parameters, which was not done in the cited studies such as that proposed by Letellier (Letellier et al., 2011) who processed by the same approach, the case of uncertain linear systems and he ignored the case of nonlinear uncertain systems.
The proposed technique has proven its effectiveness in the detection and isolation of faults affecting the system parameters, which has been shown in the previous example where the fault indicator (residuals) remains insensitive to bounded uncertainty and detects a fault as soon as his appearance on one of the sub models.

7. Conclusion

Through this paper a method for fault detection and localization in case of fuzzy systems with parametric uncertainties is established. The proposed method is based on the use of the bounded approach by exploiting the interval analysis as innovative diagnosis technique to ensure the residual insensitivity for the parametric model uncertainties. The proposed method is applied for the case of nonlinear systems described by multiple model structure. The effectiveness of the proposed method is shown by its application to a numerical example. This method has interesting prospects for research if the generalization of uncertainty is managed; i.e. whereas all matrices of system are uncertain, which is the actual case in industrial systems and will be the subject of future works.

References

[1] O. Adrot, "Diagnostic à base de modèles incertains utilisant l'analyse par intervalles: l'approche bornante," PhD thesis, Centre de Recherche en Automatique de Nancy, Institut National Polytechnique de Lorraine, INIST-CNRS, 2000.
[2] C. Letellier, G. Hoblos, and H. Chafouk, "Robust Fault Detection based on Multimodel and Interval Approach. Application to a Throttle Valve," in *IEEE MED*, 2011.
[3] C. Letellier, "Diagnostic robuste des systèmes incertains. Application à un système mécatronique pour l'automobile," PhD thesis, Université de Rouen, France, 2012.

[4] H. Bedoui, A. Khedher, and K. Ben Othman, "State estimation of uncertain nonlinear systems using the interval approach: Application to the diagnostic," *Wulfenia Journal,* vol. 21, pp. 369 - 379, 2014.
[5] H. Bedoui, A. Khedher, and K. B. Othman, "Interval analysis for the detection of sensors defects of a nonlinear system," presented at the The International Conference on Control, Engineering & Information Technology (CEIT'14), Monastir, Tunisia, 2014.
[6] M. Witczak, L. Dziekan, V. Puig, and J. Korbicz, "A fault-tolerant control strategy for Takagi-Sugeno fuzzy systems," in *17th IFAC World Congress*, COEX, South Korea, , 2008, pp. 7387-7392.
[7] J. Yoneyama, "Robust H∞ control of uncertain fuzzy systems under time-varying sampling," *Fuzzy sets and systems,* vol. 161, pp. 859-871, 2010.
[8] K. Tanaka, T. Ikeda, and H. O. Wang, "Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs," *Fuzzy Systems, IEEE Transactions on,* vol. 6, pp. 250-265, 1998.
[9] R. Murray-Smith and T. Johansen, *Multiple Model Approaches to Nonlinear Modelling and Control*: CRC press, 1997.
[10]E. Walter and H. Piet-Lahanier, "Exact and recursive description of the feasible parameter set for bounded error models," in *Decision and Control, 1987. 26th IEEE Conference on*, 1987, pp. 1921-1922.
[11]E. Walter, J. Norton, H. Piet-Lahanier, and M. Milanese, *Bounding approaches to system identification*: Perseus Publishing, 1996.
[12]L. Jaulin, M. Kieffer, O. Didrit, and E. Walter, "Applied interval analysis. 2001," ed: Springer, London.
[13]R. E. Moore and C. Yang, "Interval analysis I," *Applied Mathematics. Technical Document. Lockheed Aircraft Corporation, Missiles and Space division, Sunnyvale, California, USA,* 1959.
[14]T. Sunaga, "Theory of an interval algebra and its application to numerical analysis," *Japan Journal of Industrial and Applied Mathematics,* vol. 26, pp. 125-143, 1958.
[15]F. Lydoire and P. Poignet, "Nonlinear model predictive control via interval analysis," in *44th IEEE Conference on Decision and Control, and European Control Conference. CDC-ECC'05.* , 2005, pp. 3771-3776.
[16]D. Efimov, L. Fridman, T. Raissi, A. Zolghadri, and R. Seydou, "Application of Interval Observers and HOSM Dierentiators for Fault Detection," in *8th International Federation of Automatic Control (IFAC) Symposium SAFEPROCESS-2012*, 2012.
[17]S. Chandrasekaran, G. Golub, M. Gu, and A. Sayed, "Parameter estimation in the presence of bounded data uncertainties," *SIAM Journal on Matrix Analysis and Applications,* vol. 19, pp. 235-252, 1998.
[18]O. Adrot and J. Flaus, "Fault detection based on uncertain models with bounded parameters and bounded parameter variations," *identity,* vol. 1, p. 1, 2008.
[19]R. Seydou, T. Raissi, A. Zolghadri, and D. Efimov, "Robust fault diagnosis with interval continuous-time parity equations," in *The 8th IFAC International Symposium on Fault Detection, Supervision and Safety of Technical Processes*, 2012.
[20]R. M. Ferrari, T. Parisini, and M. M. Polycarpou, "A robust fault detection and isolation scheme for a class of uncertain input-output discrete-time nonlinear systems," in *American Control Conference, 2008*, 2008, pp. 2804-2809.
[21]A. Azar, "Fuzzy Systems," ed: Intech, 2010.
[22]A. T. Azar, "Overview of Type-2 Fuzzy Logic Systems," *International Journal of Fuzzy System Applications,* vol. 2, pp. 1-28, 2012.
[23]D. Ichalal, B. Marx, J. Ragot, and D. Maquin, "State and unknown input estimation for nonlinear systems described by Takagi-Sugeno models with unmeasurable premise variables," in *Control and Automation, 2009. MED'09. 17th Mediterranean Conference on*, 2009, pp. 217-222.