Global exponential stability of FCNNs with distributed delays

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Abstract: This paper is concerned with fuzzy cellular neural networks with distributed delays. Sufficient conditions on the existence, uniqueness and global exponential stability of equilibrium point are established by using the Homeomorphism theory and applying elementary inequality $2ab \leq a^2 + b^2$. Moreover an example is given to illustrate results obtained.

Key–Words: Fuzzy cellular neural networks, Global exponential stability, Homeomorphism theory, Lyapunov functional, Distributed delays

1 Introduction

Since cellular neural networks(CNNs) was first introduced by Chua and Yang in 1998 [1, 2]. The dynamical behaviors of CNNs and CNNs with delays (DCNNs) have received much attention due to their potential applications in associated memory, parallel computing , pattern recognition, signal processing and optimization problems(see [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]).The existence of equilibrium point and global stability are one of the most important fields investigated by some researchers. When a neural circuit is employed as associated memory, the existence of many equilibrium points is a necessary feature . However, in application to solve optimization problems , the networks must possess a unique and globally asymptotically stable (GAS) or globally exponential stable(GES) equilibrium point for every input vector. Based on traditional CNNs, Yang and Yang [17, 18] proposed another type-fuzzy cellular neural networks (FCNNs), which integrates fuzzy logic into the structure of cellular neural networks . Unlike CNNs structure, FCNNs has fuzzy logic between its template input and/or output besides the sum of product operations. Studies have shown that FCNNs has its potential in image processing and pattern recognition. Like the traditional CNNs, the stability of the system is very important in the design of the FCNNs. In recent years some results on stability for FCNNs have been derived(see [17, 18, 19, 20, 21, 22, 23, 24, 25]). To the best of our knowledge. FCNNs with delays are seldom considered. Authors in reference([21, 22]) gave some conditions to guarantee the globally stability of FCNNs with constant delays and time-varying delays. In formulating the network model, the time delay is assumed to be discrete. This assumption is reasonable. However a more satisfactory hypothesis is that the time delays are continuously distributed over a certain duration of time such that the distant past has less influence compared to the recent behavior of the state. Author in [19] considered FCNNs with finite distributed delays and found some sufficient conditions to ensure the existence and global exponential stability of equilibrium point. Based on these considerations above, in this paper we consider the following FCNNs with infinite distributed delays.

$$\begin{aligned}\dot{x}_i(t) &= -d_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=i}^{n} b_{ij} u_j \\ &\quad + \bigwedge_{j=1}^{n} \alpha_{ij} g_j\left(\int_0^\infty k_{ij}(s) x_j(t-s) ds\right) \\ &\quad + \bigvee_{j=1}^{n} \beta_{ij} g_j\left(\int_0^\infty k_{ij}(s) x_j(t-s) ds\right) \\ &\quad + \bigwedge_{j=1}^{n} T_{ij} u_j + \bigvee_{j=1}^{n} H_{ij} u_j + I_i \qquad (1)\end{aligned}$$

$i = 1, 2, \cdots, n$, where $d_i > 0$ represents the passive decay rates to the state of ith unit at time t. $\alpha_{ij}, \beta_{ij}, T_{ij}$ and H_{ij} are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed forward MIN template and fuzzy feed forward MAX template, respectively. a_{ij} and b_{ij} are elements of feedback template and feed forward template. $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively. x_i, u_j and I_i denote state, input and bias of the ith neurons, respectively. $f_j(\cdot)$ and $g_j(\cdot)$ are the activation functions. k_{ij} denote delay kernels.

Suppose that system (1) has an initial condition with $x_i(t) = \phi_i(t)(i = 1, 2\cdots, n), t \geq 0$. Denote a continuous solution of system (1.1) with initial condition by $x(t, 0, \phi)$. For convenience denote solution by $x(t)$ if no confusion should occur, where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ (T denote transpose). For $x \in R^n$, we define the vector norm $\|x\| = (\sum_{i=i}^n \mid x_i \mid^2)^{\frac{1}{2}}$. For any $\phi = (\phi_1, \phi_2, \ldots, \phi_n)^T \in C$ (where $C = C([-\tau, 0], R^n)$), we define a norm in C by $\|\phi\| = \left[\sum_{i=1}^n \left(\sup_{-\infty \leq s \leq 0} |\phi_i(s)|^2\right)\right]^{\frac{1}{2}}$

Definition 1 *The equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ of system (1.1) is said to be GES, if there are constants $\lambda > 0$ and $M \geq 1$ such that, for any $t \geq 0$*

$$\|x(t) - x^*\| \leq M\|\phi - x^*\|e^{-\lambda t}.$$

Definition 2 *If $f(t) : R \to R$ is a continuous function, then the upper right derivative of f is defined as*

$$D^+f(t) = \lim_{l \to 0^+} \sup \frac{1}{l}(f(t + l) - f(t)).$$

Definition 3 *[11]. A map $H : R^n \to R^n$ is a homeomorphism of R^n onto itself if H is continuous and one-to-one and its inverse map H^{-1} is also continuous.*

Lemma 4 *[11]. Let $H : R^n \to R^n$ be continuous. If H satisfies the following conditions*
(1) $H(x)$ is injective on R^n
(2) $\|H(x)\| \to \infty$ as $\|x\| \to \infty$
Then H is homeomorphism.

Lemma 5 *[17]. Suppose x and y are two states of system (1), then we have*

$$\left|\bigwedge_{j=1}^n \alpha_{ij}g_j(x_j) - \bigwedge_{j=1}^n \alpha_{ij}g_j(y_j)\right| \leq \sum_{j=1}^n |\alpha_{ij}||g_j(x_j) - g_j(y_j)|, \quad (2)$$

and

$$\left|\bigvee_{j=1}^n \beta_{ij}g_j(x_j) - \bigvee_{j=1}^n \beta_{ij}g_j(y_j)\right| \leq \sum_{j=1}^n |\beta_{ij}||g_j(x_j) - g_j(y_j)|. \quad (3)$$

To obtain our results, we make the following assumptions.
(A1) $f_j(\cdot)$ and $g_j(\cdot)(j = 1, 2, \ldots, n)$ are globally Lipschitz continuous, i. e., there exist positive constant μ_j and σ_j such that

$$|f_j(x)-f_j(y)| \leq \mu_j|x-y|, \quad |g_j(x)-g_j(y)| \leq \sigma_j|x-y|, \quad (4)$$

and $f_j(0) = g_j(0) = 0$ for any $x, y \in R$ and $j = 1, 2, \ldots, n$.
(A2) The delay kernels $k_{ij} : [0, \infty] \to [0, \infty]$ are continuous functions and they are assumed to satisfy

$$\int_0^\infty k_{ij}(s)ds = 1, \int_0^\infty k_{ij}(s)e^{\lambda s}ds < \infty. \quad (5)$$

where $\lambda > 0$.

When the activation functions are bounded, the existence of an equilibrium point of a neural network can be guaranteed by the use of the Brouwer's fixed point theorem. This, however, is not the case if the functions are unbounded and it could happen that there are no equilibrium points. Thus it becomes the main objective of this paper. In Section 2, we will give the existence and uniqueness of equilibrium point for fuzzy cellular neural networks. Results for global exponential stability of fuzzy cellular neural networks with distributed delays will be given and proved in Section 3. An illustrative example will be given to show effectiveness of our results in Section 4. Conclusion will be given in Section 5.

2 Existence and uniqueness of equilibrium point

In this section, we will discuss the existence and uniqueness of equilibrium point for fuzzy cellular neural networks (1).

Firstly, we study the existence and uniqueness of the equilibrium point, considering the following equation associated with system(1)

$$-d_ix_i(t) + \sum_{j=1}^n a_{ij}f_j(x_j(t)) + \sum_{j=i}^n b_{ij}u_j + \bigwedge_{j=1}^n \alpha_{ij}g_j(x_j(t)) + \bigvee_{j=1}^n \beta_{ij}g_j(x_j(t))$$

$$+\bigwedge_{j=1}^{n} T_{ij}u_j + \bigvee_{j=1}^{n} H_{ij}u_j + I_i = 0.$$

Define the map H as follows

$$H(x) = (h_1(x_1), h_2(x_2), \ldots, h_n(x_n))^T, \quad (6)$$

in which

$$\begin{aligned} h_i(x_i) &= -d_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=i}^{n} b_{ij}u_j \\ &\quad + \bigwedge_{j=1}^{n} \alpha_{ij} g_j(x_j(t)) + \bigvee_{j=1}^{n} \beta_{ij} g_j(x_j(t)) \\ &\quad + \bigwedge_{j=1}^{n} T_{ij}u_j + \bigvee_{j=1}^{n} H_{ij}u_j + I_i \quad (7) \end{aligned}$$

Theorem 6 *Assume that* $(A1)$ *and* $(A2)$ *hold, Suppose there exist real numbers* $\delta_i > 0, (i = 1, 2, \ldots, n)$ *such that*

$$\begin{aligned} d_i &> \frac{1}{2}\sum_{j=1}^{n} |a_{ij}|\mu_j + \frac{1}{2}\sum_{j=1}^{n} (|\alpha_{ij} + |\beta_{ij}|)\sigma_j \\ &\quad + \frac{1}{2}\sum_{j=1}^{n} \frac{\delta_j}{\delta_i}|a_{ji}|\mu_i \\ &\quad + \frac{1}{2}\sum_{j=1}^{n} \frac{\delta_j}{\delta_i}(|\alpha_{ji}| + |\beta_{ji}|)\sigma_i \quad (8) \end{aligned}$$

then system (1) has a unique equilibrium point x^*

Proof: We define map H as (6) and (7), we only need to show that H satisfies two conditions of Lemma 1.

Firstly, we will show that If $x \neq \overline{x}$ then $H(x) \neq H(\overline{x})$ holds for any $x, \overline{x} \in R^n$. Suppose, by contradiction, that map H is not injective on R^n. In other words, there exist $x \neq \overline{x}$ such that $H(x) = H(\overline{x})$; i. e., for $i = 1, 2, \ldots, n$.

$$\begin{aligned} h_i(x_i) &- h_i(\overline{x}_i) \\ &= -d_i(x_i - \overline{x}_i) + \sum_{j=1}^{n} a_{ij}(f_j(x_j)) - f_j(\overline{x}_j)) \\ &\quad + \bigwedge_{j=1}^{n} \alpha_{ij} g_j(x_j) - \bigwedge_{j=1}^{n} \alpha_{ij} g_j(\overline{x}_j) \\ &\quad + \bigvee_{j=1}^{n} \beta_{ij} g_j(x_j) - \bigvee_{j=1}^{n} \beta_{ij} g_j(\overline{x}_j) \\ &= 0 \quad (9) \end{aligned}$$

Applying assumption (A1) and Lemma 5, we obtain

$$\begin{aligned} d_i|x_i - \overline{x}_i| &\leq \sum_{j=1}^{n} a_{ij}|f_j(x_j) - f(\overline{x}_j)| \\ &\quad + \left|\bigwedge_{j=1}^{n} \alpha_{ij} g_j(x_j) - \bigwedge_{j=1}^{n} \alpha_{ij} g_j(\overline{x}_j)\right| \\ &\quad + \left|\bigvee_{j=1}^{n} \beta_{ij} g_j(x_j) - \bigvee_{j=1}^{n} \beta_{ij} g_j(\overline{x}_j)\right| \\ &\leq \sum_{j=1}^{n} |a_{ij}|\mu_j|x_j - \overline{x}_j| \\ &\quad + \sum_{j=1}^{n} (|\alpha_{ij} + \beta_{ij}|)\sigma_j|x_j - \overline{x}_j| \end{aligned}$$

and from which we get

$$\begin{aligned} 2\textstyle\sum_{i=1}^{n} &\delta_i d_i|x_i - \overline{x}_i|^2 \\ \leq &\sum_{i=1}^{n} \delta_i \Bigg[\sum_{j=1}^{n} |a_{ij}|\mu_j 2|x_i - \overline{x}_i||x_j - \overline{x}_j| \\ &+ \sum_{j=1}^{n} (|\alpha_{ij}| + |\beta_{ij}|)\sigma_j 2|x_i - \overline{x}_i||x_j - \overline{x}_j|\Bigg] \end{aligned}$$

where $\delta_i > 0$. Using elementary inequality $2ab \leq a^2 + b^2$ in the above, it follows that

$$\begin{aligned} 2\textstyle\sum_{i=1}^{n} &\delta_i d_i|x_i - \overline{x}_i|^2 \\ \leq &\sum_{i=1}^{n} \delta_i \Bigg\{\sum_{j=1}^{n} |a_{ij}|\mu_j|x_i - \overline{x}_i|^2 \\ &+ \sum_{j=1}^{n} |a_{ij}|\mu_j|x_j - \overline{x}_j|^2 \\ &+ \sum_{j=1}^{n} (|\alpha_{ij}| + |\beta_{ij}|)\sigma_j|x_i - \overline{x}_i|^2 \\ &+ \sum_{j=1}^{n} (|\alpha_{ij}| + |\beta_{ij}|)\sigma_j|x_j - \overline{x}_j|^2\Bigg\} \\ = &\sum_{i=1}^{n} \delta_i \Bigg\{\sum_{j=1}^{n} |a_{ij}|\mu_j|x_i - \overline{x}_i|^2 \\ &+ \sum_{j=1}^{n} \frac{\delta_j}{\delta_i}|a_{ji}|\mu_i|x_i - \overline{x}_i|^2 \\ &+ \sum_{j=1}^{n} (|\alpha_{ij}| + |\beta_{ij}|)\sigma_j|x_i - \overline{x}_i|^2 \\ &+ \sum_{j=1}^{n} \frac{\delta_j}{\delta_i}(|\alpha_{ji}| + |\beta_{ji}|)\sigma_i|x_i - \overline{x}_i|^2\Bigg\} \end{aligned}$$

which in turn gives

$$\begin{aligned} 2\sum_{i=1}^{n} & \delta_i \left\{ d_i - \frac{1}{2}\sum_{j=1}^{n} |a_{ij}|\mu_j \right. \\ & -\frac{1}{2}\sum_{j=1}^{n} \frac{\delta_j}{\delta_i}|a_{ji}|\mu_i \\ & -\frac{1}{2}\sum_{j=1}^{n}(|\alpha_{ij}| + |\beta_{ij}|)\sigma_j \\ & \left. -\frac{1}{2}\sum_{j=1}^{n} \frac{\delta_j}{\delta_i}(|\alpha_{ji}| + |\beta_{ji}|)\sigma_i \right\} |x_i - \overline{x}_i|^2 \\ \leq \quad & 0 \end{aligned}$$

By applying (8) to the above we can lead to the equality $x_i = \overline{x}_i$, for each $i = 1, 2, \ldots, n$ implying that $x = \overline{x}$. This contradicts our choice of $x \neq \overline{x}$. Hence the map H is injective on R^n. Now we only need to show that $\|H(x)\| \to \infty$ as $\|x\| \to \infty$. for sake of convenience.
Set

$$H^*(x) = (h_1^*(x_1), h_2^*(x_2), \ldots, h_n^*(x_n))^T$$

where

$$\begin{aligned} h_i^*(x_i) \quad = \quad & -d_i x_i + \sum_{j=1}^{n} a_{ij}(f_j(x_j) - f_j(0)) \\ & + \bigwedge_{j=1}^{n} \alpha_{ij} g_j(x_j) - \bigwedge_{j=1}^{n} \alpha_{ij} g_j(0) \\ & + \bigvee_{j=1}^{n} \beta_{ij} g_j(x_j) - \bigvee_{j=1}^{n} \beta_{ij} g_j(0) \quad (10) \end{aligned}$$

We consider

$$\begin{aligned} \sum_{i=1}^{n} \quad & \delta_i |x_i| \mathrm{sgn}(x_i) h_i^*(x_i) = \sum_{i=1}^{n} \delta_i |x_i| \mathrm{sgn}(x_i) \\ & \times \left[-d_i x_i + \sum_{j=1}^{n} a_{ij}(f_j(x_j) - f_j(0)) \right. \\ & + \bigwedge_{j=1}^{n} \alpha_{ij} g_j(x_j) - \bigwedge_{j=1}^{n} \alpha_{ij} g_j(0) \\ & \left. + \bigvee_{j=1}^{n} \beta_{ij} g_j(x_j) - \bigvee_{j=1}^{n} \beta_{ij} g_j(0) \right] \\ \leq \quad & \sum_{i=1}^{n} \delta_i \left[-d_i |x_i|^2 + \frac{1}{2}\sum_{j=1}^{n} \mu_j |a_{ij}| 2|x_i||x_j| \right. \\ & \left. +\frac{1}{2}\sum_{j=1}^{n} \sigma_j(|\alpha_{ij}| + |\beta_{ij}|) 2|x_i||x_j| \right] \\ \leq \quad & \sum_{i=1}^{n} \delta_i \left[-d_i |x_i|^2 + \frac{1}{2}\sum_{j=1}^{n} \mu_j |a_{ij}||x_i|^2 \right. \\ & +\frac{1}{2}\sum_{j=1}^{n} \mu_j |a_{ij}||x_j|^2 \\ & +\frac{1}{2}\sum_{j=1}^{n} \sigma_j(|\alpha_{ij}| + |\beta_{ij}|)|x_i|^2 \\ & \left. +\frac{1}{2}\sum_{j=1}^{n} \sigma_j(|\alpha_{ij}| + |\beta_{ij}|)|x_j|^2 \right] \\ = \quad & \sum_{i=1}^{n} \delta_i \left[-d_i + \frac{1}{2}\sum_{j=1}^{n} \mu_j |a_{ij}| \right. \\ & +\frac{1}{2}\sum_{j=1}^{n} \frac{\delta_j}{\delta_i}|a_{ji}|\mu_i + \frac{1}{2}\sum_{j=1}^{n} \sigma_j(|\alpha_{ij}| + |\beta_{ij}|) \\ & \left. +\frac{1}{2}\sum_{j=1}^{n} \frac{\delta_j}{\delta_i}(|\alpha_{ji}| + |\beta_{ji}|)\sigma_i \right] |x_i|^2 \\ \leq \quad & -\eta \sum_{i=1}^{n} \delta_i |x_i|^2. \end{aligned}$$

where

$$\begin{aligned} \eta \quad = \quad & \min_{1\leq i\leq n, 1\leq j\leq n} \left\{ d_i - \frac{1}{2}\sum_{j=1}^{n} \mu_j |a_{ij}| \right. \\ & -\frac{1}{2}\sum_{j=1}^{n} \frac{\delta_j}{\delta_i}|a_{ji}|\mu_i - \frac{1}{2}\sum_{j=1}^{n} \sigma_j(|\alpha_{ij}| + |\beta_{ij}|) \\ & \left. -\frac{1}{2}\sum_{j=1}^{n} \frac{\delta_j}{\delta_i}(|\alpha_{ji}| + |\beta_{ji}|)\sigma_i \right\} \geq 0 \end{aligned}$$

Applying Holder's inequality to the above, we obtain

$$\begin{aligned} \eta \quad & \min_{1\leq i\leq n} (\delta_i) \|x_i\|^2 \\ \leq \quad & \eta \sum_{i=1}^{n} \delta_i |x_i|^r \leq \left| \sum_{i=1}^{n} \delta_i \mathrm{sgn}(x_i) h_i^*(x_i) |x_i| \right| \\ \leq \quad & \max_{1\leq i\leq n} (\delta_i) \left(\sum_{i=1}^{n} |h_i^*(x_i)|^2 \right)^{\frac{1}{2}} \left(\sum_{i=1}^{n} |x_i|^2 \right)^{\frac{1}{2}} \\ = \quad & \max_{1\leq i\leq n} (\delta_i) \|H^*(x)\| \|x\| \end{aligned}$$

Therefore it follows that $\|H^*(x)\| \geq \eta \frac{\min_{1\leq i\leq n}(\delta_i)}{\max_{1\leq i\leq n}(\delta_i)} \|x\|$. Which directly implies that $\|H^*(x)\| \to \infty$ as $\|x\| \to +\infty$. Hence there exists a unique equilibrium point $x = x^*$ such that $H(x^*) = 0$. The proof of Theorem 6 is complete. □

3 Global exponential stability of fuzzy cellular neural networks

In this section, we will analyze the global exponential stability of the unique equilibrium point x^* of system (1) under sufficient condition (8) .

Let $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ be the equilibrium point of system(1.1), we make a transform for system (1.1): $z_i(t) = x_i(t) - x_i^*(i = 1, 2, \ldots, n)$, we have

$$\begin{aligned} \dot{z}_i(t) &= -d_i z_i(t) + \sum_{j=1}^{n} a_{ij} \left[f_j(z_j(t) + x_j^*) - f_j(x_j^*) \right] \\ &+ \bigwedge_{j=1}^{n} \alpha_{ij} g_j \left(\int_0^\infty k_{ij}(s)(z_j(t-s) + x_j^*) ds \right) \\ &- \bigwedge_{j=1}^{n} \alpha_{ij} g_j \left(\int_0^\infty k_{ij}(s) x_j^* ds \right) \\ &+ \bigvee_{j=1}^{n} \beta_{ij} g_j \left(\int_0^\infty k_{ij}(s)(z_j(t-s) + x_j^*) ds \right) \\ &- \bigvee_{j=1}^{n} \beta_{ij} g_j \left(\int_0^\infty k_{ij}(s) x_j^* ds \right) \end{aligned} \tag{11}$$

where $z_i(t) = \Phi_i(t), \Phi_i(t) = \phi_i(t) - x_i^*, i, j = 1, 2, \ldots, n.$ for $t \in (-\infty, 0]$.

Clearly, the equilibrium point x^* of system (1) is GES if and only if the equilibrium point O of system (11) is GES. In the following, we only study global exponential stability of the equilibrium point O for system (11).

Theorem 7 *Let the conditions $(A1) - (A2)$ hold, If (8) is satisfied, then the system (11) has a unique equilibrium point O of system which is GES.*

Proof: The existence and uniqueness of an equilibrium point O follow from Theorem 1. Let $z(t)$ denote an arbitrary solution of system (11). Calculating the upper right derivative $D^+|z_i(t)|$ and noting that assumptions (A1)-(A2) above and Lemma 5, we have

$$\begin{aligned} D^+ \quad & |z_i(t)| \le -d_i |z_i(t)| + \sum_{j=1}^{n} |a_{ij}| \mu_j |z_j(t)| \\ & + \sum_{j=1}^{n} (|\alpha_{ij}| + |\beta_{ij}|) \sigma_j \\ & \times \int_0^\infty k_{ij}(s) |z_j(t-s)| ds \end{aligned} \tag{12}$$

for $i = 1, 2, \ldots, n; t > 0$.
since (8) holds, we can choose a small constant $\lambda > 0$ such that

$$\begin{aligned} \lambda &- d_i + \frac{1}{2} \sum_{j=1}^{n} |a_{ij}| \mu_j + \frac{1}{2} \sum_{j=1}^{n} \frac{\delta_j}{\delta_i} |a_{ji}| \mu_i \\ &+ \frac{1}{2} \sum_{j=1}^{n} \left((|\alpha_{ij} + |\beta_{ij}|) \int_0^\infty k_{ij}(s) e^{\lambda s} ds \right) \sigma_j \\ &+ \frac{1}{2} \sum_{j=1}^{n} \frac{\delta_j}{\delta_i} \left((|\alpha_{ji}| + |\beta_{ji}|) \right. \\ &\times \left. \int_0^\infty k_{ji}(s) e^{\lambda s} ds \right) \sigma_i \le 0 \end{aligned} \tag{13}$$

Set

$$w_i(t) = e^{\lambda t} |z_i(t)| = e^{\lambda t} |x_i(t) - x_i^*|; \; t \ge 0. \tag{14}$$

for $i = 1, 2, \cdots, n$. Estimating the upper right derivative $D^+ w_i(t)$ and noting that (12), we have

$$\begin{aligned} D^+ \quad & w_i(t) = \lambda e^{\lambda t} |z_i(t)| + e^{\lambda t} D^+ |z_i(t)| \\ \le \quad & \lambda e^{\lambda t} |z_i(t)| - d_i e^{\lambda t} |z_j(t)| \\ & + \sum_{j=1}^{n} |a_{ij}| \mu_j e^{\lambda t} |z_j(t)| + \sum_{j=1}^{n} (|\alpha_{ij}| + |\beta_{ij}|) \\ & \times \sigma_j \int_0^\infty k_{ij}(s) e^{\lambda t} |z_j(t-s)| ds \end{aligned}$$

and this gives

$$\begin{aligned} D^+ \quad & w_i(t) \le (\lambda - d_i) w_i(t) + \sum_{j=1}^{n} |a_{ij}| \mu_j w_j(t) \\ & + \sum_{j=1}^{n} (|\alpha_{ij}| + |\beta_{ij}|) \sigma_j \\ & \times \int_0^\infty k_{ij}(s) e^{\lambda s} w_j(t-s) ds \end{aligned} \tag{15}$$

Now we consider the following Lyapunov functional

$$\begin{aligned} V(t) &= \sum_{i=1}^{n} \delta_i \left(w_i^2(t) + \sum_{j=1}^{n} (|\alpha_{ij}| + |\beta_{ij}|) \sigma_j \right. \\ & \times \left. \int_0^\infty k_{ij}(s) e^{\lambda s} \left(\int_{t-s}^{t} w_j^2(r) dr \right) ds \right) \end{aligned} \tag{16}$$

Calculating the upper right derivative $D^+ V(t)$ along the solution of (15), we obtain

$$\begin{aligned} D^+ \quad & V(t) \\ = \quad & \sum_{i=1}^{n} \delta_i \left(2 w_i(t) D^+ w_i(t) + \sum_{j=1}^{n} (|\alpha_{ij}| + |\beta_{ij}|) \sigma_j \right. \\ & \times \left. \int_0^\infty k_{ij}(s) e^{\lambda s} [w_j^2(t) - w_j^2(t-s)] ds \right) \\ \le \quad & \sum_{i=1}^{n} \delta_i \Big(2(\lambda - d_i) w_i^2(t) \\ & + \sum_{j=1}^{n} 2 |a_{ij}| \mu_j w_i(t) w_j(t) \end{aligned}$$

$$
\begin{aligned}
& +\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\sigma_j \\
& \times\int_0^\infty 2k_{ij}(s)e^{\lambda s}w_i(t)w_j(t-s)ds \\
& +\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\sigma_j \\
& \times\int_0^\infty k_{ij}(s)e^{\lambda s}[w_j^2(t)-w_j^2(t-s)]ds\Big) \\
\leq\; & \sum_{i=1}^{n}\delta_i\Big(2(\lambda-d_i)w_i^2(t) \\
& +\sum_{j=1}^{n}|a_{ij}|\mu_j[w_i^2(t)+w_j^2(t)] \\
& +\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\sigma_j \\
& \times\int_0^\infty k_{ij}(s)e^{\lambda s}[w_i^2(t)+w_j^2(t-s)]ds \\
& +\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\sigma_j \\
& \times\int_0^\infty k_{ij}(s)e^{\lambda s}[w_j^2(t)-w_j^2(t-s)]ds\Big) \\
=\; & \sum_{i=1}^{n}\delta_i\left(2(\lambda-d_i)w_i^2(t)+\sum_{j=1}^{n}|a_{ij}|\mu_jw_i^2(t)\right. \\
& +\sum_{j=1}^{n}\frac{\delta_j}{\delta_i}|a_{ji}|\mu_iw_i^2(t)+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\sigma_j \\
& \times\left(\int_0^\infty k_{ij}(s)e^{\lambda s}ds\right)w_i^2(t) \\
& +\sum_{j=1}^{n}\frac{\delta_j}{\delta_i}(|\alpha_{ji}|+|\beta_{ji}|)\sigma_i \\
& \left.\times\left(\int_0^\infty k_{ji}(s)e^{\lambda s}ds\right)w_i^2(t)\right) \\
=\; & \sum_{i=1}^{n}2\delta_i\left(\lambda-d_i+\frac{1}{2}\sum_{j=1}^{n}|a_{ij}|\mu_j\right. \\
& +\frac{1}{2}\sum_{j=1}^{n}\frac{\delta_j}{\delta_i}|a_{ji}|\mu_i+\frac{1}{2}\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\sigma_j \\
& \times\left(\int_0^\infty k_{ij}(s)e^{\lambda s}ds\right) \\
& +\frac{1}{2}\sum_{j=1}^{n}\frac{\delta_j}{\delta_i}(|\alpha_{ji}|+|\beta_{ji}|)\sigma_i \\
& \left.\times\left(\int_0^\infty k_{ji}(s)e^{\lambda s}ds\right)\right)w_i^2(t)
\end{aligned}
$$

Noting (13) that it follows that $D^+V(t)\leq 0$, for $t>0$. This in turn implies $V(t)\leq V(0)$, for all $t>0$.

Furthermore

$$
\begin{aligned}
V(0) &= \sum_{i=1}^{n}\delta_i\left(w_i^2(0)+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\sigma_j\right. \\
&\quad \left.\times\int_0^\infty k_{ij}(s)e^{\lambda s}\left(\int_{-s}^{0}w_j^2(r)dr\right)ds\right) \\
&= \sum_{i=1}^{n}\delta_i\left(w_i^2(0)+\sum_{j=1}^{n}\frac{\delta_j}{\delta_i}(|\alpha_{ji}|+|\beta_{ji}|)\sigma_i\right. \\
&\quad \left.\times\int_0^\infty k_{ji}(s)e^{\lambda s}\left(\int_{-s}^{0}w_i^2(r)dr\right)ds\right) \\
&\leq \sum_{i=1}^{n}\delta_i\left(1+\sum_{j=1}^{n}\frac{\delta_j}{\delta_i}(|\alpha_{ji}|+|\beta_{ji}|)\sigma_i\right. \\
&\quad \left.\times\int_0^\infty k_{ji}(s)e^{\lambda s}sds\right)\left(\sup_{-\infty<s\leq 0}w_i^2(s)\right)
\end{aligned}
$$

Recall that $\sup_{-\infty<s\leq 0}|z_i(s)|^2<\infty(i=1,2,\ldots,n)$, and from assumption (A2) it is clear that $V(0)<\infty$. Hence $V(t)\leq V(0)<\infty$ for all $t>0$. From (16) we have

$$
\begin{aligned}
\textstyle\sum_{j=1}^{n} & \delta_iw_i^2(t)\leq V(t) \\
& \leq\sum_{i=1}^{n}\left(\delta_i+\sum_{j=1}^{n}\delta_j(|\alpha_{ji}|+|\beta_{ji}|)\sigma_i\right. \\
& \left.\times\int_0^\infty k_{ji}(s)e^{\lambda s}sds\right)\left(\sup_{-\infty<s\leq 0}w_i^2(s)\right)
\end{aligned}
$$

for $t>0$. Applying (14) in the above, we obtain

$$
\begin{aligned}
\textstyle\sum_{i=1}^{n} & \delta_ie^{2\lambda t}|x_i(t)-x_i^*|^2 \\
\leq & \sum_{i=1}^{n}\left(\delta_i+\sum_{j=1}^{n}\delta_j(|\alpha_{ji}|+|\beta_{ji}|)\sigma_i\right. \\
& \left.\times\int_0^\infty k_{ji}(s)e^{\lambda s}sds\right)\left(\sup_{-\infty<s\leq 0}|x_i(s)-x_i^*|^2\right)
\end{aligned}
$$

Which in turn gives

$$
\sum_{i=1}^{n}|x_i(t)-x_i^*|^2\leq\xi e^{-2\lambda t}\sum_{i=1}^{n}\left(\sup_{-\infty<s\leq 0}|x_i(s)-x_i^*|^2\right) \tag{17}
$$

for $t>0$, where

$$
\begin{aligned}
\xi &= \max_{1\leq i\leq n,1\leq j\leq n}\left\{\delta_i+\sum_{j=1}^{n}\delta_j(|\alpha_{ji}|+|\beta_{ji}|)\sigma_i\right. \\
&\quad \left.\times\int_0^\infty k_{ji}(s)e^{\lambda s}sds\right\}/\min_{1\leq i\leq n}\{\delta_i\}\geq 1
\end{aligned}
$$

Hence from (17) and noting that $x_i(s) = \phi_i(s), -\infty < s \le 0$, it follows that

$$\|x(t) - x^*\| \le Me^{-\lambda t}\|\phi - x^*\|$$

for $t > 0$, where $M = \xi^{\frac{1}{2}}$. This means the equilibrium point of system (1) is global exponential stability. The proof of Theorem 7 is complete. □

Remark 1 If we don't consider fuzzy AND and OR operation, System (1) becomes traditional cellular neural networks. It is obvious that the results in [13] are the corollary of Theorem 7. Therefore the results of this paper extend the previous known publication.

Remark 2 In this paper, we don't assume the boundedness, monotonicity, and differentiable of activation functions. Clearly, these functions satisfying assumption (A1) are more general. For example, the Gaussian and inverse Gaussian functions have been used in the circuit designs and applications of cellular neural networks.

4 An example

In this section, we give an example to illustrate effectiveness of our results.

Example 4.1 Consider the following fuzzy cellular neural networks with distributed delays.

$$\begin{cases} \frac{dx_1}{dt} &= -d_1x_1(t) + \sum_{j=1}^{2} a_{1j}f_j(x_j(t)) \\ & + \bigwedge_{j=1}^{2} \alpha_{1j}g_j\left(\int_0^\infty k_{1j}(s)x_j(t-s)ds\right) \\ & + \bigvee_{j=1}^{2} \beta_{1j}g_j\left(\int_0^\infty k_{1j}(s)x_j(t-s)ds\right) \\ & + \sum_{j=1}^{2} b_{1j}u_j \\ & + \bigwedge_{j=1}^{2} T_{1j}u_j + \bigvee_{j=1}^{2} H_{1j}u_j + I_1 \\ \frac{dx_2}{dt} &= -d_2x_2(t) + \sum_{j=1}^{2} a_{2j}f_j(x_j(t)) \\ & + \bigwedge_{j=1}^{2} \alpha_{2j}g_j\left(\int_0^\infty k_{2j}(s)x_j(t-s)ds\right) \\ & + \bigvee_{j=1}^{2} \beta_{2j}g_j\left(\int_0^\infty k_{2j}(s)x_j(t-s)ds\right) \\ & + \sum_{j=1}^{2} b_{2j}u_j \\ & + \bigwedge_{j=1}^{2} T_{2j}u_j + \bigvee_{j=1}^{2} H_{2j}u_j + I_2 \end{cases} \tag{18}$$

where

$$D = \begin{pmatrix} 10 & 0 \\ 0 & 7 \end{pmatrix}, \quad A = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix},$$

$$B = \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}, \quad \alpha = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix},$$

$$\beta = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

$f_i(x_i) = g_i(x_i) = -|x_i|, i = 1, 2, I_1 = I_2 = 4, u_1 = u_2 = 1, k_{ij}(t) = \frac{2}{\pi(1+t^2)}(i, j = 1, 2)$ and the matrices $T = (T_{ij})_{2\times 2}, H = (H_{ij})_{2\times 2}$ are identity matrices. Clearly $f_i(\cdot), g_i(\cdot)$ are unbounded and Lipschitz continuous with the Lipschitz constants $\mu_i = \sigma_i = 1$. set $\delta_1 = 1, \delta_2 = 2$.

By simply calculating, we can easily check that system (18) satisfy assumptions (A1)-(A2) and (8). This illustrates the global exponential stability of system (18).

5 Conclusion

In this paper employing the elementary inequality $2ab \le a^2 + b^2$, constructing a new Lyapunov functional, and applying the Homeomorphism theory, we have derived a new conditions of the existence, uniqueness of the equilibrium point. The fuzzy cellular neural networks with distributed delays is GES under these conditions. In contrast with the previous papers, these conditions are independent of delays. Furthermore, the removal of the bounded-ness condition on activation functions gives the networks a wider scope of applicability particularly to optimization problems which contain unbounded constraints.

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