Variable Structure Attitude Controller Design for Solely Magnetically Actuated Small Satellites Subject to Environmental Disturbances

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Abstract: - Sliding mode control algorithms with classical and modified discontinuous reaching laws for purely magnetic attitude control of small satellites are proposed and compared with each other in this work. They are designed by following the steps characteristic to the variable structure control method and the magnetic attitude control problem. The asymptotical stability of the control laws is proven by using theoretical and intuitive approaches together. The stability of the controllers is verified by converging simulation results of the states. The aim is to obtain a controller superior to the sliding mode magnetic controller with continuous reaching law in terms of steady state error under environmental disturbance effects. The time responses of Euler angles and angular velocities indicate that the aim is reached. The necessity of modifying the classical discontinuous reaching law is made clear by comparing states' responses obtained by using the classical and the modified sliding mode algorithms.

Key-Words: - Magnetic attitude control, Satellite dynamics, Variable structure control, Sliding manifold

1 Introduction

Since the beginning of the eighties, small satellites have been increasingly preferred for space missions. These platforms are restricted in terms of size, mass and power, so they require attitude control actuators that are smaller in size, weigh less and consume less energy than the conventional ones. Consequently, the attitude control problem of small satellites has become a topic of interest on which many solutions have been proposed so far in literature. One of them is the "purely magnetic attitude control" method, which enables controlling the attitude in three axes by using only electromagnetic actuation.

The magnetic actuators are highly suitable for small satellites employed in missions with relatively low pointing accuracy requirements because they are low mass electromagnetic coils or rods fitting small volumes easily and require less energy during the nominal operation mode. They are free of degradation thanks to their non-mechanical structure. In addition, they are driven by controlled currents, which makes them much more suitable than mechanical momentum exchange devices for use with switching control algorithms such as sliding mode control.

It has been worked on benefiting from magnetic torquers as auxiliary actuators since 1961. One of the first important papers that deal with purely magnetic attitude control is published in 1989 [1]. In that work, a finite-time horizon linear quadratic regulator is proposed, and it is claimed that the application of the proposed method is possible due to the increasing processing capability of the spacecraft onboard computers. In a Ph.D. thesis dated to 1996, many linear and nonlinear control laws, one of which is based on sliding mode control method, are designed to control a small satellite in low Earth orbit by using only three mutually perpendicular magnetic actuators [2]. In there, it is claimed that the attitude cannot be stabilized asymptotically if a discontinuous reaching law is used. Therefore, the controller is designed based on a continuous reaching law, which eventually leads to the loss of disturbance rejection capability of the sliding mode controller [3,4]. Based on [1] and depending on the fact that the variation of the geomagnetic field in nearly polar orbits is periodic, infinite- and finite-time horizon periodic controllers and a constant gain controller are also designed in [2] benefiting from the linear periodic systems theory [5,6]. Moreover, energy based nonlinear controllers are proposed to control the nonlinear periodic system with a better performance [7,8]. Two more of the fewer nonlinear solutions to purely magnetic attitude control problem are given in [9] and [10], where nearly global asymptotic stability is achieved by designed nonlinear controllers. In a similar work [11], uniformly global stability result is obtained for the nonlinear problem. In [12], time-optimal controllers are proposed for the case with constrained inputs. In the first literature survey [13] of the purely magnetic attitude control problem, new results by the model-based predictive control approach are presented. In another study [14], a nearly globally stable solution is obtained by using an adaptive, PD-like controller. The problem is also solved by an intuitive magnetic control law in a globally asymptotic manner for any nearly polar orbits in [15]. In [16], fuzzy logic is applied to the nonlinear magnetic attitude control problem subject to environmental disturbances. Two examples of application of sliding mode control method to purely magnetic attitude control problem can be found in [17] and [18]. The problem is also solved by a nonlinear passivity-based sliding mode controller, which is integrated with an algorithm that makes the direct implementation of the controller to the system possible [19]. In a recent attempt to solve the problem, a nonlinear sliding manifold and a second-order sliding mode controller are used [20]. In a previous study by the authors of this paper, it is shown that the nonlinear attitude dynamics can be stabilized asymptotically by using the classical discontinuous reaching law on the contrary to the result in [2], and a new modified discontinuous sliding mode controller is designed, which is shown to be superior to the classical one [21]. In a most recent paper [22], by using neural network approach, the stabilization is achieved for the nonlinear system with parameter uncertainties and under disturbance effects. The presented literature summary on the purely magnetic attitude control problem depicts the need for robust, moreover insensitive control solutions. Variable structure controllers have the potential to provide the problem with solutions insensitive to external disturbances and model parameter uncertainties.

Motivated by [2] and based on [21], the three axis attitude control problem of a small satellite is considered in this paper regarding the attitude acquisition phase of its mission. In the acquisition phase, the initial attitude is far from the equilibrium, therefore the highly nonlinear attitude dynamics have to be handled without linearization. It is aimed to carry the states in the closest possible vicinity of the equilibrium under the effect of environmental disturbances. These two properties of the problem lead to the preference of sliding mode control method to solve the problem. As previously cited, the same problem is solved in the literature by the sliding mode control method by using a continuous reaching law. Two sliding mode controllers are proposed in this work to control the nonlinear attitude dynamics in three axes by using only three magnetic torquers. The first one composes of the equivalent control term and the classical discontinuous reaching law, which is the signum function of the sliding vector multiplied by a constant gain. The second controller has a discontinuous reaching law that is a modified version of the classical law. It is shown for the second controller theoretically that the attitude can be stabilized asymptotically even if beginning from an angular state that is far from the zero equilibrium state and under the effect of environmental disturbance torques. That analysis also indicates the stability of the classical controller. The performance of the designed modified controller is compared to the continuous controller proposed in [2] to indicate its superiority in terms of steady state error. It is also compared with the classical controller to present the necessity of the modification of the classical reaching law for the magnetic attitude control problem.

The paper is organized as follows: In Section 2, the nonlinear spacecraft attitude dynamics are presented, and the mathematical model for the environmental disturbance torques is given. In the 3rd section, both the classical and the modified control algorithms are introduced, and it is shown how the magnetic control torque is derived by modifying the control signal following the necessary steps instructed in [2], which are characteristic to the magnetic attitude control problem. To make the comparison possible, the continuous control algorithm from [2] is also given in that section. Then the reachability of the used sliding manifold by the proposed controller is proven. The section is concluded with the theoretical analysis of the designed controller with modified reaching law to show its asymptotically stable character. The simulation results obtained with three given controllers are presented in Section 4 after giving the used satellite model and the initial conditions. Responses of Euler angles, angular velocities, quaternions, magnetic control moment components, and sliding vector's magnitude obtained in the same simulation environment and for the same conditions are given comparatively. In conclusion, the steady state performances of the continuous and modified discontinuous control laws are evaluated, and it is concluded that the sliding mode magnetic control algorithm with modified reaching law may be preferable due to the lower steady state error margin it achieves thanks to its partial disturbance rejection capability.

2 Satellite Attitude Dynamics

2.1 Equations of Motion

For a rigid spacecraft in a circular orbit, the rotational motion is described by three dynamic (1) and four kinematic equations (2) in terms of Euler parameters, which define the orientation of the principal body reference system B of the satellite with respect to the orbit reference system A (see Fig.1) [23].

$$I\dot{\vec{\omega}}^{B/N} + \vec{\omega}^{B/N} \times I\vec{\omega}^{B/N} = \vec{T} \tag{1}$$

$$\dot{\vec{q}} = \frac{1}{2}(\vec{\omega}q_4 - \vec{\omega}\times\vec{q}) \text{ and } \dot{q}_4 = -\frac{1}{2}(\vec{\omega}\bullet\vec{q}) \tag{2}$$

where $\vec{q}$ is the 3x1 quaternion vector, q_4 is the scalar quaternion component, $\vec{\omega}$ and $\vec{\omega}^{B/N}$ are 3x1 angular velocity vectors of the satellite's body axis system with respect to the orbit reference system and Earth-centered, inertial reference system N, respectively (see Fig.1). I consists of nonzero diagonal elements, I_1, I_2, I_3, which are the principal moments of inertia [23]. $\vec{T}$ is the total torque acting on the satellite, which is the addition of the environmental disturbance torque components and the control torque. Finally, "$\times$" and "$\bullet$" are the cross and dot product signs, respectively.

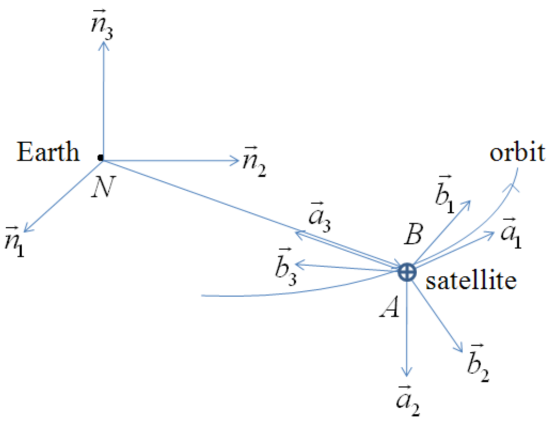

Fig.1 Reference systems A, B, and N (adapted from [23])

2.2 Environmental Disturbances

The gravity-gradient torque is calculated on each iteration step using its analytical formula given in [23] as

$$\vec{T}_{gg} = 3n^2\left(\vec{a}_3 \times I\vec{a}_3\right) \tag{3}$$

where

$$\vec{a}_3 = \begin{bmatrix} 2\left(q_1q_3 - q_2q_4\right) \\ 2\left(q_2q_3 + q_4q_1\right) \\ 1 - 2\left(q_1^2 + q_2^2\right) \end{bmatrix} \tag{4}$$

and n is the angular velocity of the satellite around the Earth.

The environmental disturbance torques emerging from aerodynamic drag and solar radiation pressure are inputted to the simulation environment by employing the following simple harmonic model [24].

$$\vec{T}_d = \begin{bmatrix} A_d\left[3\cos\left(nt\right)+1\right] \\ A_d\left[1.5\sin\left(nt\right)+3\cos\left(nt\right)\right] \\ A_d\left[3\sin\left(nt\right)\right] \end{bmatrix} \tag{5}$$

Here, the amplitude of the harmonic function, A_d, corresponding to both disturbance components at the altitude of satellite's orbit is estimated from a graph in [25].

3 Sliding Mode Controller

3.1 Controller Design

The proposed sliding mode controller with the modified discontinuous reaching law is going to be introduced in this subsection. Firstly, the sliding manifold will be defined. Then the equivalent control term and the designed discontinuous reaching law will be presented. Finally, the necessary manipulation steps will be accomplished to derive the actual magnetic control torque, $\vec{T}_{mc}$, from the ideal (desired) control signal.

The following sliding manifold

$$\vec{s} = \vec{\omega} + K_q\vec{q} \tag{6}$$

which is optimal as shown in [26] by referring to [27], is going to be used. It is theoretically shown in [2] that the sliding mode on the manifold (6) is stable in an asymptotic manner to the equilibrium, which is the origin of the state space. Here, K_q is the sliding manifold design parameter matrix and is defined as

$$K_q = k_q \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (7)$$

where k_q is a positive constant, so K_q is positive definite.

The control signal, $\vec{T}_{mc}$, is equal to the "equivalent control vector", $\vec{T}_{eq}$, in sliding mode, in which (8) holds according to the Filippov approach [27]

$$\vec{s} = \vec{0} \text{ and } \dot{\vec{s}} = \vec{0} \quad (8)$$

That forcing vector keeps the state vector on the manifold when ideal sliding occurs. By manipulating and then substituting (1) properly into the second equation in (8), the equivalent control term can be derived as

$$\begin{aligned} \vec{u}_{eq} = \vec{\omega}^{B/N} \times I\vec{\omega}^{B/N} - \frac{1}{2} I \Lambda_q \left(\vec{\omega} q - \vec{\omega} \times \vec{q} \right) \\ -3n^2 \left(\vec{a}_3 \times I\vec{a}_3 \right) - nI \left(\vec{a}_2 \times \vec{\omega} \right) \end{aligned} \quad (9)$$

Here,

$$\vec{a}_2 = \begin{bmatrix} 2\left(q_1 q_2 + q_3 q_4\right) \\ 1 - 2\left(q_3^2 + q_1^2\right) \\ 2\left(q_3 q_2 - q_1 q_4\right) \end{bmatrix} \quad (10)$$

and

$$\vec{\omega}^{B/N} = \vec{\omega} - n\vec{a}_2 \quad (11)$$

The detailed derivation of $\vec{u}_{eq}$ can be found in [2] or [28].

The reaching law corresponds to the term that carries the states onto the manifold making the system reach the sliding mode. In the reaching mode,

$$\vec{s} \neq \vec{0} \text{ and } \dot{\vec{s}} \neq \vec{0} \quad (12)$$

hold. The discontinuous reaching law proposed in [27] is as follows

$$I\dot{\vec{s}} = \vec{T}_{control} - \vec{u}_{eq} = -K_s \operatorname{sign}\left(\vec{s}\right) \quad (13)$$

Regarding the differences emerging from the nature of the magnetic attitude control problem,

$$\vec{u} - \vec{u}_{eq} = -K_s \operatorname{sign}\left(\vec{s}\right) \quad (14)$$

can be written. The positive definite reaching law design parameter matrix K_s is defined as follows with the positive constant k_s

$$K_s = k_s \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (15)$$

The classical sliding mode control law (16) can be written from (14) as [29]

$$\vec{u} = \vec{u}_{eq} - K_s \operatorname{sign}\left(\vec{s}\right) \quad (16)$$

It is a direct result of the stability analysis of the proposed modified controller, which will be given in the next subsection, that (16) is asymptotically stable. Thus that is a solution for the magnetic attitude control problem. Time responses obtained using (16) oscillate with constant amplitude for the cases without and with disturbance effect acting on the system. The sameness of responses indicates the expected insensitivity of the switching control law to disturbances. However, the performance of the controller (16) is unsatisfactory due to the unacceptable steady state error margin. The time responses of Euler angles obtained by (16) will be presented in the next section. Therefore a tuning has been carried out between (16) and the continuous control law (17)

$$\vec{u} = \vec{u}_{eq} - K_s \vec{s} \quad (17)$$

which is shown to be asymptotically stable in [2], to come up with a controller that has not complete, but partial disturbance rejection capability and therefore a lower steady state error margin under disturbance effect compared to (17). The resulting control algorithm is given in (18)

$$\vec{u}_{des} \equiv \vec{u} = \vec{u}_{eq} - K_s \left| \left(\|\vec{\omega}\| - k_{q\omega} \|\vec{q}\| \right) \right| \operatorname{sign}\left(\vec{s}\right) \quad (18)$$

where k_{qw} is a positive constant. As seen from (18), the designed control signal is defined as the desired one. Because only the component of (18) that is parallel to the sliding vector, $\vec{s}$, leads the states to the sliding manifold, only that component should be produced by the control law. So,

$$\vec{u}_{des\_ps} = \frac{\left(\vec{u}_{des} \bullet \vec{s}\right)}{s^2}\vec{s} \qquad (19)$$

has to be evaluated by the attitude control computer before the signal is fed to the actuators. In addition, (19) should be manipulated so that the resultant magnetic control moment vector, $\vec{M}$, produced by three magnetic torquers is equal to its maximum value when $\vec{s}$ is perpendicular to the local geomagnetic field vector, $\vec{B}$, and equal to zero when $\vec{s}$ is parallel to $\vec{B}$. This final manipulation can be realized by

$$\vec{M} = \frac{\vec{B} \times \vec{u}_{des\_ps}}{B^2} \qquad (20)$$

which prevents the actuators from producing unnecessary excessive control outputs in cases when $\vec{s}$ approaches $\vec{B}$. That is related with the fact that a magnetic attitude control system can produce a resultant control torque vector, $\vec{T}_{mc}$, only in the plane perpendicular to $\vec{B}$ according to the following relation

$$\vec{T}_{mc} = \vec{M} \times \vec{B} \qquad (21)$$

As a result, $\vec{T}_m$ can be written as

$$\vec{T}_{mc} = \left[\frac{\vec{B} \times \left(\frac{\left(\vec{u}_{des} \bullet \vec{s}\right)}{s^2}\vec{s}\right)}{B^2}\right] \times \vec{B} \qquad (22)$$

which will take its final form if (18) is substituted into (22). The detailed explanation of the logic in manipulations (19) and (20) is available in [2] or [28].

3.2 Reachability Analysis

In [2], the asymptotical stability of the motion on the sliding manifold is proven by using the kinematic relations (2). This means that the trajectory in the state space remains on the manifold and tends to the equilibrium (zero) state once the manifold is reached by the states. In the same reference, it is also theoretically shown that the trajectory in the state space converges to the sliding manifold, $\vec{s}$, in an asymptotical manner while $\vec{s}$ converges to the zero vector if the reaching law is asymptotically stable or the following inequality (reachability condition) holds [27,30,31].

$$\vec{s} \bullet \dot{\vec{s}} < 0 \qquad (23)$$

Therefore, a stability analysis is required to learn if (23) is satisfied or not.

The positive definite Lyapunov function candidate (24) is suggested as

$$V = \frac{1}{2}\left[\vec{s} \bullet \left(I\vec{s}\right)\right] \qquad (24)$$

The inertia matrix I is positive definite by definition. The time derivative of V is

$$\dot{V} = \vec{s} \bullet \left(I\dot{\vec{s}}\right) \qquad (25)$$

Since $\vec{u}_{eq}$ is derived for the sliding mode conditions (8), and because (12) is valid instead of (8) in the reaching mode, the equivalent control torque can be assumed to be zero. Then, by using (13) and (22) together with the vector algebra property,

$$\vec{A} \bullet \left(\vec{B} \times \vec{C}\right) = \vec{C} \bullet \left(\vec{A} \times \vec{B}\right) = \vec{B} \bullet \left(\vec{C} \times \vec{A}\right) \qquad (26)$$

the following equation can be obtained

$$\dot{V} = \vec{s} \bullet \left(\left[\frac{\vec{B} \times \left(\frac{\left(\vec{u}_{des} \bullet \vec{s}\right)}{s^2}\vec{s}\right)}{B^2}\right] \times \vec{B}\right) = \frac{1}{B^2 s^2}\left\{\left[\vec{B} \times \left(\left(\vec{u}_{des} \bullet \vec{s}\right)\vec{s}\right)\right] \bullet \left[\vec{B} \times \vec{s}\right]\right\} \qquad (27)$$

Substituting (18) for $\vec{u}_{eq}$ = 0 into (27) gives

$$\dot{V} = -\frac{k_s}{B^2 s^2}\left|\left(\|\vec{\omega}\| - k_{q\omega}\|\vec{q}\|\right)\right| \cdot \left\{\left[\vec{B} \times \left(\left(\text{sign}\left(\vec{s}\right) \bullet \vec{s}\right)\vec{s}\right)\right] \bullet \left[\vec{B} \times \vec{s}\right]\right\} = -\frac{k_s\left[\text{sign}\left(\vec{s}\right) \bullet \vec{s}\right]}{B^2 s^2}\left|\left(\|\vec{\omega}\| - k_{q\omega}\|\vec{q}\|\right)\right| \cdot \left\{\left[\vec{B} \times \vec{s}\right] \bullet \left[\vec{B} \times \vec{s}\right]\right\} \qquad (28)$$

Since

$$\left[\operatorname{sign}(\vec{s})\right]\bullet\vec{s} > 0 \tag{29}$$

and

$$\left|\left(\|\vec{\omega}\| - k_{q\omega}\|\vec{q}\|\right)\right| > 0 \tag{30}$$

$\dot{V}$ is negative semidefinite due to the fact that it becomes zero also when $\vec{B}$ and $\vec{s}$ are parallel. It is obvious from (19) and (20) that $\vec{M}$ becomes zero in this case. As mentioned in [2], it can be concluded that in any case other than this case, the total rotational energy of the satellite will decrease to zero by referring to the Krasovskii-LaSalle theorem, and this case is not permanent because $\vec{B}$ rotates in the reference systems *A* and *B* whatever the angular state of the satellite in space is. Based on this intuitive conclusion, it has been shown that the reachability condition is satisfied.

3.3 Stability Analysis

It can be shown by using the Lyapunov's direct method adopted to satellite attitude dynamics that the attitude motion on or nearby the sliding manifold is asymptotically stable to the equilibrium at the origin of the state space [2]. First, a positive definite Lyapunov candidate function is selected as follows

$$V = \vec{q}\bullet\vec{q} + (1-q_4)^2 \tag{31}$$

By using the kinematic equations (2) and the sliding manifold equation (6), its time derivative is derived as

$$\dot{V} = \vec{q}\bullet\vec{\omega} = \vec{q}\bullet\left(\vec{s} - K_q\vec{q}\right) \tag{32}$$

It can be concluded that $\dot{V}$ is bounded from above as

$$\dot{V} \le \|\vec{q}\|\|\vec{s}\| - \lambda_q\|\vec{q}\|^2 \tag{33}$$

where λ_q is the minimum singular value of the matrix K_q. If both sides of (33) are integrated,

$$V(t) - V(t_0) \le -\lambda_q\int_{t_0}^{t}\|\vec{q}\|^2\,dt + \int_{t_0}^{t}\|\vec{q}\|\|\vec{s}\|\,dt \tag{34}$$

is obtained, which can be manipulated as

$$\lambda_q\int_{t_0}^{t}\|\vec{q}\|^2\,dt + -\int_{t_0}^{t}\|\vec{q}\|\|\vec{s}\|\,dt - V(t_0) \le 0 \tag{35}$$

by regarding the positive definitiveness of *V*. The following inequality can be derived by using Hölder's inequality and the definition of L_2-norm [2]

$$\lambda_q\|\vec{q}\|_2^{\,2} - \|\vec{q}\|_2\|\vec{s}\|_2 - V(t_0) \le 0 \tag{36}$$

According to the quadratic formula,

$$\|\vec{q}\|_2 \le \frac{\|\vec{s}\|_2 + \sqrt{\|\vec{s}\|_2^{\,2} + 4\lambda_q V(t_0)}}{2\lambda_q} \tag{37}$$

can be written. Since the reachability condition is satisfied as shown in the previous subsection, it is guaranteed that $\vec{s}$ converges to zero. According to (37), $\vec{q}$ also converges to zero. Thus the attitude motion on or nearby the sliding manifold is asymptotically stable to the origin.

4 Simulation and Comparison Results

The equations of motion (1) and (2) are simulated for the control laws (16), (17), and (18) in Matlab/Simulink (see Fig.A1 and Fig.A2) by using the principal moments of inertia given in (38)

$$I_1 = 1.1\ \text{kgm}^2\ ;\ I_2 = 1.0\ \text{kgm}^2\ ;\ I_3 = 1.2\ \text{kgm}^2 \tag{38}$$

which indicates that the satellite model is gravity-gradiently unstable. The satellite's circular orbit has an altitude of 740 km, for which

$$\begin{aligned} &n = 1.05141\times10^{-3}\ \text{rad/s} \\ &\quad\Rightarrow\ T = 2\pi/n = 5976\ \text{s} = 99.6\ \text{min} \end{aligned} \tag{39}$$

where *T* is the orbital period, and

$$A_d = 3.5\times10^{-9}\ \text{Nm} \tag{40}$$

The initial conditions are input to the simulation environment as Euler angles and their rates

$$\begin{aligned} &\left[\phi(0)\quad \dot{\phi}(0)\quad \theta(0)\quad \dot{\theta}(0)\quad \psi(0)\quad \dot{\psi}(0)\right]^T \\ &\quad = \left[160^\circ\quad 0^\circ/\text{s}\quad -80^\circ\quad 0^\circ/\text{s}\quad 160^\circ\quad 0^\circ/\text{s}\right]^T \end{aligned} \tag{41}$$

Through simulation trials, the best value of the sliding manifold design parameter is determined as

$$k_q = 0.00125 \text{ rad/s} \quad (42)$$

The reaching law design parameter is taken as

$$k_s\big|_{cont.} = 0.003 \text{ Nms/rad} \quad (43)$$

for the controller (17) with continuous reaching law and

$$k_s\big|_{discont.} = 3\times10^{-7} \text{ Nm} \quad (44)$$

for the controller (16) with classical discontinuous reaching law for the best performance. The value of $k_s\big|_{discont.}$ given in (44) satisfies the necessary condition for disturbance rejection [26], which is

$$k_s\big|_{discont.} > \max\left(T_d\big|_i\right) = 4A_d \quad ; \; i = 1,2,3 \quad (45)$$

Finally, the two constants of the designed controller (18) with modified discontinuous reaching law have the following values

$$k_s = 0.003 \text{ Nms/rad}$$
$$k_{qw} = 0.00175 \text{ rad/s} \quad (46)$$

The values of the design parameters k_s and k_{qw} seen in (46) have to satisfy the following necessary condition for disturbance rejection

$$k_s\left|\left(\|\vec{\omega}\| - k_{q\omega}\|\vec{q}\|\right)\right| > 4A_d \quad (47)$$

which leads to the condition

$$\left|\left(\|\vec{\omega}\| - k_{q\omega}\|\vec{q}\|\right)\right| > 4.67\times10^{-6} \text{ rad/s} \quad (48)$$

To make the comparison possible, time responses of Euler angles and angular velocities obtained by (16), (17), and (18) are presented in Fig.2-4 and Fig.5-7, respectively.

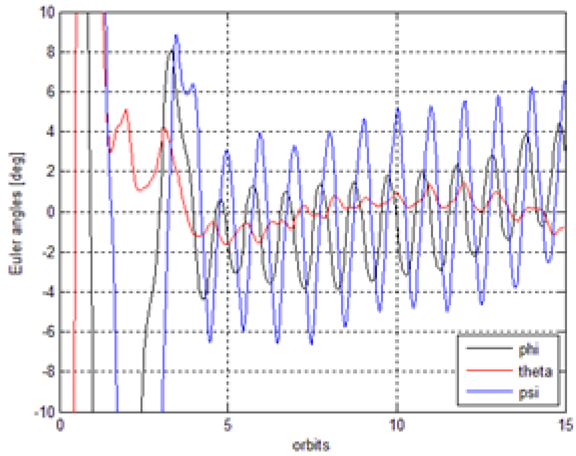

Fig.2 Time responses of Euler angles by the control law (16)

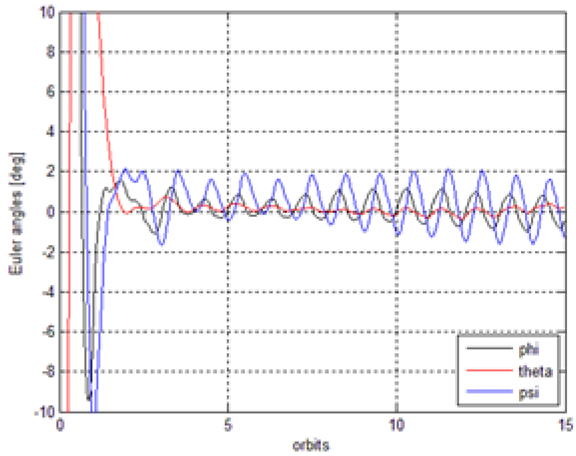

Fig.3 Time responses of Euler angles by the control law (17)

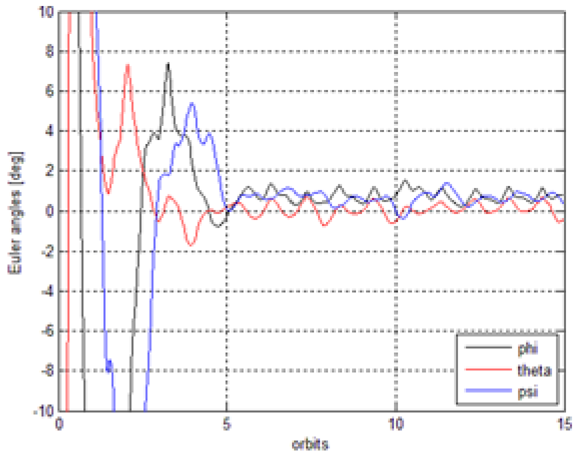

Fig.4 Time responses of Euler angles by the proposed control law (18)

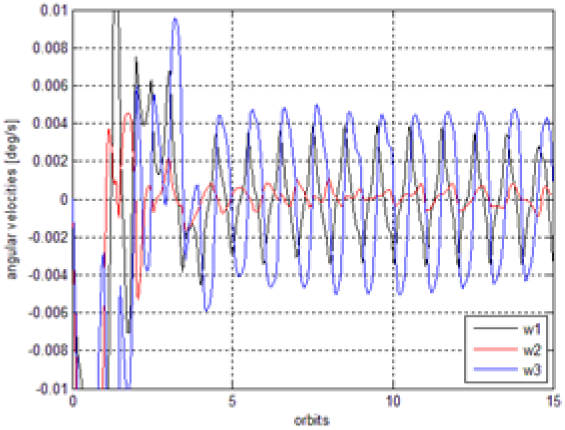

Fig.5 Time responses of angular velocities by the control law (16)

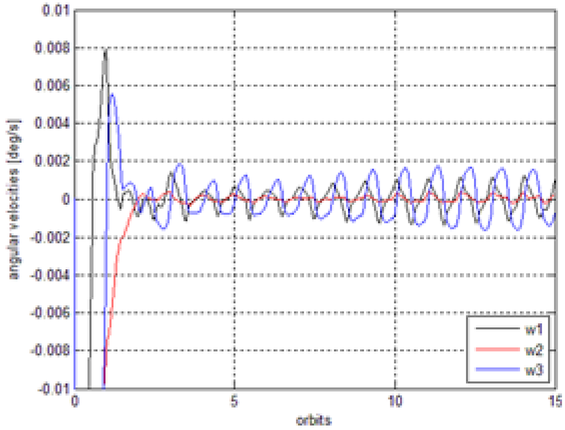

Fig.6 Time responses of angular velocities by the control law (17)

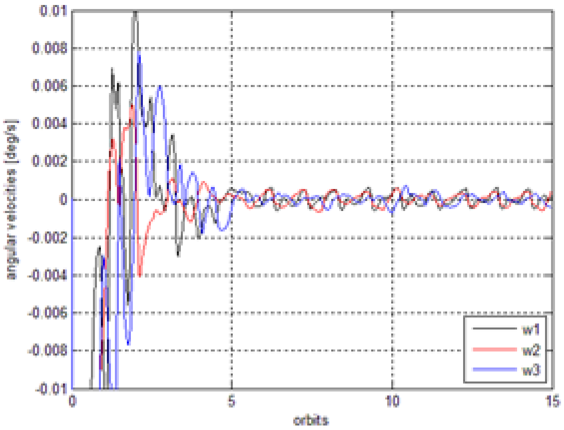

Fig.7 Time responses of angular velocities by the proposed control law (18)

The superiority of the designed controller in terms of steady regime performance can be clearly seen if Fig.4 and Fig.7 are compared with Fig.3 and Fig.6, respectively. In the following six figures, time responses of quaternions and magnetic control moment components are compared.

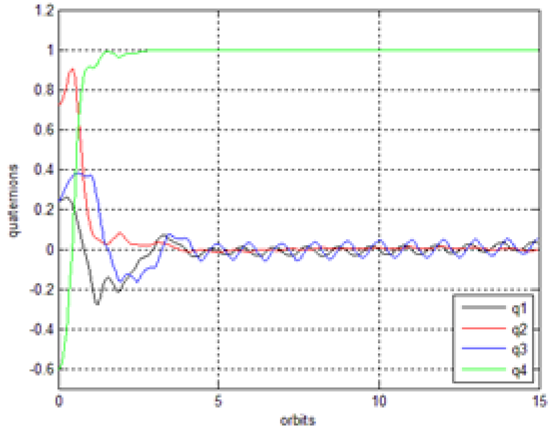

Fig.8 Time responses of quaternions by the control law (16)

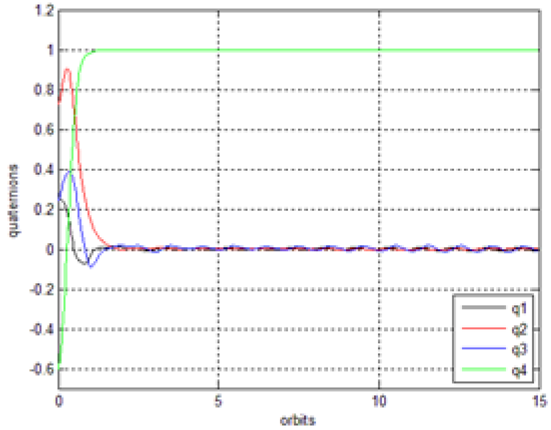

Fig.9 Time responses of quaternions by the control law (17)

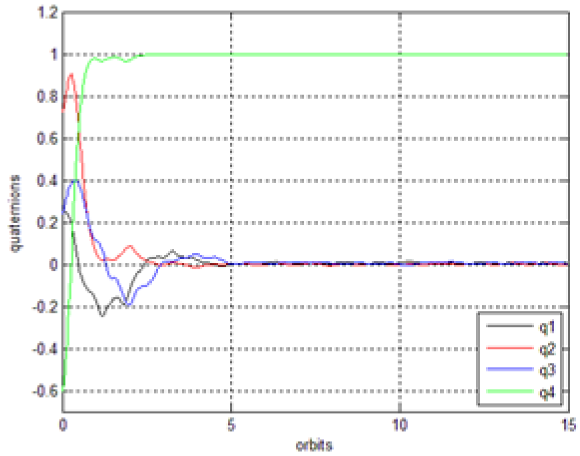

Fig.10 Time responses of quaternions by the proposed control law (18)

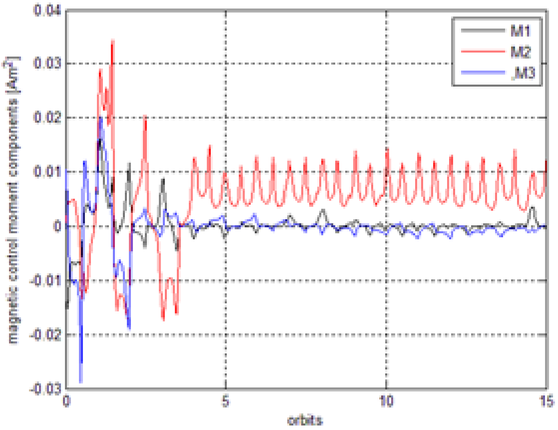

Fig.11 Time responses of magnetic control moment components by the control law (16)

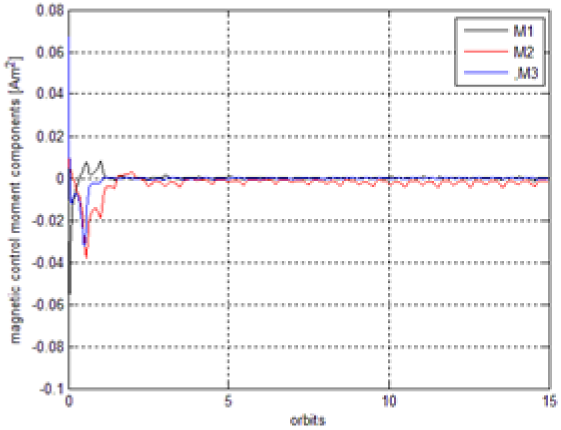

Fig.12 Time responses of magnetic control moment components by the control law (17)

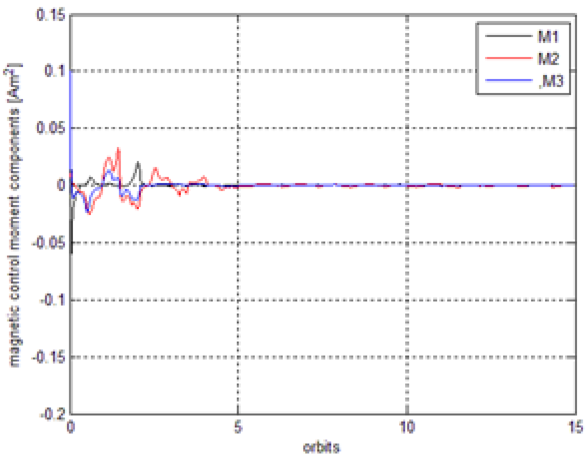

Fig.13 Time responses of magnetic control moment components by the proposed control law (18)

A saturation value of 1 Am² is accepted for the magnetic torquers that are suitable for small satellites. None of the magnetic control moment components produced by the proposed controller exceeds that threshold according to Fig.13 even if they are higher than the moment outputs by the laws (16) and (17). The overall control effort can be minimized by using approaches similar to the one proposed in [32]. Variations of sliding vector's magnitude, which converges to a steady state value different than zero, by three controllers are depicted in Fig.14-16.

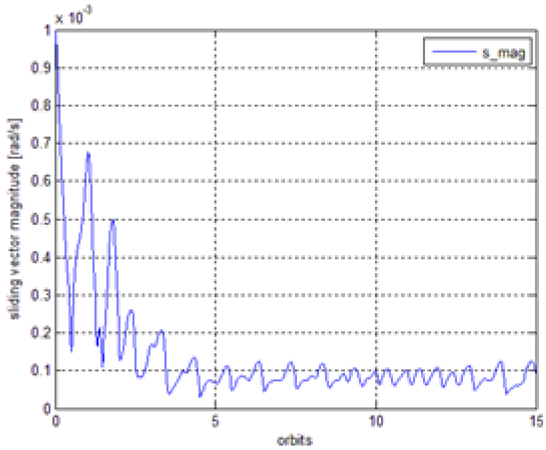

Fig.14 Time response of sliding vector's magnitude by the control law (16)

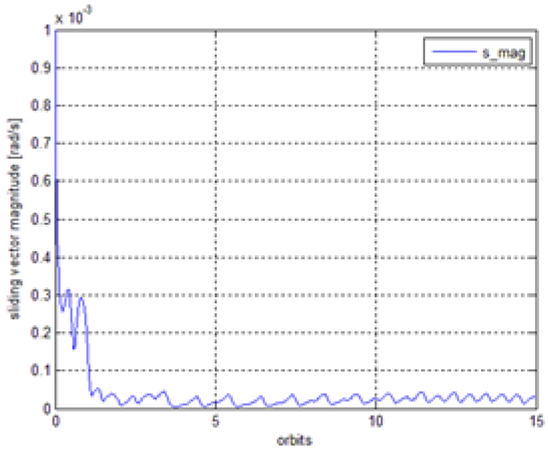

Fig.15 Time response of sliding vector's magnitude by the control law (17)

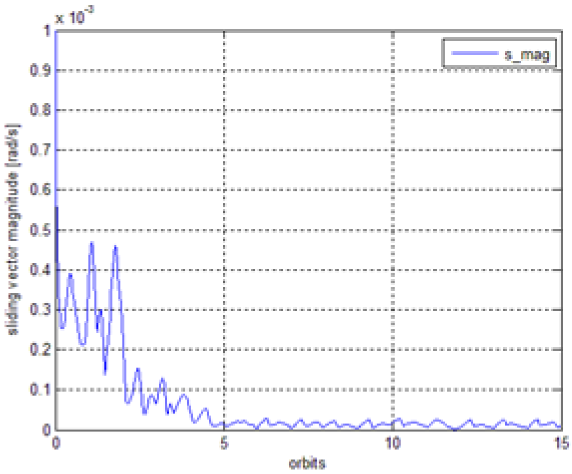

Fig.16 Time response of sliding vector's magnitude by the proposed control law (18)

Finally, time responses of Euler angles by (17) and (18) under five times higher disturbance effect, which is

$$5\times A_d = 5\times 3.5\times 10^{-9}\ \text{Nm} = 1.75\times 10^{-8}\ \text{Nm} \quad (49)$$

are shown in the following two figures, respectively.

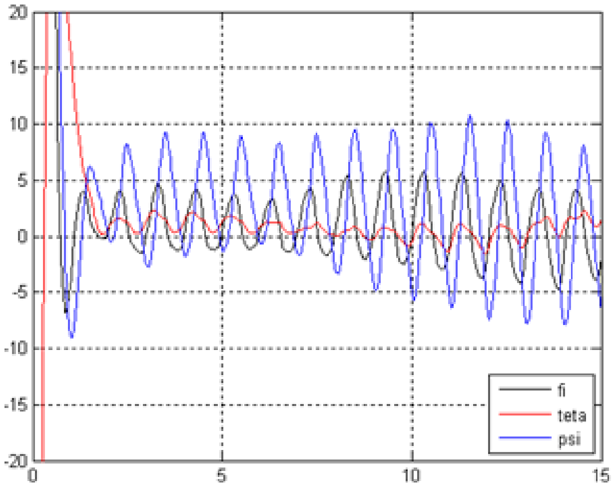

Fig.17 Time responses of Euler angles in degrees by the control law (17) subject to $5\times A_d$ along 15 orbits

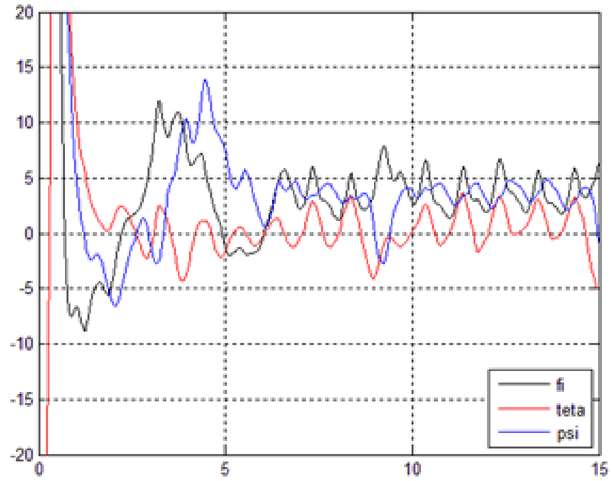

Fig.18 Time responses of Euler angles in degrees by the proposed control law (18) subject to $5\times A_d$ along 15 orbits

The condition (48) should be rewritten for this case as

$$\left|\left(\|\vec{\omega}\| - k_{q\omega}\|\vec{q}\|\right)\right| > 2.33\times 10^{-5}\ \text{rad/s} \quad (50)$$

When compared to Fig.3, the attitude angles' responses obtained by the proposed controller indicate a weaker transient regime performance. However, if the simulation is started with initial conditions that are closer to the equilibrium than the conditions (41) such as

$$\begin{bmatrix}\phi(0) & \dot{\phi}(0) & \theta(0) & \dot{\theta}(0) & \psi(0) & \dot{\psi}(0)\end{bmatrix}^T = \begin{bmatrix}60^\circ & 0^\circ/\text{s} & 40^\circ & 0^\circ/\text{s} & 30^\circ & 0^\circ/\text{s}\end{bmatrix}^T \quad (51)$$

then the controller (18) seems to perform superior to the controller (17) in both steady state and transient regime, which can be observed by comparing Fig.20 with Fig.19.

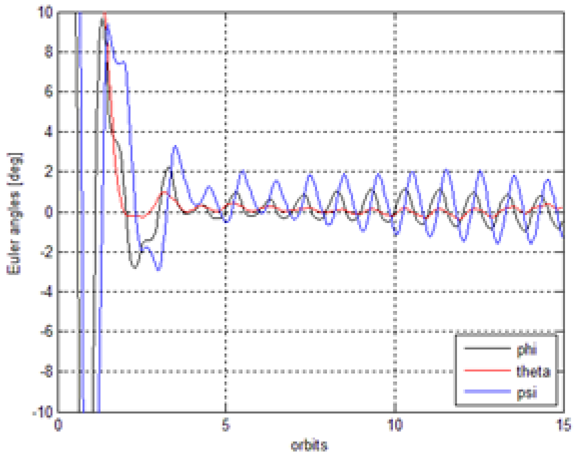

Fig.19 Time responses of Euler angles by the control law (17) for lower initial conditions

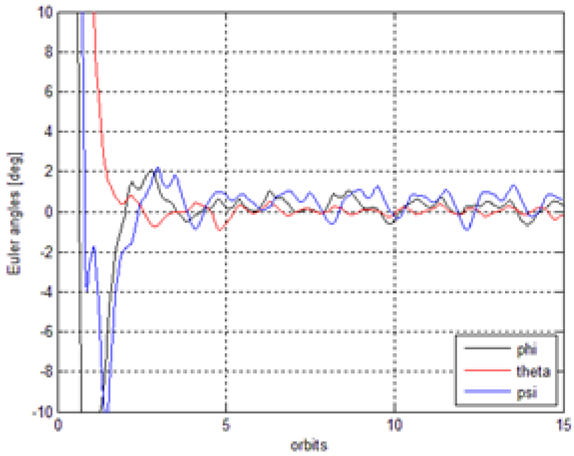

Fig.20 Time responses of Euler angles by the proposed control law (18) for lower initial conditions

5 Conclusion

The simulation results verify the theoretical result that the designed classical (16) and modified (18) sliding mode control algorithms stabilize the attitude asymptotically. The implementation of the classical sliding controller by simply amplifying the signum function of the sliding vector leads to unacceptable steady state errors that are about twice the errors that the continuous sliding controller (17) gives (see Fig.2 and Fig.3). That is why the modification of the classical discontinuous reaching law given in (13) is necessary. According to the comparison of Fig.4 with Fig.3, the aim seems to be reached by the proposed controller (18) because the steady state error margin is made narrower as between -1 and +1.5 degrees whereas the margin achieved by the continuous controller (17) is between -1.5 and +2 degrees.

If the amplitude of the harmonic disturbance function increases at a different altitude, the steady state performance of the proposed controller becomes significantly superior. Under disturbance torque components with a five times higher amplitude, Euler angle responses by (18) remain between -5 and +8 degrees while (17) can keep the responses only between -8 and +11 degrees.

The oscillating character of state responses emerges from the necessary manipulation of $\vec{u}_{des}$ according to (19) and (20). Currently, different manipulation approaches that will eliminate the oscillations are tried to be found. A different sliding manifold design may also help to remove the residual oscillations in system responses. If such approaches work, the complete rejection of disturbance effect can be accomplished, and as a result, the sliding vector's magnitude will converge to zero.

Responses of magnetic control moment components indicate that the modified control law (18) requires more control effort especially in the beginning of the simulation interval where the states are far from zero.

The drawback of the designed modified sliding mode controller (18) seems to be its low transient regime performance according to the comparison of Fig.4 with Fig.3. The same weakness is valid for the classical sliding mode controller (16). However, if initial conditions that are less far from the equilibrium state are used, the simulation results depict that the proposed controller performs better than the controller (17) in both steady state and transient regime. Moreover, to obtain a response at least as fast as the continuous one for any initial conditions, the time derivative of the sliding vector can be positively fed back to the control system, which is already accomplished in [33] by the authors of this paper.

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Appendix:

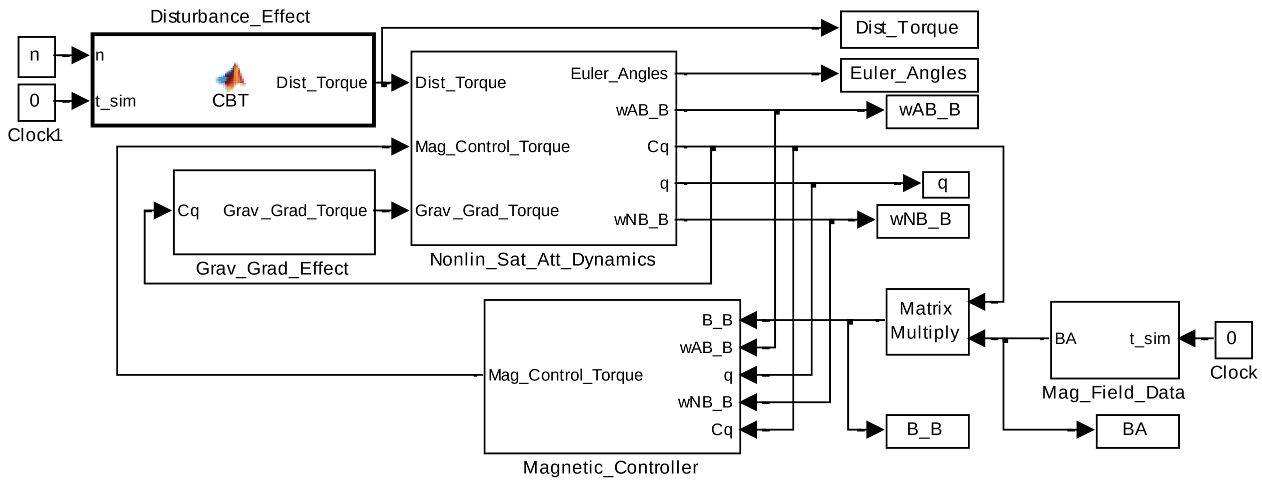

Fig.A1 Matlab/Simulink Model of the Magnetic Attitude Control System

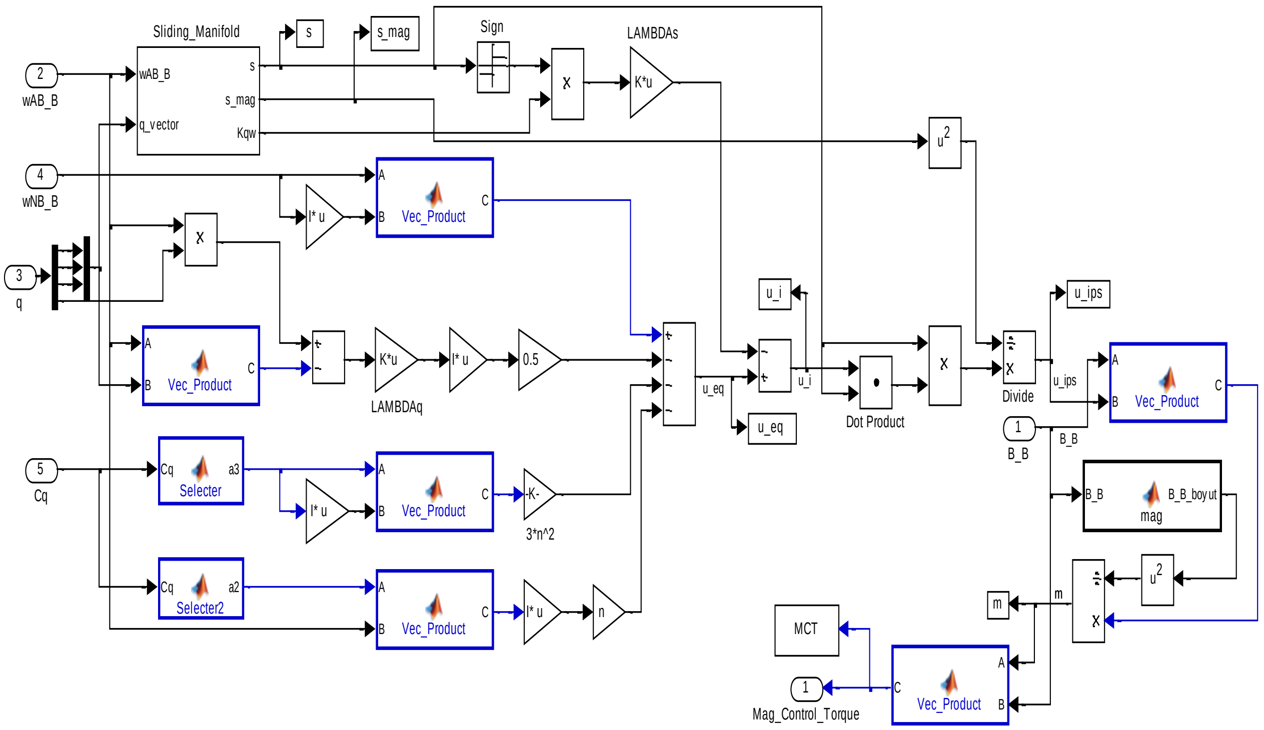

Fig.A2 Matlab/Simulink Block for the Proposed Sliding Mode Magnetic Attitude Controller