Structure and Extrapolation of a Complex Error Function of Control System with Feedback.

Oleg N. Agamalov
Electrical Department
Tashlyk Pumped-Storage Power Plant
st. Drugby Narodov 33/152, Yushnoukrainsk, Mykolayv region, 55002
UKRAINE
aon@ukrsat.mk.ua, o_agamalov@sunpp.atom.gov.ua

Abstract: - in paper considered a new type of the feedback, based on structure and, subsequent, an extrapolation of a complex error function that used for design of a control system

Key-Words: - Hilbert transform; structure of a complex error function; extrapolation of a complex error function; Wirtinger derivative

1 Introduction

Now, in the theory and practice of the control systems, applied in various branches of an economy, a science and a society, the basic role played the feedback systems. The feedback (FB) usually realized on a plant output or it state variables, which are accessible to measurement or which values can be estimated used the various state observers, simulated processes or plant [1-6]. The next synthesis of control as in simple systems, similar to PID-controllers [7-8], or in the complex systems, similar to systems with variable structure (SVS) [9], linear quadratic regulator (LQR) [1-3], H^2 or H^∞ control [10] and another, based on calculation a linear scalar error between reference (needed value) and measured (simulated) output or state variable of a plant. Thus, the control algorithms with FB realized in the form: simple calculation of a scalar linear error with next it transformation (sometimes complex) for achievement the needed purposes of a control. For many input many output (MIMO) systems used the similar algorithm with the set scalar one-dimensional control errors (the given set named as a vector of an error). A calculated control error is linear as a usual scalar difference, that causes application the non-simple control algorithms for plant or process (especially for nonlinear and with delay). The given control algorithms based on preliminary identifications of a plant in it working point or in it linear vicinity. Therefore, for nonlinear plants, application of traditional FB, demands their exact identification and the next application of complex control laws.

Therefore for control engineering, design of a new nonlinear feedback [9], which structure reflects as internal properties of a plant and also the features of it operating mode it is important. That will allow to weakening the requirements to accuracy of identification and aprioristic information about plant. According to the given thesis in paper the control problem solving by increase a dimension of a control error. The increase of a dimension control error allows defining it internal structure. Parameters of it internal structure should reflect the properties of a plant (delays, constant time, order, etc.) and to be considered as state parameters of a plant.

Guided by the above-stated, in paper considered the definition of a structure and extrapolation of a complex error function for control of a one-dimensional non-stationary nonlinear plant with delay, so as the given case is a key for understanding a control for MIMO plant and a discrete control system with use a complex error function.

For this purpose, the complex plane C is entered. The origin and a positive direction of a real axis thus complex plane C is defined by reference or input $\bar{r}$ of a control system. The positive direction of an imaginary axis of a complex plane C allows defining an orientation of a measured plant output $\bar{y}$ with delay. Thus, the structure of a bidimentional control error [11-13] for one-dimensional non-stationary nonlinear plant with delay, as a complex difference between input (reference) $\bar{r}$ and an output $\bar{y}$, is considered. For MIMO plant the set of the similar complex planes, defined in coordinates "input-output" can be considered.

Paper includes sections with the following maintenance. In the second section definition of a structure complex error function for control system is shown. The measurement method of complex error function based on representation the inputs-outputs of plant in the form of analytical signals and usage of Hilbert transform for calculation the differences of their instant phases is considered. In the third section the model of control system using as state variables the parameters of the structure a complex error function is presented. Synthesis of control system feedback by extrapolation a complex error function in point of initial stationary mode (an equilibrium point) by a power series, which gains are defined using a Wirtinger derivative for not analytical function of a complex error, is considered. Examples of application are resulted in the fourth section. In conclusions the maintenance of given paper is briefly reflected.

2 Complex Error Function Structure.

Description of a plant and it control system depends at a choice of the state variables and a coordinate system in which they are considered. If to observe an output of plant through some time intervals Δt it is possible to receive some information concerning all states. As FB defined by interaction the elements of control system there is a phase (time) delay between it input, state variables and an output. That allows defining a complex error function of a control system and it structure:

Definition 1. The difference between an input (reference) $\bar{r}$ and an output $\bar{y}$ of a control system, presented on a complex plane C is a ***complex error function*** $\bar{e}$ of a control. For complex plane C, it real axis R is co-directed with an input (reference) vector $\bar{r}$, and imaginary axis I is orthogonal to axis R, that allows to consider phase (time) delay of an output vector $\bar{y}$. This definition of a bidimentional structure of a complex error function for a one-dimensional non-stationary plants with delay in a vector and in a complex forms is: $\bar{e} = \bar{r} - \bar{y} = e_R + je_I$, where $j = \sqrt{-1}$ - imaginary unit, e_R and e_I - real and imaginary components of a structure complex error function in the Cartesian coordinate system. Phase delay $\Delta\varphi$, real e_R and imaginary e_I components of a complex error function in the Cartesian coordinate system, and also the module e and argument δ of a complex error function in a polar coordinate system, define it structure according to fig. 1:

$$\bar{e} = \bar{y} - \bar{r} = e_R + je_I, e_R = r - y\cdot\cos(\Delta\varphi) = e\cdot\cos\delta;$$
$$e_I = y\cdot\sin(\Delta\varphi) = e\cdot\sin\delta, e^2 = r^2 + y^2 - 2\cdot r\cdot y\cdot\cos(\Delta\varphi),$$
$$\Delta\varphi = arctg\left(\frac{e_I}{r+e_R}\right) = \arcsin\left(\frac{e_I}{y}\right),$$
$$\delta = \arcsin\left[\frac{y\cdot\sin(\Delta\varphi)}{\sqrt{r^2+y^2-2\cdot r\cdot y\cdot\cos(\Delta\varphi)}}\right]; \tag{1}$$

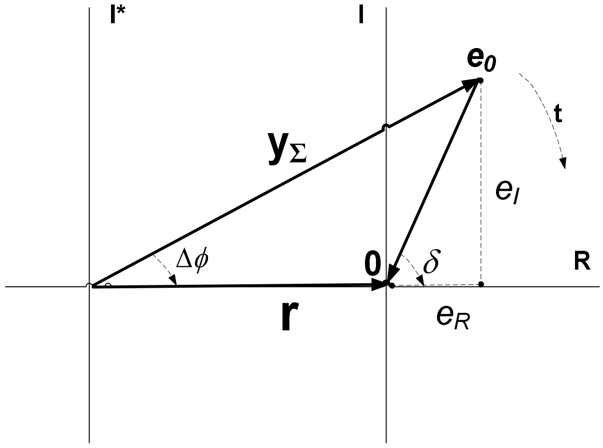


Fig.1. Structure of a complex error function.

where δ - argument of a complex error function in the polar coordinate system $\bar{e} = e\exp(j\delta)$. Phase delay $\Delta\varphi$ is calculated by means of the Hilbert transform (HT) [14-15], defined any signals input $\bar{r}$ and output $\bar{y}$ as analytical signals. Analytical signal $Z_s(t)$ represents the sum of two orthogonal signals, and for which the instant phase and frequency can be certain [15]. Imaginary part of an analytical signal $\operatorname{Im} Z_s(t) = \tilde{s}(t)$ is analytically conjugated with it real part $\operatorname{Re} Z_s(t) = s(t)$ through HT: $\tilde{s}(t) = HT[s(t)]$. An analytical signal:

$$Z_s(t) = s(t) + j \cdot \tilde{s}(t) = S(t) e^{ja(t)},$$
$$S(t) = \sqrt{s^2(t) + \tilde{s}^2(t)}, a(t) = arctg \frac{\tilde{s}(t)}{s(t)} \qquad (2)$$

The difference of instant phases for two any signals $s_1(t), s_2(t)$ can be defined as [16]:

$$\Delta\varphi_{12} = arctg \frac{\tilde{s}_1(t) \cdot s_2(t) - s_1(t) \cdot \tilde{s}_2(t)}{s_1(t) \cdot s_2(t) + \tilde{s}_1(t) \cdot \tilde{s}_2(t)} \qquad (3)$$

Accordingly, for single input, single output (SISO), non-stationary plant with delay:

$$\Delta\varphi_{yr} = arctg \frac{\tilde{y}(t) \cdot r(t) - y(t) \cdot \tilde{r}(t)}{y(t) \cdot r(t) + \tilde{y}(t) \cdot \tilde{r}(t)} \qquad (4)$$

For many input, many output (MIMO), non-stationary plant with delay:

$$\Delta\varphi_{mn} = arctg \frac{\tilde{y}_n(t) \cdot r_m(t) - y_n(t) \cdot \tilde{r}_m(t)}{y_n(t) \cdot r_m(t) + \tilde{y}_n(t) \cdot \tilde{r}_m(t)} \qquad (5)$$

where *m* – input, *n* – output of control system.

The set of all radius-vector of a complex error function $\bar{e}$ forms a phase space. We shall define it properties:

1. Approachability - an opportunity movement a control system at control $u^*(t)$ from a current state point of a plant, described a complex error function $\bar{e}_0$, to the area (point), defining the necessary state of a plant $\bar{\varepsilon}(0)$, where $\bar{\varepsilon}$ - the vicinity of an equilibrium point 0 in phase space;
2. Observability - for each value of a complex error function $\bar{e}_i$ there is control u_i at which the state $\bar{\varepsilon}(0)$ is reached, that allows to distinguish the given point in phase space of a complex error function between all nearest points $\bar{e}_j$.

Using the above-stated properties, the control system for plant with phase delay and extrapolation of a complex error function in a point of the initial stationary mode (an equilibrium point) by a power series, which gains are defined as Wirtinger derivatives for not analytical function of a complex error function, further is considered.

3 Complex Error Function Extrapolation.

We shall consider the control system with phase delays defined by integrators of a state variable and an output of a plant, fig. 2:

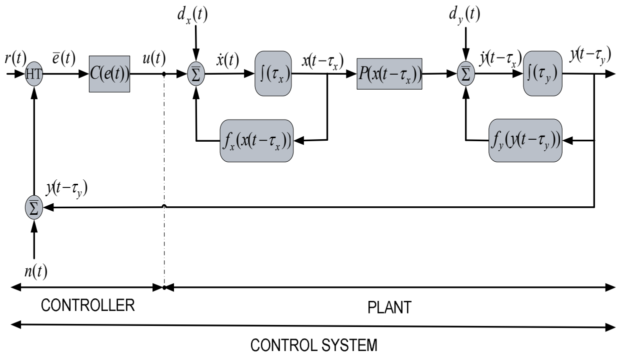


Fig. 2. Block diagram of a control system.

were: $r(t)$ - reference (input), $\bar{e}(t)$ - complex error function, $u(t)$ - control input, $\dot{x}(t)$ - derivative of a state variable, $x(t-\tau_x)$ - state variable with delay τ_x, $\dot{y}(t-\tau_x)$ - derivative of a plant output with delay τ_x, $y(t-\tau_y)$ - plant output with full delay τ_y, $P(x(t-\tau_x))$ - map of a state variable to a derivative of an output $x(t-\tau_x) \to \dot{y}(t-\tau_x)$, $f_x(x(t-\tau_x))$ - feedback between a late state variable $x(t-\tau_x)$ and it derivative $\dot{x}(t-\tau_e)$, defined by the structure and parameters of the plant and disturbances on state variable, $f_y(y(t-\tau_y))$ - feedback between a plant output $y(t-\tau_y)$ and it derivative $\dot{y}(t-\tau_x)$, defined by the structure and parameters of the plant, output disturbances and noises, $\int(\tau_x)$ - integrator of a state variable derivative $\dot{x}(t)$ with delay τ_x, $\int(\tau_y)$ - integrator of an output derivative $\dot{y}(t-\tau_x)$ with delay $\tau_y - \tau_x$, Σ - adder, HT - Hilbert transform for an input $r(t)$ and a late output $y(t-\tau_y)$, $C(e(t))$ - map of controller $\bar{e}(t) \to u(t)$, $d_x(t)$ - state variable disturbances of a plant, $d_y(t)$ - output disturbances of a plant, $n(t)$ - noises of a plant output, $\tau_y > \tau_x$ - parities between delays in a control system. Then model of a control system for one-dimensional plant with phase delay can be written as a system of the algebraic and differential equations with late argument [17]:

1. The equation of a complex error function for a plant with phase delay:

$$\begin{aligned} e(t) &= HT[r(t) - (y(t-\tau_y) + n(t))] = \\ &= r(t) - y(t)e^{-j(\Delta\varphi)} = e_R + je_I = e\angle(-\Delta\varphi) \end{aligned},$$

where

$$\tau_y = \frac{\Delta\varphi}{\Delta\dot{\varphi}}, y(t-\tau_y) = y(t)e^{-j(\Delta\varphi)}$$

2. The equation of a controller:

$$u(t) = C(e(t)) = C(r(t) - y(t)e^{-j(\Delta\varphi)});$$

3. The equation of a plant state:

$$\begin{aligned} &\dot{x}(t) = f_x(x(t-\tau_x)) + u(t) + d_x(t) = \\ &f_x(x(t-\tau_x)) + C(r(t) - y(t)e^{-j(\Delta\varphi)}) + d_x(t) \end{aligned};$$

4. The equation of plant output:

$$\dot{y}(t-\tau_x) = f_y(y(t-\tau_y)) + P(x(t-\tau_x)) + d_y(t). \quad (9)$$

So as, we consider a complex error function of a complex argument (a complex output of a plant):

$$\begin{aligned} &y = y_R + iy_I = y\cos(\Delta\varphi) + iy\sin(\Delta\varphi), \\ &e = f(r,y) = e_R + je_I = \\ &= r - y\cos(\Delta\varphi) + jy\sin(\Delta\varphi), 0 < t < T \end{aligned} \quad (10)$$

For a complex error function (1) we shall check up the feasibility of the Cauchy-Riemann conditions [18] defining of it differentiation on a complex output of a plant:

$$\begin{aligned} &\frac{\partial e_R(y_R,y_I)}{\partial y_R} = \frac{\partial e_I(y_R,y_I)}{\partial y_I} \rightarrow \\ &\frac{\partial(r - y(\cos(\Delta\varphi))}{\partial(y\cos(\Delta\varphi)} = -1 \neq 1 = \frac{\partial(y\sin(\Delta\varphi))}{\partial(y\sin(\Delta\varphi))} \end{aligned} \quad (11)$$

$$\begin{aligned} &\frac{\partial e_R(y_R,y_I)}{\partial y_I} = -\frac{\partial e_I(y_R,y_I)}{\partial y_R} \rightarrow \\ &\frac{\partial(r - y(\cos(\Delta\varphi))}{\partial(y\sin(\Delta\varphi))} = tg(\Delta\varphi) \neq -ctg(\Delta\varphi) = \frac{\partial(y\sin(\Delta\varphi))}{\partial(y\cos(\Delta\varphi))} \end{aligned}$$

The equations (11) shows, that for a complex error function Cauchy-Riemann conditions are not carried out, and their derivatives possesses property of "anisotropic" at change of the real and imaginary components of a complex output of a plant and depends on the conjugate complex output of a plant $\widetilde{y} = y_R - iy_I = y\cos(\Delta\varphi) - jy\sin(\Delta\varphi)$:

$$\frac{\partial e}{\partial \widetilde{y}} \neq 0 \quad (12)$$

For calculation of the complex error function derivatives, we shall enter the definition of a real differentiated [19-20] complex error function:

Definition 2: The complex error function $e(y) = e_R(y_R,y_I) + je_I(y_R,y_I)$ is a **real differentiated** if it real and imaginary components $e_R(y_R,y_I), e_I(y_R,y_I)$ defined as a differentiated functions of real variables $y_R = \frac{y+\widetilde{y}}{2}$ and $y_I = \frac{y-\widetilde{y}}{2j}$ where $y = y\cos(\Delta\varphi) + jy\sin(\Delta\varphi)$ - a complex output, and $\widetilde{y} = y\cos(\Delta\varphi) - iy\sin(\Delta\varphi)$ - a conjugate complex output of a plant, and so:

$$e(y) = e_R\left(\frac{y+\widetilde{y}}{2}, \frac{y-\widetilde{y}}{2j}\right) + je_I\left(\frac{y+\widetilde{y}}{2}, \frac{y-\widetilde{y}}{2j}\right).$$

Thus a complex error function of a control system (9) is considered as map $e: R \times R = R^2 \mapsto C$ and can use properties of a real area R^2 for the proof of the theorem [21]:

THEOREM 1: Let control error function of a control system $e: R \times R = R^2 \mapsto C$ is defined by real variables $y_R = \frac{y+\widetilde{y}}{2}$ and $y_I = \frac{y-\widetilde{y}}{2j}$ so, that $e = f(y_R,y_I) = f(y,\widetilde{y})$, where $y = y\cos(\Delta\varphi) + jy\sin(\Delta\varphi) = ye^{j(\Delta\varphi)}$ and $\widetilde{y} = y\cos(\Delta\varphi) - jy\sin(\Delta\varphi) = ye^{-j(\Delta\varphi)}$ complex and conjugate complex, independent from each other outputs of control system. Then:

1) Can be defined the Wirtinger derivatives [20] of a complex error function:

$$\frac{de(y)}{dy}=\frac{1}{2}\left(\frac{\partial e(y)}{\partial y_R}-j\frac{\partial e(y)}{\partial y_I}\right);$$
$$\frac{de(y)}{d\tilde{y}}=\frac{1}{2}\left(\frac{\partial e(y)}{\partial y_R}+j\frac{\partial e(y)}{\partial y_I}\right) \tag{13}$$

2) Necessary and sufficient conditions of an equilibrium point $(\bar{r}_0,\bar{y}_0,\bar{e}_0)$ of a complex error function it is defined by equalities:

$$\frac{de(r_0,y_0,\tilde{y}_0)}{dy}=0\ ,\ \frac{de(r_0,y_o,\tilde{y}_0)}{d\tilde{y}}=0 \tag{14}$$

Proof: Using definition of a real differentiated complex error function and a chain rule, we shall receive:

$$\frac{de(y)}{dy}=\frac{\partial e(y)}{\partial y_R}\frac{\partial y_R}{\partial y}+\frac{\partial e(y)}{\partial y_I}\frac{\partial y_I}{\partial y}=$$
$$=\frac{1}{2}\left(\frac{\partial e(y)}{\partial y_R}-j\frac{\partial e(y)}{\partial y_I}\right);$$
$$\frac{de(y)}{d\tilde{y}}=\frac{\partial e(y)}{\partial y_R}\frac{\partial y_R}{\partial \tilde{y}}+\frac{\partial e(y)}{\partial y_I}\frac{\partial y_I}{\partial \tilde{y}}=$$
$$=\frac{1}{2}\left(\frac{\partial e(y)}{\partial y_R}+j\frac{\partial e(y)}{\partial y_I}\right)$$

The key moment at this theorem is independence of the complex and the conjugate complex outputs of a plant that is carried out for all real plants.

In an equilibrium point $(\bar{r}_0,\bar{y}_0,\bar{e}_0=0)$ we define a decomposition of a complex error function in a power series, used for extrapolation of the future control error at change reference (input) or a complex output of a plant and the subsequent formation of a controller feedback, in view, that for an equilibrium point $y_0=r_0$. As the space C of a control error function has corresponding structure of a real space R^2, $C\cong R^2$ we shall define following equivalence for a complex output of a plant:

$$(y=y_R+jy_I)\in C\leftrightarrow Y=\begin{pmatrix}y_R\\ y_I\end{pmatrix}\in R^2 \tag{15}$$

where Y - real vector of components of a complex output of a plant. Also, for complex outputs (15) the set of the conjugated vectors can be certain:

$$c=\begin{pmatrix}y\\ \tilde{y}\end{pmatrix}=\begin{pmatrix}y_R+jy_I\\ y_R-jy_I\end{pmatrix}\in C^2\cong R^4 \tag{16}$$

Thus, we have three vector spaces of coordinates for representation of a complex output of a plant:

A) Complex vectors $y\in C$;

B) Real vectors of a component of a complex output $Y\in R^2$;

C) Complex or real vectors defined through complex and complex-conjugated outputs of a plant $c\in C^2\cong R^4$.

For decomposition the complex error function in the real power series it is necessary to define isomorphism between the vector spaces (15), (16). Components of the complex and the complex-conjugated outputs of a plant are connected by the matrix equation:

$$\begin{pmatrix}y\\ \tilde{y}\end{pmatrix}=\begin{pmatrix}1 & j\\ 1 & -j\end{pmatrix}\begin{pmatrix}y_R\\ y_I\end{pmatrix}=J\begin{pmatrix}y_R\\ y_I\end{pmatrix},J\equiv\begin{pmatrix}1 & j\\ 1 & -j\end{pmatrix}$$

or $c=JY$ (17)

The inverse matrix of coordinate transform:

$$J^{-1}=\frac{1}{2}\begin{pmatrix}1 & 1\\ j^{-1} & -j^{-1}\end{pmatrix}\rightarrow Y=J^{-1}c \tag{18}$$

Then for the given coordinate transform the Jacobians can be defined:

$$J_c\equiv\frac{\partial}{\partial Y}c=\frac{\partial}{\partial Y}JY=J,J_Y=J_c^{-1}=J^{-1} \tag{19}$$

It allows defining differentials of a complex output of a plant in various coordinate systems:

$$dc=\frac{\partial c}{\partial Y}dY=J_c dY=JdY,$$
$$dY=\frac{\partial Y}{\partial c}dc=J_Y dc=J^{-1}dc \tag{20}$$

Presented (17) through components of a complex output:

$$\begin{pmatrix}\Delta y\\ \Delta\tilde{y}\end{pmatrix}=\begin{pmatrix}1 & j\\ 1 & -j\end{pmatrix}\begin{pmatrix}\Delta y_R\\ \Delta y_I\end{pmatrix}=\begin{pmatrix}1 & j\\ 1 & -j\end{pmatrix}\begin{pmatrix}\Delta(y\cos(\Delta\varphi))\\ \Delta(y\sin(\Delta\varphi))\end{pmatrix}$$

$$\begin{pmatrix}\Delta y_R\\ \Delta y_I\end{pmatrix}=\frac{1}{2}\begin{pmatrix}1 & 1\\ j^{-1} & -j^{-1}\end{pmatrix}\begin{pmatrix}\Delta y\\ \Delta\tilde{y}\end{pmatrix} \qquad (21)$$

The vector $Y=\begin{pmatrix}y_R\\ y_I\end{pmatrix}\in R^2$ is real and is a complex-conjugate to itself $\tilde{Y}=Y$. The complex-conjugated output of a plant is a *nonlinear map* to a complex plane C: $y=y_R+jy_I\rightarrow\tilde{y}=y_R+j(-y_I)$, however at representation on a real plane R^2 is *linear map*:

$$Y=\begin{pmatrix}y_R\\ y_I\end{pmatrix}\rightarrow\tilde{Y}\equiv\begin{pmatrix}y_R\\ -y_I\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}y_R\\ y_I\end{pmatrix}=\tilde{E}Y \qquad (22)$$

where $\tilde{E}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ - a conjugate matrix, for which $\tilde{E}=\tilde{E}^T=\tilde{E}^{-1}$. Therefore for a vector $c\in C^2\cong R^4$:

$$c=JY\leftrightarrow\tilde{c}=J\tilde{Y} \qquad (23)$$

Thus, the complex output of a plant y can be presented as vectors Y or c, and the complex-conjugated output $\tilde{y}$ as vectors $\tilde{Y}$ or $\tilde{c}$. Thus to nonlinear operation of complex conjugate on a complex plane C corresponds the linear operator $\tilde{E}$ on an isomorphic real plane R^2. Then real scalar complex error function can be considered as function of a real vector Y, a real or complex vector c, or a complex output y: $e=f(Y)=f(c)=f(y)$ and decomposition in a series at all given arguments.

Then real scalar complex error function can be considered as function of a real vector Y, a real or complex vector c, or a complex output y: $e=f(Y)=f(c)=f(y)$ and decomposition in a series at all given arguments. As by definition: $\frac{\partial}{\partial c}=\begin{pmatrix}\frac{\partial}{\partial y} & \frac{\partial}{\partial\tilde{y}}\end{pmatrix}^T,\Delta c=\begin{pmatrix}\Delta y\\ \Delta\tilde{y}\end{pmatrix}$, for real scalar function of a complex error it is received:

$$\begin{aligned}&\frac{\partial f(c)}{\partial c}\Delta c=\frac{\partial f}{\partial y}\Delta y+\frac{\partial f}{\partial\tilde{y}}\Delta\tilde{y}=\\ &=\frac{\partial f}{\partial y}\Delta y+\overline{\frac{\partial f}{\partial y}\Delta y}=2\operatorname{Re}\left\{\frac{\partial f}{\partial y}\Delta y\right\}\end{aligned} \qquad (24)$$

Thus, for complex error function of control system for various representation of a complex output of a plant it is received following values of a linear part at decomposition in a series:

$$\frac{\partial f}{\partial Y}\Delta Y=\frac{\partial f}{\partial c}\Delta c=2\operatorname{Re}\left\{\frac{\partial f}{\partial y}\Delta y\right\} \qquad (25)$$

where the first expression is the real derivative, the second derivative can be interpreted as a real derivative and in the third expression defined a complex derivative for which it real part is used. For the second member of a series it gain is Hessian:

$$\begin{aligned}&H_{yy}\equiv\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right),H_{\tilde{y}y}\equiv\frac{\partial}{\partial\tilde{y}}\left(\frac{\partial f}{\partial y}\right),\\ &H_{y\tilde{y}}\equiv\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial\tilde{y}}\right),H_{\tilde{y}\tilde{y}}\equiv\frac{\partial}{\partial\tilde{y}}\left(\frac{\partial f}{\partial\tilde{y}}\right)\end{aligned} \qquad (26)$$

Using transform (15) - (26) we shall consider extrapolation of a real scalar complex error function by a power series, as really differentiated function of a real variable $r\in R$ and a real vector $Y\in R^2$:

$$\begin{aligned}&e=f(r_0+\Delta r,y_0+\Delta y)=f(r_{0,}y_0)+\\ &+2\operatorname{Re}\left\{\frac{\partial f(r)}{\partial r}(r-r_0)\right\}+2\operatorname{Re}\left\{\frac{\partial f(y)}{\partial y}(y-y_0)\right\}+\\ &+\operatorname{Re}\left\{H_{rr}(r-r_0)^2+H_{r\tilde{r}}(r-r_0)(\tilde{r}-\tilde{r}_0)\right\}+\\ &\operatorname{Re}\left\{H_{yy}(y-y_0)^2+H_{y\tilde{y}}(y-y_0)(\tilde{y}-y_0)\right\}+h.o.t\end{aligned} \qquad (27)$$

Let's transform equality (27), using (1) for extrapolation of a complex error function $e=r-y\cos(\Delta\varphi)+jy\sin(\Delta\varphi)$ for complex $y=y\cos(\Delta\varphi)+jy\sin(\Delta\varphi)$ and complex-conjugated $\tilde{y}=y\cos(\Delta\varphi)-jy\sin(\Delta\varphi)$ outputs of a plant and accepting for an equilibrium point $y_0=r$, then:

$$\begin{aligned}&\Delta y=y-r=y\cos(\Delta\varphi)+jy\sin(\Delta\varphi)-r,\\ &\Delta\tilde{y}=\tilde{y}-r=y\cos(\Delta\varphi)-jy\sin(\Delta\varphi)-r\end{aligned} \qquad (28)$$

Then, the linear extrapolation of a complex error function of a control system:

$$
\begin{aligned}
&e = f(r_0 + \Delta r, y_0 + \Delta y) \approx \\
&f(r_{0,} y_0) + 2\,\mathrm{Re}\left\{\frac{\partial f(r)}{\partial r}(r - r_0)\right\} + \\
&+ 2\,\mathrm{Re}\left\{\frac{df(y)}{dy}(y - y_0)\right\} = \\
&= 2(r - r_0) + (ctg(\Delta\varphi) + tg(\Delta\varphi))\sin(\Delta\varphi)\,y = \\
&2(r - r_0) + y\sec(\Delta\varphi) = 2(r - r_0) + \frac{2\cos(\Delta\varphi)}{\cos(2\Delta\varphi) + 1}\,y = \\
&= 2\left(\frac{\cos(\Delta\varphi)}{\cos(2\Delta\varphi) + 1}\,y + (r - r_0)\right) \qquad (29)
\end{aligned}
$$

Further example of application the control systems, used a linear extrapolation of a complex error function are considered.

4. Application the extrapolation of a complex error function for control system

As example for application of extrapolation a complex error function, used the complex power system (file *MATLAB* “power_PSS.mdl”) [22]. The simulated system consists of two fully symmetrical areas linked together by two *230 kV* lines of *220 km* length. It was specifically designed in [23] to study low frequency electromechanical oscillations in large interconnected power systems. Each area is equipped with two identical round rotor generators rated *20 kV/900 MVA*. The synchronous machines have identical parameters, except for inertias which are *H = 6.5s* in area *1* and *H = 6.175s* in area *2*. Thermal plants having identical speed regulators are further assumed at all locations, in addition to static exciters with an automatic voltage regulator (AVR) that extrapolated the complex error function (22). The load is represented as constant impedances and split between the areas in such a way that area 1 is exporting *413MW* to area 2. Since the surge impedance loading of a single line is about *140 MW*, the system is somewhat stressed, even in steady-state. Transients in a power system are simulated at submission a step signal with value 5 % on an input of a reference at terminal voltage of the first turbogenerator by duration of 12 periods (60 Hz) and occurrence a three-phase short circuit on a ground on line duration up to 0.5 s. The circuit of a model is shown on fig. 3:

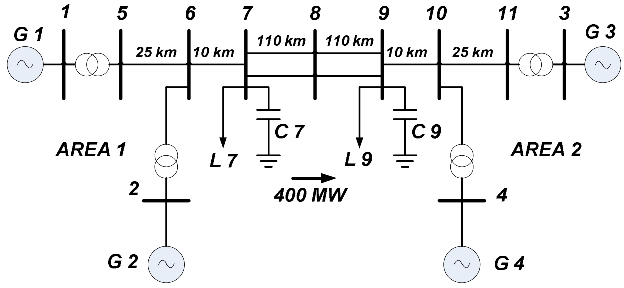


Fig. 3. The simulated model.

The synchronous generators was simulated by system of the differential and algebraic equations in the Park coordinate system [23], considering dynamics of the windings stator, rotor and dampers, linking a voltage V_i, a current i_i, a flux ψ_i, an active resistance R_i and an inductance L_i for *i-th* contour at neglect the leakage inductance:

$$
\begin{aligned}
&STATOR: \\
&V_d = R_s i_d + \frac{d\psi_d}{dt} - \omega_R \psi_q, V_q = R_s i_q + \frac{d\psi_q}{dt} + \omega_R \psi_d, \\
&ROTOR: V_{fd} = R_{fd} i_{fd} + \frac{d\psi_{fd}}{dt}, \\
&DAMPERS: \\
&V_{kd} = R_{kd} i_{kd} + \frac{d\psi_{kd}}{dt}, V_{kq1} = R_{kq1} i_{kq1} + \frac{d\psi_{kq1}}{dt}, \\
&V_{kq2} = R_{kq2} i_{kq2} + \frac{d\psi_{kq2}}{dt}, \\
&\qquad\qquad\qquad\qquad\qquad\qquad (30) \\
&FLUXES: \\
&\psi_d = L_d i_d + L_{md}(i_{fd} + i_{kd}), \psi_q = L_q i_q + L_{mq} i_{kq}, \\
&\psi_{fd} = L_{fd} i_{fd} + L_{md}(i_d + i_{kd}), \\
&\psi_{kd} = L_{kd} i_{kd} + L_{md}(i_d + i_{fd}), \\
&\psi_{kq1} = L_{kq1} i_{kq1} + L_{mq} i_q, \psi_{kq2} = L_{kq2} i_{kq2} + L_{mq} i_q
\end{aligned}
$$

where indexes are: d, q - parameters on axes *d* and *q* of Park coordinate systems, s, f, k - parameters of the windings stator, rotor and dampers, m - parameters of a magnetizing inductance. Movement of rotors of the synchronous generators was defined by the equations:

$$\omega(t) = \omega_0 + \Delta\omega(t),$$
$$\Delta\omega(t) = \frac{1}{2H}\int_0^t (P_m - P_e)dt - K_d\Delta\omega(t) \quad (31)$$

where ω_0 - rated rotor speed, $\Delta\omega(t)$ - increment of a rotor speed, $P_m = const$ - mechanical power, $P_e = \frac{V_t E}{X}\sin\delta$ - electrical power, $V_t = \sqrt{V_d^2 + V_q^2}$ - terminal voltage, that is a controlled parameter of the excitation system, E - e. m. f. of the synchronous generator, δ - rotor angle between V_t and E, X - internal inductive resistance of the synchronous generator, H - inertia constant, K_d - damper gain. The excitation system type ST1 of the synchronous generator was simulated by the standard *IEEE Std 421.5-1992* [24]. The cores elements of the excitation control system are an automatic voltage regulator (AVR) and the exciter with transfer function:

$$\frac{V_{fd}}{e_f} = \frac{1}{T_e s + K_e} \quad (32)$$

where V_{fd} - field voltage, e_f - AVR output, T_e, K_e - time constant and gain of the exciter, s - complex variable. The linear extrapolation of a complex error function (29) was used in AVR:

$$u = Ke = K \cdot 2\left(V_{ref} - \frac{\cos(\Delta\varphi)}{\cos(2\Delta\varphi)+1}V_t\right) \quad (33)$$

For measurement of phase delay $\Delta\varphi$ between controlled variable – terminal voltage V_t and it is reference V_{ref} develops block SIMULINK «delta fi », simulating the equation (4), fig. 4:

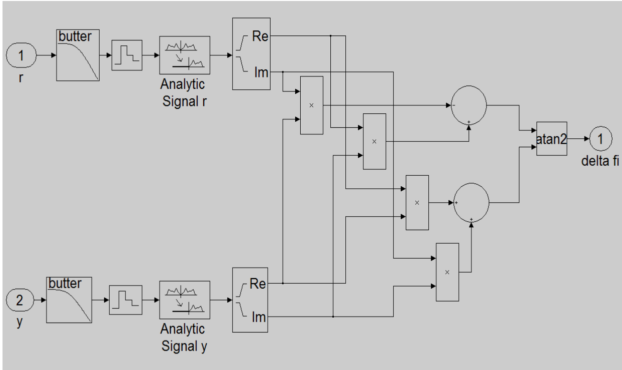


Fig. 4. Measuring instrument for phase delay $\Delta\varphi$.

Efficiency of excitation control and power swing suppression is considered for AVR (33) with accepted gain $K = 3.7$ for all turbogenerators. The results of a simulation are in the application and shown, that AVR (33) provides a steady state of a power system without use of additional stabilizing signals as PSS (power system stabilizer) and stabilizing of a terminal voltage. It is connected by that in structure of a complex error function at a terminal voltage it is considered the phase delay, that defined an adaptive gain $\frac{\cos(\Delta\varphi)}{\cos(2\Delta\varphi)+1}$.

5.Conclusion

Defined with use of a Hilbert transform the structure and extrapolation of a complex error function of a control system allows generating an adaptive feedback on a complex output of a plant. The given feedback allows defining the phase delay arising at disturbances on a plant. In paper the definition of a structure complex error function of a control system and the method of it measurement, based on representation the inputs and outputs of a plant in the form of an analytical signals is considered. Properties of a phase space of a complex error function, defined by it structure are formulated. The opportunity of extrapolation the complex error function for control of SISO and MIMO plants is certain. The model of a control system in which parameters of a structure of a complex error function are considered as it state variables, which linear extrapolation allows to generate a feedback on it predicted value is presented. The application examples show an opportunity of development the industrial controllers using for a feedback extrapolated value of a complex error function. Also it is to note, that the controller with a feedback on the extrapolated value of a complex error function adaptive, so as it feedback gain changes on-line, depending at measured delay according to (33).

References:

[1]. G. C. Goodwin, S. F. Grebe, M. E. Salgado, *Control system design*. Prentice-Hall, 2001.

[2]. R. C. Dorf, R. H. Bishop, *Modern control systems*. Prentice-Hall, 2011.

[3]. S. H. Zak, *Systems and Control*. New York: Oxford University Press. 2003.

[4]. K. J. Astrom, R. M. Murray, *Feedback Systems. An Introduction for Scientists and Engineers*. Princeton University Press. 2008.

[5]. *The Control Handbook*. Editor William S. Levine. Jaico Publishing House. 2005.

[6]. C. L. Phillips, J. Parr, *Feedback control systems*, Prentice-Hall, 2010.

[7]. K. H. Ang, G. C. Y. Chong, Y. Li, *PID Control System Analysis and Design and Technology*. IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, Vol.13, No.4, July 2005, pp. 559-576.

[8]. K. H. Ang, G. C. Y Chong, Y. Li, PID Control System Analysis and Design. Problems, remedies and future directions. IEEE CONTROL SYSTEMS MAGAZINE, February 2006, pp. 32-41.

[9]. S. V. Emelyanov, S. K. Korovin, Control of Complex and Uncertain Systems: New Types of Feedback. Springer, 2000.

[10]. J. R. Leigh, *Control theory*, The Institution of Electrical Engineer, 2004.

[11]. O. N. Agamalov, *Design of Control Systems based on Vector Error,* selected papers from WSEAS Conferences “Computers and Simulation in Modern Science, Volume V ”, pp. 176-184.

[12]. O. N. Agamalov*, Geometrical Model of Plant Presented on a State Surface of a Complex Error*, 10th International Conference on Signal Processing, Robotics and Automation (ISPRA-2011), Cambridge, UK, February 20-22, 2011.

[13]. Oleg N. Agamalov, *Geometrical Model of Control System Presented on Complex Error Surface,* Journal of Automation and Information Sciences, Vol. 43, 2011/Issue 5, pp. 23-39, Begell Digital Library.

[14]. A. D. Poularikas, The Transforms and Applications Handbook, CRC Press, 2000.

[15]. R. Rabiner, B. Gold, Theory and Application of Digital Signal Processing. Prentice-Hall, 1975.

[16]. M. Rosenblum, J. Kurths. *Analyzing Synchronization Phenomena from Bivariate Data by Means of the Hilbert Transform*, Nonlinear Analysis of Physiological Data, Springler, 1998, pp. 91-99

[17]. L. F. Shampine, I. Gladvel, S. Tomson, *Solving ODEs with MATLAB*, Cambridge University Press, 2003.

[18]. A. D. Polyanin, A. V. Manzhirov, *Handbook of mathematics for engineers and scientists,* Chapman&Hall/CRC, 2007.

[19]. R. Remmert, *Theory of Complex Functions*, Springer-Verlag, 1991.

[20]. K. Kreutz-Delgado, *The complex gradient operator and the CR-calculus*, in: Lecture Supplement ECE275A, 2006, pp. 1–74.

[21]. D. H. Brandwood, *“A complex gradient operator and its application in adaptive array theory,”* IEE Proceedings, F: Communications, Radar and Signal Processing, vol. 130, no. 1, pp. 11–16, 1983.

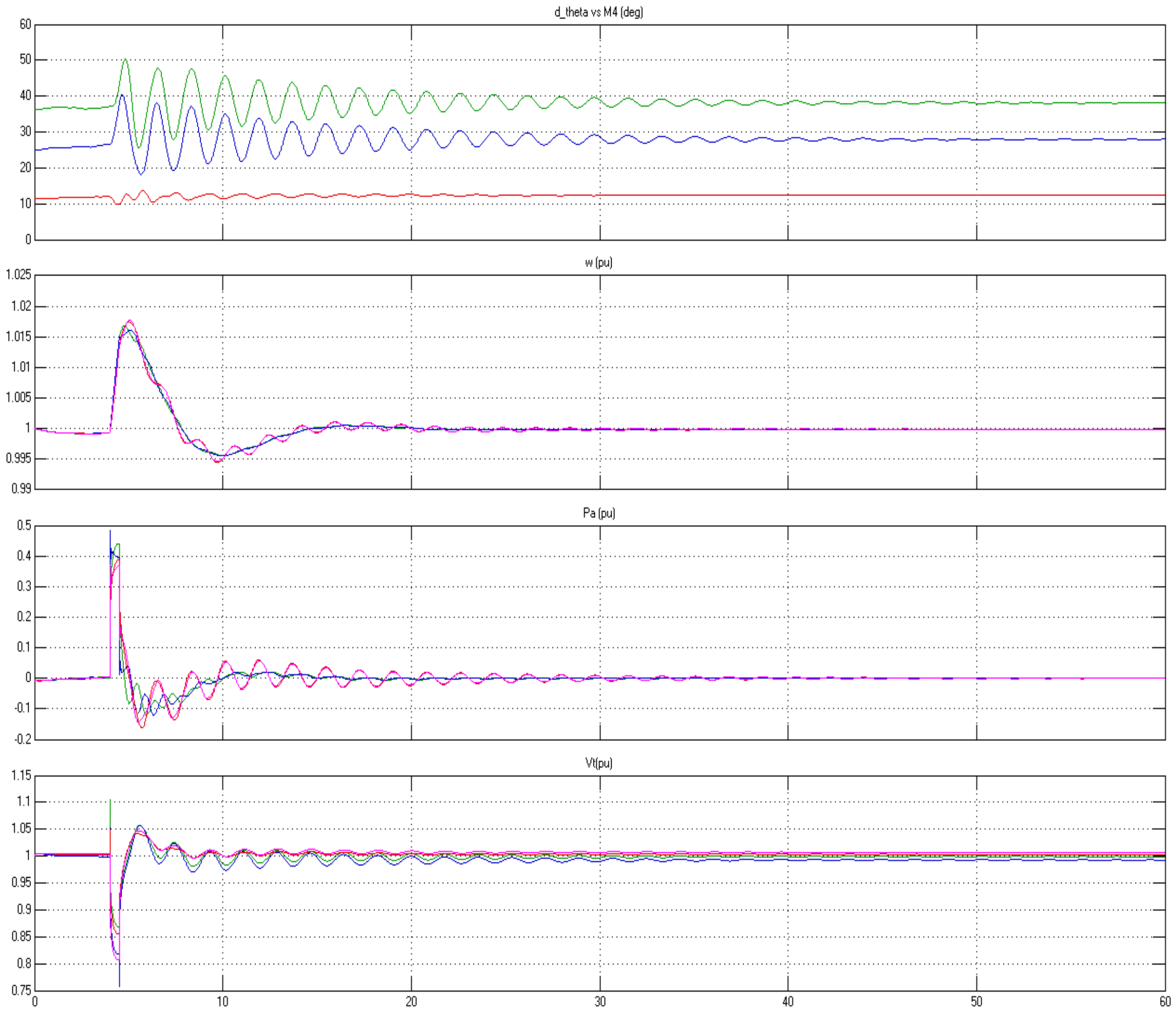
[22]. SimPowerSystems for use with Simulink. User’s guide. Version 4. 2005.

[23]. P. Kundur, Power System Stability and Control, McGraw-Hill, 1994.

[24]. *Recommended Practice for Excitation System Models for Power System Stability Studies*, IEEE Standard 421.5-1992, 1992.

Application

Transients for model of a complex power system with AVR, using extrapolation of a complex error function.



Schedule 1 – rotor angle for 1-st (blue), 2-nd (green) and 3-rd (red) turbogenerators concerning 4-th turbogenerator.

Schedule 2 – speed.

Schedule 3 – accelerating power.

Schedule 4 – terminal voltage.