Some Implications of a Scale Invariant Model of Statistical Mechanics to Boltzmann versus Shannon Entropy in Thermodynamics and Information Theory

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Abstract: - A scale invariant model of statistical mechanics is applied for a comparative study of *Boltzmann's* entropy in thermodynamics versus *Shannon's* entropy in information theory. The implications of the model to the objective versus subjective aspects of entropy as well as *Nernst-Planck* statement of the third law of thermodynamics are also discussed.

Key-Words: - Boltzmann and Shannon entropy; Information theory; Entropy; Third law of thermodynamics.
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1 Introduction

Stochastic quantum fields [1-17] and classical hydrodynamic fields [18-29] viewed as ensembles of weakly coupled oscillators resulted in the introduction of a scale-invariant model of statistical mechanics [30] and its application to the fields of thermodynamics [31], fluid mechanics [32-33], statistical mechanics [34], and quantum mechanics [35]. In the present study, some implications of the model to the physical foundation of classical and statistical thermodynamics and *Boltzmann* thermodynamic entropy versus *Shannon* information entropy as well as *Nernst-Planck* statement of the third law of thermodynamics are examined.

2 A Scale-Invariant Model of Statistical Mechanics

The scale-invariant model of statistical mechanics for equilibrium galacto-, planetary-, hydro-system-, fluid-element-, eddy-, cluster-, molecular-, atomic-, subatomic-, kromo-, and tachyon-dynamics corresponding to the scale β = g, p, h, f, e, c, m, a, s, k, and t is schematically shown in Fig. 1 [31]. Each statistical field is identified as the "system" and is composed of an ensemble of "elements" described by a distribution function $f_\beta(\mathbf{u}_\beta) = f_\beta(\mathbf{r}_\beta, \mathbf{u}_\beta, t_\beta)\, d\mathbf{r}_\beta d\mathbf{u}_\beta$. Each element is composed of an ensemble of small particles called the "atoms" of the field and are viewed as *point-mass*. The most probable element (system) velocity of the smaller scale *(j)* becomes the velocity of the atom (element) of the larger scale *(j+1)*.

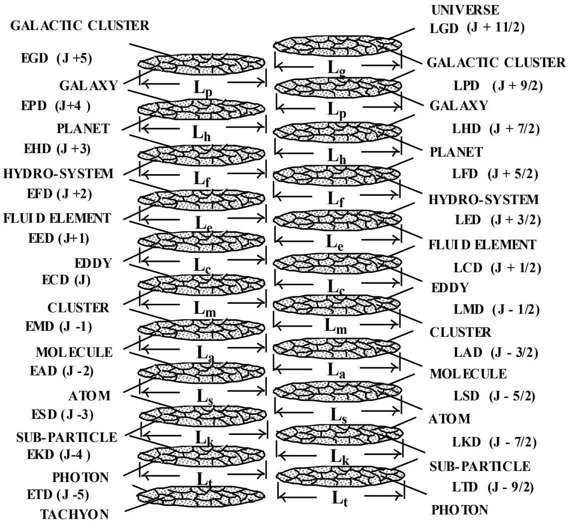

Fig. 1 A scale invariant view of statistical mechanics from cosmic to tachyon scales.

Following the classical methods [36-41] the invariant definition of density ρ_β, and velocity of *element* $\mathbf{v}_\beta$, *atom* $\mathbf{u}_\beta$, and *system* $\mathbf{w}_\beta$ at the scale β are [35, 42]

$$\rho_\beta = n_\beta m_\beta = m_\beta \int f_\beta du_\beta \quad , \quad \mathbf{u}_\beta = \mathbf{v}_{mp\beta-1} \tag{1}$$

$$\mathbf{v}_\beta = \rho_\beta^{-1} m_\beta \int \mathbf{u}_\beta f_\beta d\mathbf{u}_\beta \quad , \quad \mathbf{w}_\beta = \mathbf{v}_{mp\beta+1} \tag{2}$$

Similarly, the invariant definition of the peculiar and diffusion velocities are introduced as

$$\mathbf{V}'_{\beta} = \mathbf{u}_{\beta} - \mathbf{v}_{\beta} \quad , \quad \mathbf{V}_{\beta} = \mathbf{v}_{\beta} - \mathbf{w}_{\beta} \tag{3}$$

such that

$$\mathbf{V}_{\beta} = \mathbf{V}'_{\beta+1} \tag{4}$$

When the model is applied to social structures one arrives at the cascade shown in Fig. 2.

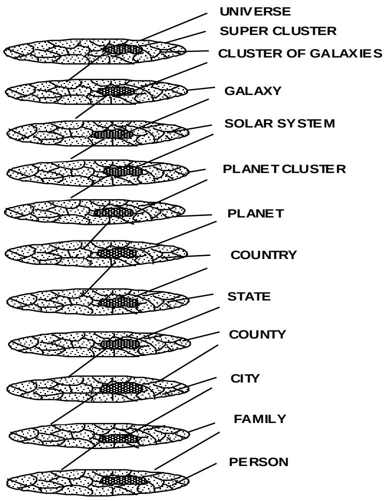

Fig. 2 Hierarchy of social structures from cosmic to individual or atomic scales.

Interestingly, as signs for capacity of number of people in elevators in Athens show, in Greek a person is referred to as "atom" that is the smallest unit of social structure shown in Fig. 2. The correspondence between statistical fields in Figs. 1-2 clearly show the physical and objective basis of information and its communication between "atoms" or individuals be it in the form of particle exchange or exchange of more complex symbols such as words or numbers. In this sense, the information theory like all other branches of science must of course be a subset of the theory of everything TOE (Fig. 1).

3 Stochastic Natures of Planck and Boltzmann Constants and De Pretto Number 8338

Because at thermodynamic equilibrium the mean velocity of each particle, *Heisenberg-Kramers* virtual oscillator [43], vanishes $\langle \mathbf{u}_{\beta} \rangle = 0$ the translational kinetic energy of particle oscillating in two directions (x+, x–) is expressed as

$$\begin{aligned}\varepsilon_{\beta} &= m_{\beta}\langle u^{2}_{\beta x+}\rangle / 2 + m_{\beta}\langle u^{2}_{\beta x-}\rangle / 2 \\ &= m_{\beta}\langle u^{2}_{\beta x+}\rangle = \bar{p}_{\beta}\langle \lambda^{2}_{\beta}\rangle^{1/2}\langle \nu^{2}_{\beta}\rangle^{1/2}\end{aligned} \tag{5}$$

where $\bar{p}_{\beta} = m_{\beta}\langle u^{2}_{\beta x+}\rangle^{1/2}$ is the root-mean-square momentum of particle and $\langle u^{2}_{\beta x+}\rangle = \langle u^{2}_{\beta x-}\rangle$ by *Boltzmann* equipartition principle. At any scale β, the result in Eq. (5) can be expressed in terms of either frequency or wavelength

$$\varepsilon_{\beta} = m_{\beta}\langle u^{2}_{\beta}\rangle = \bar{p}_{\beta}\langle \lambda^{2}_{\beta}\rangle^{1/2}\langle \nu^{2}_{\beta}\rangle^{1/2} = h_{\beta}\nu_{\beta} \tag{6a}$$

$$\varepsilon_{\beta} = m_{\beta}\langle u^{2}_{\beta}\rangle = \bar{p}_{\beta}\langle \nu^{2}_{\beta}\rangle^{1/2}\langle \lambda^{2}_{\beta}\rangle^{1/2} = k_{\beta}\lambda_{\beta} \tag{6b}$$

when the definition of stochastic *Planck* and *Boltzmann* factors are introduced as [33]

$$h_{\beta} = \bar{p}_{\beta}\langle \lambda^{2}_{\beta}\rangle^{1/2} \tag{7a}$$

$$k_{\beta} = \bar{p}_{\beta}\langle \nu^{2}_{\beta}\rangle^{1/2} \tag{7b}$$

At the important scale of EKD (Fig. 1) corresponding to *Casimir* [44] vacuum composed of photon gas, the universal constants of *Planck* [45, 46] and *Boltzmann* [31] are identified from equations (6)-(7) as

$$h = h_{k} = m_{k}c\langle \lambda^{2}_{k}\rangle^{1/2} = 6.626\times 10^{-34} \text{ J-s} \tag{8a}$$

$$k = k_{k} = m_{k}c\langle \nu^{2}_{k}\rangle^{1/2} = 1.381\times 10^{-23} \text{ J/K} \tag{8b}$$

Next, following *de Broglie* hypothesis for the wavelength of matter waves [2]

$$\lambda_{\beta} = h / \bar{p}_{\beta} \tag{9a}$$

the frequency of matter waves is defined as [31]

$$\nu_{\beta} = k / \bar{p}_{\beta} \tag{9b}$$

When matter and radiation are in the state of thermodynamic equilibrium equations (9a) and (9b) can be expressed as

$$h_{\beta} = h_{k} = h \quad , \quad k_{\beta} = k_{k} = k \tag{10}$$

The definitions in equations (8a) and (8b) result in the gravitational mass of photon [31]

$$m_{k} = (hk / c^{3})^{1/2} = 1.84278\times 10^{-41} \text{ kg} \tag{12}$$

that is much larger than the reported [47] value of 4×10^{-51} kg. The finite gravitational mass of photons was anticipated by *Newton* [48] and is in accordance with *Einstein-de Broglie* theory of light [49-53]. *Avogardo-Loschmidt* number was predicted as [31]

$$N^{o} = 1/(m_{k}c^{2}) = 6.0376\times 10^{23} \tag{13}$$

leading to the modified value of the universal gas constant

$$R^o = N^o k = 8.338 \quad \text{kJ/(kmol-K)} \qquad (14)$$

Also, by Eq. (13) the atomic mass unit becomes

$$\text{amu} = m_k c^2$$
$$= (hkc)^{1/2} = 1.6563 \times 10^{-27} \quad \text{kg/kmol} \qquad (15)$$

Since all baryonic matter is known to be composed of atoms, equations (12) and (15) suggest that all matter in the universe is composed of light [54]. From Eqs. (8a)-(8b) the wavelength and frequency of photon in vacuum $\langle\lambda_k^2\rangle^{1/2}\langle\nu_k^2\rangle^{1/2} = c$ are

$$\lambda_k = \langle\lambda_k^2\rangle^{1/2} = 1/R^o = 0.119935 \text{ m} \quad ,$$
$$\nu_k = \langle\nu_k^2\rangle^{1/2} = 2.49969 \times 10^9 \text{ Hz} \qquad (16)$$

In a recent study [35] a modified definition of thermodynamic temperature $T' = 2T$ was introduced that resulted in the modified value of *Joule-Mayer* mechanical equivalent of heat J [35]

$$J = 2J_c = 2 \times 4.169 = 8338 \quad \text{Joule/kcal} \qquad (17)$$

where the classical value $J_c = 4.169 \simeq 4.17$ [kJ/kcal] is the average of two values $J_c = (4.15, 4.19)$ reported by *Pauli* [55]. The number in Eq. (17) is thus identified as the universal gas constant in Eq. (14) when expressed in appropriate MKS system of units

$$R^o = kN^o = J = 8338 \quad \text{Joule/(kmol-K)} \qquad (18)$$

The modified value of the universal gas constant in Eq. (14) was recently identified [56] as *De Pretto* number 8338 that appeared in the mass–energy equivalence equation of *De Pretto* [57]

$$E = mc^2 \quad \text{Joule} \quad = mc^2 / 8338 \quad \text{kcal} \qquad (19)$$

Unfortunately, the name of *Olinto De Pretto* in the history of evolution of mass energy equivalence is little known. Ironically, *Einstein's* best friend *Michele Besso* was a relative and close friend of *Olinto De Pretto's* brother *Augusto De Pretto*. The relativistic form of equation (19) was first introduced in 1900 by *Poincaré* [58]

$$E = m_r c^2 \qquad (20)$$

where $m_r = m_o / \sqrt{1 - v^2 / c^2}$. Since the expression (19) is the only equation in the paper by *De Pretto* [57], the exact method by which he arrived at the number 8338 is not known even though one possible method was recently suggested [56]. The important contributions by *Hasenöhrl* [59] and *Einstein* [60] as well as the equivalence principle, equivalence of the rest or gravitational mass and the inertial mass were discussed in a recent study [54].

According to Eq. (15) mass is simply associated with numbers in harmony with the perceptions of *Sommerfeld* [61]

"Our spectral series, dominated as they are by integral quantum numbers, correspond, in a sense, to the ancient triad of the lyre, from which the Pythagoreans 2500 years ago inferred the harmony of the natural phenomena; and our quanta remind us of the role which the Pythagorean doctrine seems to have ascribed to the integers, not merely as attributes, but as the real essence of physical phenomena."

as well as the ideas of *Weyl* [62].

4. Objective versus Subjective Nature of Thermodynamic Entropy

Possible subjective versus objective nature of entropy has been subject of much debate ever since 1948 when *Shannon* [63-64] used the name entropy in his information theory. The objective nature of thermodynamic entropy is evident form its classical definition first introduced by *Clausius*

$$dS = dQ_{\text{rev}} / T \qquad (21)$$

In other words, direct connection between entropy S and heat Q clearly establishes the objective nature of the former.

The misunderstandings concerning possible subjective nature of entropy could be due to its statistical definition first introduced by *Boltzmann*

$$S = k \ln W \quad , \qquad S_j = k \ln W_j \qquad (22)$$

The total number of complexions for independent energy levels introduced by *Boltzmann* [65-67] and *Planck* [45] is

$$W = \prod_j W_j \qquad (23)$$

The number of complexions for distributing N_j indistinguishable particles among g_j distinguishable clusters or "quantum states" for level (element) j is

$$W_j = \frac{(N_j + g_j - 1)!}{N_j!\,(g_j - 1)!} \qquad (24)$$

According to the present model of statistical mechanics g_j denotes number of distinguishable clusters (elements) in a particular energy level j of the system [35]. However, the smallest cluster contains only a single particle and is therefore considered to be full since no other particle can be added to this smallest cluster. Because an empty cluster has no physical significance, the total number of *available* clusters or quantum states will be $(g_j - 1)$. Therefore, *Planck-Boltzmann* formula (24) is the exact probability of distribution of N_j indistinguishable oscillators amongst $(g_j - 1)$ distinguishable *available* clusters. The invariant model of statistical mechanics (Fig. 1) provides new perspectives on the probabilistic nature of Eq. (24) and the problem of distinguishability discussed by *Darrigol* [68].

Under the realistic assumptions

$$g_j \gg N_j \quad , \quad N_j \gg 1 \tag{26}$$

it is known that the number of complexions for *Bose-Einstein* statistics in Eq. (24) simplifies such that all three types namely *Corrected Maxwell-Boltzmann*, *Bose-Einstein*, and *Fermi-Dirac* statistics will have [69]

$$W_j = N_j^{N_j} / N_j! \quad , \quad W = N^N / N! \tag{27}$$

According to the classical methods the maximization of W in Eq. (23) will result in the *Boltzmann* distribution [42]

$$N_j = g_j e^{-\beta(\varepsilon_j - \alpha\hat{\mu}_j)} = g_j e^{-(\varepsilon_j - \alpha\hat{\mu}_j)/kT} \tag{28}$$

The coefficient α multiplying chemical potential $\hat{\mu}_j$ was shown to be unity α = 1 [42] and the partition function for normalization is defined as

$$Z = \sum g_j e^{-\beta(\varepsilon_j - \alpha\hat{\mu}_j)} \tag{29}$$

According to the information theory of *Shannon* [67-68] the amount of information H also referred to as "entropy" is expressed by *Shannon* formula

$$H_{max} = -K \sum_j p_j \ln p_j \tag{30}$$

where K is a positive constant. The above definition is based on conditional probability for occurrence of event j given a message A [69]

$$\hat{i} = K \ln[p_j(j/A)/p_j] \tag{31}$$

Only with maximum value of the conditional probability

$$p_j(j/A) = 1 \tag{32}$$

equation (31) reduces to

$$\hat{i}_{max} = -K \ln p_j \tag{33}$$

thus leading to the mean value of information entropy [69]

$$H_{max} = \sum_j \hat{i}_{max} p_j = -K \sum_j p_j \ln p_j \tag{34}$$

The thermodynamic "entropy" on the other hand is often derived on the basis of number of complexions for distribution of particles amongst various quantum states expressed as

$$W = N! / \prod_j N_j! \tag{35}$$

The W in Eq. (35) along with the definition of probability

$$p_j = N_j / N \tag{36}$$

when substituted in *Boltzmann* equation (22) lead to "entropy" [69]

$$S = Nk \ln W = -Nk \sum_j p_j \ln p_j \tag{37}$$

Clearly, Eq. (37) does not represent the true thermodynamic entropy of *Boltzmann* and the discrepancy arises from the fact that W in Eq. (35) does not correspond to that in Eq. (23).

It is desirable to avoid the above mentioned discrepancy between Eq. (37) and the true thermodynamic entropy of *Boltzmann*. In other words, one wishes to establish the exact relationship between *Shannon's* entropy of information in Eq. (30) and the true thermodynamic entropy that represents the degree of randomness of energy distributions of particles amongst particle clusters, *de Broglie* wave packets, in statistical fields as measured by the number of complexions according to *Boltzmann* equation (23). Therefore, rather than the conventional definition in Eq. (36) the thermodynamic probability p_j is defined as

$$p_j = W_j / W \tag{38}$$

Next, since *Boltzmann* entropy represents a measure of uncertainty or randomness from Eq. (22) and in harmony with ideas of *Brillouin* [70] the difference between maximum entropy and entropy of state j is defined as a measure of information that by Eq. (38) gives

$$I_{jmax} = S_{max} - S_j = -k \ln p_j \tag{39}$$

Since reduction of uncertainty from its maximum value to that of state j is equivalent to an increase of certainty it constitutes information as schematically shown in Fig. 3.

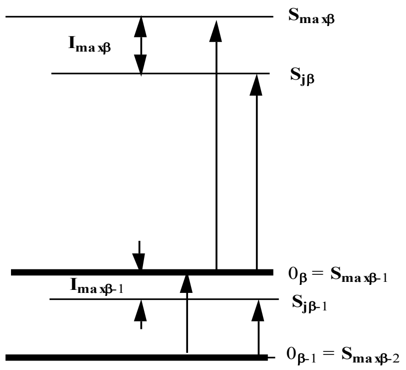

Fig. 3 Scale-invariant definition of thermodynamic information defined as reduction of absolute uncertainty or entropy $I_{max} = S_{max\beta} - S_{j\beta}$.

Hence, parallel to Eq. (34) the mean value of thermodynamic information becomes

$$I_{max} = \sum_j I_{jmax} p_j = -k \sum_j p_j \ln p_j \qquad (40)$$

The equations (30) and (40) now only differ by the universal *Boltzmann* constant k.

The occurrence of the universal constant k has fundamental significance. According to equations (7)-(8) *Planck* and *Boltzmann* constants are two fundamental constants of nature that relate to spatial and temporal aspect of *Casimir* [44] vacuum fluctuations. Because all conceivable information must be transmitted by some physical entity, such as electron, photon, or neutrino, etc. in space-time, it is reasonable to expect that both constants (h, k) will play a central role is transmission of information. Therefore, it is not advisable to modify thermodynamic entropy in Eq. (40) to achieve correspondence with Eq. (30). Instead one can simply define dimensionless *Boltzmann* thermodynamic information entropy as

$$\tilde{H}_{max} = I_{max} / k = -\sum_j p_j \ln p_j \qquad (41)$$

in exact correspondence with dimensionless *Shannon* information entropy in Eq. (30).

The classical definition of entropy by *Boltzmann* is based on the number of complexions according to equations (22)-(24). However, it is possible to introduce a *modified definition of thermodynamic entropy* as

$$S_j = -k \ln p_j \qquad (42)$$

that is exactly equivalent to thermodynamic entropy of *Boltzmann* described in an earlier study [42]. To arrive at the modified definition in Eq. (42) one first introduces the thermodynamic probability as

$$p_j = N_j / N \qquad (43)$$

that will be directly related to thermodynamic equilibrium distribution function of *Boltzmann* in Eq. (28) normalized as

$$N_j = Ne^{-(E_j - E)/kT} = Ne^{-H_j/kT} \qquad (44)$$

The simplification in Eq. (44) is due to the vanishing of chemical potential $\hat{\mu}_j = 0$ of ideal gas at equilibrium by to the equilibrium conditions $G = dG = 0$ [42]. Next, one identifies the energy (E_j – E) in Eq. (44) as the transition energy or enthalpy H_j of energy level j that for an ideal gas is given by

$$H_j = U_j + p_j V = 3N_j kT + N_j kT = 4N_j kT \qquad (45)$$

Because *Boltzmann* distribution in Eq. (28) is obtained from maximization of W_j one can relate entropy to the probability p_j based on *Boltzmann* distribution obtained from equations (43) - (45) as

$$p_j = N_j / N = e^{-H_j/kT} = e^{-4N_j} \qquad (46)$$

Finally, by substitution from Eq. (46) into Eq. (42) one obtains

$$S_j = 4N_j k \qquad (47)$$

in exact agreement with the result obtained based the number of complexions [42]. Therefore, the reason for the validity of equivalence of equations (22) and (42) is that equations (43) and (46) are connected to thermodynamic equilibrium through *Boltzmann* distribution in Eq. (44) that in turn was derived on the basis of maximization of W_j [42].

The probability function in Eq. (46) and its origination from the *Boltzmann* distribution function in Eq. (44) helps to clarify the nature of the connection between thermodynamic entropy of *Boltzmann* and the information entropy of *Shannon*. To better reveal this connection, one first notes that the concept of entropy according to *Boltzmann's* statistical definition in Eq. (22) concerns maximization of randomness by maximization of number of complexions W. For example, when applied to photon gas representing equilibrium black-body radiation maximization of W will correspond to having a spectrum of photon cluster sizes, *de Broglie* molecules of light, with energy distribution given by *Planck* energy spectrum [35, 42]. It is noted that the entropy of equilibrium photon gas given by Eq. (47) involves number N_j of *Planck* or *Heisenberg-Kramers* virtual oscillators, as well as the universal constant of *Boltzmann* such that the total energy of the energy-level j may be expressed as [42]

$$T_j S_j = 4N_j k T_j = 4N_j \bar{\varepsilon}_{tj} = 4N_j \frac{\hat{u}_j}{3} = \frac{4}{3} U_j \qquad (48)$$

In Eq. (48) the mean energy of oscillator involving the product $\bar{\varepsilon}_{tj} = kT_j$ is divided into two parts the first part $4N_j k$ is the definition of entropy by Eq. (47) and the second part T_j relates to the *de Broglie* wavelength of oscillator by Eq. (6b) as

$$\bar{\varepsilon}_{t\beta} = \bar{p}_\beta \langle \nu_\beta^2 \rangle^{1/2} \langle \lambda_\beta^2 \rangle^{1/2} = k_\beta \lambda_\beta = kT_\beta \qquad (49)$$

As a result the definitions of (k, T) lead to conjugate roles of thermodynamic properties (*S*, T) such that their product gives the oscillator energy in Eq. (48).

Now that the role of constant k in *Boltzmann* thermodynamic entropy has been identified, possible role of the universal constant k in *Shannon* information entropy will be examined. First, it is noted that all communications and exchange of information must involve some kind of physical entity such as a particle like molecule, atom, electron, photon, neutrino, … involving molecular-dynamics, … electro-dynamics, chromo-dynamics, … statistical fields. Such fundamental approach to communication theory can also address the problem of conveying information by ensembles of different types of symbols such as alphabets, A, B, C, … and alphabet clusters containing N_A, N_B, N_C, … alphabets such as encountered in the information theory of *Shannon* [67, 68]. Hence, parallel to system of ideal gas, one seeks a spectrum of "alphabet cluster" or "word" sizes that will maximize the entropy i.e. total randomness of the system such that the number of complexions expressed by corresponding *Boltzmann-Planck* Eq. (24) is maximized. Under "*equilibrium*" distribution alphabet-cluster sizes will correspond to maximum randomness and hence entropy and thus minimum information in accordance with Fig. 3. It is now clear that in order to insure an exact correspondence between *Boltzmann* and *Shannon* entropy one must let K = k and write *Shannon* formula Eq. (33) as

$$\hat{i}_{max} = -k \ln p_j \qquad (50)$$

Indeed, concerning the positive constant K in his formula in Eq. (33) *Shannon* writes [63- 64]

> **"The choice of the coefficient K is a matter of convenience and amounts to the choice of a unit of measure"**

Equation (50) leads to modified *Shannon* formula

$$H_{max} = \sum_j \hat{i}_{max} p_j = -k \sum_j p_j \ln p_j \qquad (51)$$

that is now in exact correspondence with *Boltzmann* formula in Eq. (40). The information entropy in Eq. (51) insures that the conjugate thermodynamic properties (*S*, T) are such that their product T*S* will relate to the system energy as expressed in Eq. (48). The *Boltzmann* constant k = K in Eq. (51) does indeed relate to *the choice of the "measure"* as was anticipated by *Shannon* [63-64]. This is because according to the present theory thermodynamic temperature is identified as a length scale $T = \langle \lambda_\beta^2 \rangle^{1/2}$ that is *de Broglie* wavelength according to equations (6) and (9). Therefore, in exact agreement with the perceptions of *Shannon* as described in the above quotation, the constant K = k is related to the measure $T = \langle \lambda_\beta^2 \rangle^{1/2}$ since the product kT gives the mean particle energy according to Eq. (49). In other words, the introduction of k in Eq. (51) is necessary since it allows the entropy of information to be related to energy through *de Broglie* wavelength of the particles being used to transmit the information.

Recently, it was suggested by *Ben-Naim* [71] that to achieve exact correspondence between *Boltzmann's* entropy in thermodynamics in Eq. (40) and *Shannon's* entropy in information theory in Eq. (34) the dimension of thermodynamics absolute temperature be changed

> **"One should redefine a new absolute temperature; denote it tentatively as $\bar{T}$ to replace kT. The new temperature $\bar{T}$ would have the units of energy and there would be no need for Boltzmann constant. The equation for entropy will be simply S = lnW,** [21] **and entropy would be rendered dimensionless"; "This will automatically expunge the Boltzmann constant k_B from the vocabulary of physics"**

According to the definitions in equations (5)-(10) of Section 3 *Planck* h and *Boltzmann* k constants are associated with the stochastically stationary aspects of *Casimir* [72] vacuum fluctuations with the dimensions

$$\varepsilon_\beta = h \langle \nu_\beta^2 \rangle^{1/2} = h_\beta \nu_\beta \qquad [\text{J-s}].[\text{Hz}] = [\text{J}] \qquad (52a)$$

$$\varepsilon_\beta = k \langle \lambda_\beta^2 \rangle^{1/2} = k_\beta \lambda_\beta \qquad [\text{J/m}].[\text{m}] = [\text{J}] \qquad (52b)$$

Therefore, just like *Planck* constant h what he named *Boltzmann* constant k cannot be expunged from the vocabulary of physics. Also, according to Eq. (6b) *Kelvin* absolute temperature is identified as a length scale associated with mean wavelength of thermal oscillations.

$$\varepsilon_\beta = k\langle\lambda_\beta^2\rangle^{1/2} = kT \tag{53}$$

The identification of dimension of absolute thermodynamic temperature as [meter] rather than [degree K] is a step towards clarification of the exact physical nature of this intensive property.

Because temperature is a measured quantity ultimately it is the theory that must determine the significance of such a variable and its dimension according to *Planck* [73].

> **"Every measurement first acquires its meaning for physical science through the significance which a theory gives it"**

Therefore, following *Planck's* suggestion, it is reasonable to change the dimension of absolute thermodynamic temperature T from [degree K] to [meter]. The exact correspondence between H_{max} and I_{max} could be achieved through the definition of dimensionless thermodynamic information entropy $\tilde{H}_{max}$ in Eq. (41).

Arguments have also been made that the very name *entropy* first coined by its discoverer *Clausius* should be changed [74]

> **"It is also time to change not only the units of entropy to make it dimensionless, but the term "entropy" altogether. Entropy, as is now recognized, does not mean "transformation, or "change" or "turn". It does mean information. Why not replace the term that means "nothing" as Cooper noted, and does not even convey the meaning it was meant to convey when selected by Clausius? Why not replace it with a simple, familiar, meaningful, and precisely defined term "information?"**

It is emphasized however that contrary to the above quotation the name entropy has been chosen most appropriately by *Clausius* to represent precisely what it should namely "transformation". According to the result in Eq. (47) for ideal gas, entropy relates to the number of *Planck* [45] or *Heisenberg-Kramers* oscillators [43]. Therefore, the second law of thermodynamics suggests that in all natural processes there is a tendency to transform energy and hence motion from ordered motions of a few large-scale oscillators to random motions of many small-scale oscillators thus increasing the entropy of the system. In other words entropy according to its macroscopic definition by *Clausius* is directly related to heat

$$dS = dQ_{rev} / T \tag{54}$$

Therefore, all types of energy dissipation induced by plastic deformation, friction, viscosity,… will lead to *transformation* of ordered (correlated) motions into disordered (uncorrelated) random motions that is heat thus leading to increased entropy by Eq. (54). It seems that if we follow the changes of dimensions and notations suggested by *Ben–Naim* [71, 74] the state of our scientific terminology may become *void* and hence approach what he calls "*Tohu Vavohu*" [71] meaning total chaos.

5. Impact on Nernst-Planck Third Law of Thermodynamics

According to *Nernst-Planck* statement of the third law of thermodynamics, entropy must approach zero as absolute thermodynamic temperature approaches zero. Another statement of the third law is [75]

> **"It is impossible to reach absolute zero using a finite number of processes"**

or the absolute zero temperature cannot be reached with a finite number of steps. Examination of Fig. 1 clearly shows that the very definitions of space and time that is applicable across the hierarchies of embedded statistical fields become complicated [35] and demand careful investigations. Recently, it was suggested [42] that the description of the hierarchy of statistical fields shown in Fig. 1 could be expressed in terms of a logarithmic coordinates schematically shown in Fig. 4.

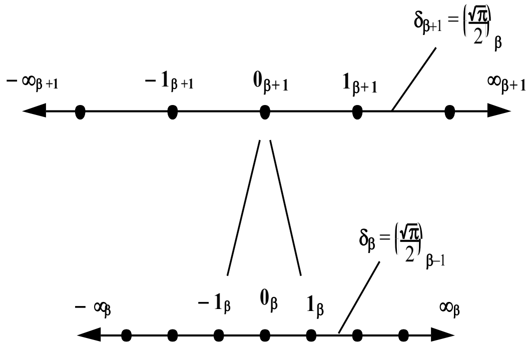

Fig. 4 Hierarchy of normalized coordinates associated with embedded statistical fields [42].

Because according to Eq. (53) thermodynamic temperature is identified as a length scale in view of Fig. 4 one arrives at a hierarchy of absolute zero temperatures [42]

$$\begin{array}{l}\ldots\\ \mathrm{T}_{\beta} = 0_{\beta} = \mathrm{T}_{\beta-1} = 1_{\beta-1}\\ \qquad\qquad \mathrm{T}_{\beta\text{-}1} = 0_{\beta\text{-}1} = \mathrm{T}_{\beta\text{-}2} = 1_{\beta\text{-}2}\\ \qquad\qquad\qquad\qquad \ldots \end{array} \qquad (55)$$

Furthermore, entropy of an ideal gas is a measure of the number of oscillators by Eq. (47). Therefore, from *Nernst-Planck* statement of the third law and Eq. (55) one arrives at the hierarchy of "absolute zero" temperature and corresponding "absolute zero" entropy schematically shown in Fig. 5.

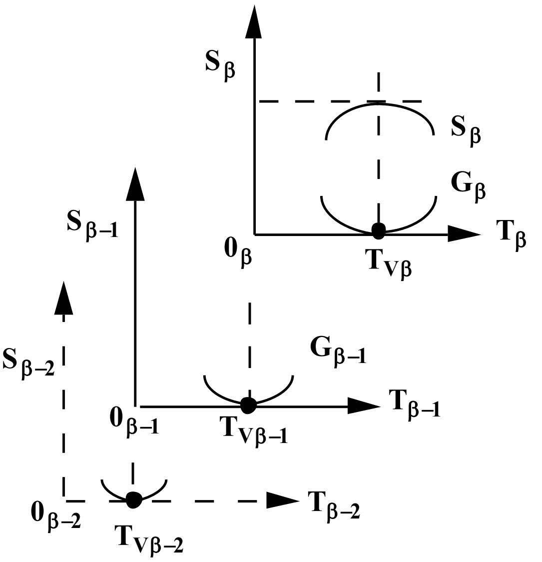

Fig. 5 Hierarchy of absolute "zeros" $T_\beta = 0_\beta$ and "vacuum" $T_{v\beta}$ temperatures and associated entropies approached at equilibrium corresponding to minimum *Gibbs* free energy G_β or maximum entropy S_β [42].

The hierarchy shown in Fig. 5 is associated with the hierarchy of vacua at *equilibrium* conditions and the associated entropy and enthalpy leading to the vanishing of invariant *Gibbs* free energy

$$G_{v\beta} = H_{v\beta} - \mathrm{T}_{v\beta} S_{v\beta} = 0_{\beta} \qquad (56)$$

as discussed in Sec. 10 of an earlier study [42].

6. Concluding Remarks

A scale-invariant model of statistical mechanics was applied to describe the nature of the connection between *Boltzmann* thermodynamic entropy and *Shannon* information entropy. Also, the application of the model to statistical fields associated with social structures was described. It was shown that the equality of the positive constant K in *Shannon's* formula for information entropy and the *Boltzmann* constant k in *Boltzmann* formula for thermodynamic entropy insure the consistency between the energy and *de Broglie* wavelengths of particles being used to transmit information. Finally, the impact of the model on the *Nernst-Planck* statement of the third law of thermodynamics was briefly discussed.

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