Functional Calculus of Concepts for Knowledge Acquisition and Processing

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Abstract: This paper addresses the problem of formal representation of categorization and concept learning from logical perspective. That way, we construct functional calculus of concepts (**FCC**) as a natural deduction system enriched with subscripts and type assignments and based on identity. The idea of this presentation stems from our previous research in areas of Intentional Theory of Concepts and formal consideration of Aristotel's paradeigma (example).
The first section clarifies the motivation and briefly outlines the guiding ideas of our approach in the broader context of related work. The second section starts with the discussion of Aristotel's ideas of example-based reasoning in connection with first principle grasping. We consider some relevant modern findings to support the claim that categorization and concept learning are based on identity rather then on similarity and comparison. That is the third section, which introduces the very functional calculus of concepts formalizing that way an Aristotelian paradeigma as a procedure of new concepts formation. The conclusion contains closing remarks and indicates directions for future work.

Key–Words: concept learning, categorization, natural deduction, identity

1 Introduction

It's with good reason that similarity is considered as a basis for categorization. We act in complete agreement with our intuition when we decide that similar objects belong in the same category. According to W. Quine [18, p.6], "for surely there is nothing more basic to thought and language than our sense of similarity; our sorting of things into kinds".

Nevertheless, as Quine further notes, when once he thinks about the relationship between the terms 'similarity' and 'kind', it turns out that their meaning is not self-explanatory, but requires clarification. In fact, what does it mean that things a and b are more similar than a and c? In logical tradition, the answer is straightforward: a and b share more properties than a and c do. Thereby, establishing similarity between two things is reduced to establishing their common properties. In turn, atomic facts that objects a and b have both the property A are based on the identity of the property A with respect to a and b. Thus, completing the circle, similarity is reduced to identity.

If one look at categorization and concept learning from cognitive perspective, then it is also not quite clear how and to what degree is similarity involved in this procedure. Thirty years have passed since a hallmark study [19] was published where L. Rips throughly considered all pros and contras and made a very compelling argument against similarity-based categorization, and it seems instructive add to the list of objections new items supported by recent findings. For example, paper [6, p.1] states that "much research has shown that when people are asked to construct their own categories they rarely do so on the basis of overall similarity, instead categorizing on the basis of a single feature or dimension of the objects.". At the same time, when both instance-(or memory-) based learning and concept (or category) learning are concerned, it is generally agreed that these procedures are based on comparison of a new stimulus with instances perceived earlier and stored in memory. In turn, comparison is typically interpreted in terms of similarity [3], [4], [11], [17]. For example, "in the simplest case, every object o is classified according to its nearest instance, according to some similarity measure or to some distance measure" [10, p.268].

We incline to the vision expressed by the authors of the article [12, p.38]: "When comparing people and the current best algorithms in AI and machine learning, people learn from less data and generalize in richer and more flexible ways. Even for relatively simple concepts such as handwritten characters, people need to see just one or a few examples of a new concept before being able to recognize new examples, generate new examples, and generate new concepts based on related ones."

The principal goal of this paper is to propose a formal system designed for explication of categorization and concept learning on a different basis than similarity. This formal machinery is a natural implementation of our phenomenological approach to cognitive activity. In particular, it grows out of two lines of research within a singular project, namely Intensional Concept Theory and formal reconstruction of Arestotelian paradeigma (example, rhetoric argument based on parallel cases, also known as parallel reasoning).

As for Intentional Concept Theory is concerned, it should be noted that it is based on worldview insights and methodology developed in phenomenology which found its applications in cognitive science. Husserlian phenomenology on its own is a very well-known and popular trend not only in philosophy but in computer science as well. Since Hubert Dreyfus's 'What computers can't do' [7] phenomenology has been tightly linked with AI. To mention but a few, consider [2], [5], [8], [13], [16], [22], and the special issue of Minds and Machines (2018, 28 (1)), presenting the results of a session of IACAP 16 on Computation and Representation in Cognitive Neuroscience, with the paper [15] as the most telling example, directly addressing the concept of intentionality.

Our conception was presented in great detail in the papers [23] and [25], so it will suffice here to confine ourselves to a brief characterization of its two key concepts relevant to current research.

First of all, it is *intentional*, because we take the concept of intentionality (directedness) to be the basis of our theory. That way, we interpret intentionality as functional relation which transforms intended objects into recognized objects and so performs the meaning-bestowal function.

Secondly, there is another one even more important phenomenological concept of *analogizing apperseption* (apprehension). Appresentation (co-presentation) provides identification of a object on the ground of experienced earlier 'analogous' object serving in this context as a model. The core and basis of apperseption is formed by more fundamental pairing procedure, being a specific form of passive synthesis in which a pair is constructed. As a result of this cognitive procedure, a new object is appercepted in accordance with the sense of the model one.

This chain of discourse leads us to a natural interpretation of concepts-as-functions close to Fregian tradition. He considered concept to be a predicate function from individuals into truth-values. We shall follow a more abstract representation of concept as mapping from its universe (domain) into its extension. Needless to say, behind this mapping there is always a predicate function that returns a value True for all elements of the corresponding concept extension. For example, the concept 'prime number' can be characterized by a mapping from the set of natural numbers into the set of such natural numbers that are greater than 1 and are divisible only by themselves and by 1. Back to intentionality, now it may be interpreted as a concept function from stimuli into intentional (meaningful) objects.

The second line of research aimed to examine and formalize rhetoric argument based on example, which is far less known and thanks to this deserves way more close consideration in a separate section.

Thus, the paper is structured in a following way. In the next 2nd section, we will consider Aristotelian conception of example in more detail and argue in favor of identity as a basis for concept learning. The 3rd section contains a formal presentation of functionl calculus of concepts as a natural deduction system. The final section summarizes the results and outlines the prospects for further research.

2 Finding regularities – even a duckling can do it

Aristotle considers example (paradeigma) two times – as a mode of convincing argument in Rhetoric (Rhet A2 1357), and as a specific sort of reasoning in Prior Analytics (APr B24), where he explains how it works and gives an appropriate illustration:

> "For example let A be evil, B making war against neighbours, C Athenians against Thebans, D Thebans against Phocians. If then we wish to prove that to fight with the Thebans is an evil, we must assume that to fight against neighbours is an evil. Conviction of this is obtained from similar cases, e.g., that the war against the Phocians was an evil to the Thebans. Since then to fight against neighbours is an evil, and to fight against the Thebans is to fight against neighbours, it is clear that to fight against the Thebans is an evil."

If we skip the details, then paradeigma in a narrow sense may be reconstructed as a non-deductive reasoning of the following form (presented in a syllogistic manner):
d is A, c is B-similar to $d \models$ (All) B are A.

However, it is only, so to say, a surface consideration of paradeigma, there is more to investigation its cognitive nature. Aristotle returns to it in a deeper cognitive context of Posterior Analytics (APo B19). In so doing, he raises a problem of first principles (primitive truths from which all things are known) grasping. These principles are non-demonstrative (just like major premises in syllogistic arguments), and the knowledge of them can be gained as a gradual transition from perception through memory and experience. A starting point in this transition is provided by 'an innate discriminatory capacity', that all animals possess. It allows further cognitive advance from perceived particulars to a 'universal in the soul' which in turn gives rise to general formulations of first principles. Thus paredeigma-like cognitive procedure has more in common with knowledge mining and finding regularities than with inductive generalization.

Interestingly, we come across almost the same cognitive machinery when consider the ability of non-human animals and human infants to represent 'same-different' relations or their capacity to such primitive forms of learning as imprinting [14], [20]. Ducklings, penguins, chicks and other not the smartest birds are capable of categorization and concept learning and oftentimes they do it spontaneously, based on a small number of supporting examples.

> "In contrast to machine-learning systems, chicks do not require explicit reinforcement, supervised learning, or thousands/millions of examples to feed learning. They are equipped with dedicated orienting and learning mechanisms that work as adaptive priors and architectural structures. These priors imply some assumptions about the external world that guide learning, but can, and must, allow errors, as was the case of goslings imprinted on Konrad Lorenz." [21, p.964]

All these considerations argue in favor of hypotheses that categorization and concept learning are based on identity rather then on similarity, where the latter suggests comparison. When we identify an object we do not compare it with some other object, but just recognize something in a novel stimulus connected with the model object. We assign the stimulus the same type which the model object previously stored in memory was assigned, and react to it in a typical way. This procedure does not require reflection and reasoning, it is carried out automatically in conformity with the fashionable nowdays idea of System 1 intuitive processing. With this in mind, we must admit the existence of embedded and embodied prototypes – categories, model objects, recognized in the primary acts of categorization-as-identification.

People and other animals do not live in the world of an infinite number of individual objects, but in the world of types. A cat responds to the mouse as to the type. The type assignment procedure is universal and basic, inherent to all animals, since it is closely connected with adaptation. It is important for an animal to identify an object, to typify it in order to react in a typical way.

Comparison is not necessary for adaptation but rather a sort of fundamental typing procedure. Comparison operation presupposes, in the first place, the existence of at least two separate objects that have already been identified, that is, we assigned them the same type. Secondly, comparing objects, we find in them common properties and on this basis assign them a type or a category. Whereby finding out common properties itself requires reflection, in which we 'reconvert' the objects as a whole to its state as a manifold of parts, moments and properties, reactivating in memory the temporal process of step-by-step, gradual construction of the object as a whole from parts or fragments. It means that, first of all, we are comparing not separate and standalone objects, but rather their properties or parts; secondly, these parts or properties appear as new independent objects in consequence of reflection presented in acts of a higher cognitive level. In this case, the process of comparison can be represented as a modification of the primary typification. This modification lies in recognition of an object, identification of this object (property) with another, serving as a sample. Therefore, typing on the basis of comparison is a multi-step procedure, which includes rational components, corresponding to analytic, System 2 stage of thinking. If an identification-as-recognition is concerned, it appears to ground this process. In other words, the operation of primary typing lies at the base of both intuitive and rational thinking.

Summing up, we stick to narrative of categorization as not similar-based. By way of suggesting itself implementation of this view, we present a logical system of functional concept calculus with types designed to formalize categorization and concept learning on the basis of identity. In so doing, we will treat paradeigma not as a form of plausible reasoning, but in more cognitive manner as a specific categorization rule.

3 Natural deduction for categorization and concept learning

In this section, we develop further the approach to the interpretation of concepts as functional abstracts. A concept is treated as a lambda term $\lambda x.F\colon A \longrightarrow B$, consisting of a functional term, a variable, and type assignment denoting a map from A to B.

In the paper [24], we proposed a version of natural deduction calculus of concepts. This system was enriched with subscripts meant to keep track of how assumptions are used. We write subscripts in square brackets to the right of the formula, and numbers in square brackets indicated the hypotheses used to prove of formula. When a hypothesis is discharged, the subscript is dropped.

This system was primarily designed to formalize the logical procedure of establishing standard relations between extensions of concepts, and first of all, the inclusion relationship, which allows to specify the remaining fundamental and derivative relations. We considered both simple and complex types of objects formed with the help of operations analogous to conjunction, disjunction, and negation. In so doing, we assumed standard restrictions on the extensions of concepts: logical relations are considered only between comparable (that is, belonging to the same universe) non-empty concepts.

Below we present an extended and more abstract version of this calculus.
The language of Functional Calculus of Concepts (**FCC**) contains:

- a non-empty set of individual variables $\{v\}$;
- a non-empty set of individual constants $\{c\}$;
- a non-empty set of functional constants with subscript $\{F_A\}$;
- a non-empty set of atomic type forming properties $\{P\}$;
- symbols for lambda-abstraction (λ), application ($\bullet$) and relative identity ($\simeq$);
- type forming connectives ($\wedge, \vee, \neg$).

Term: $t := v|c|F_A|t \bullet t|c \simeq c|\lambda v.F_A$
Type: $\tau := P|\neg B|B \wedge B|B \vee B|B \longrightarrow B$

A formula is an expression of the following form: $t\colon \tau$. In particular, a formula can express a statement about the typification of some object ($a\colon P$) or a functional representation of a concept ($\lambda x.F_Q\colon A \longrightarrow Q$).

Inference rules of **FCC**:

$[concept_{intro}]\ \ \frac{c_1 \simeq c_2 : B[\Gamma],\ c_2 : A[\Delta]}{\lambda v.F_B : A \longrightarrow B[\Gamma,\Delta]}$

$[concept_{elim}]\ \ \frac{\lambda v.F_B : A \longrightarrow B[\Gamma],\ t:\neg B[\Delta,\ t:A]}{t:\neg(A \longrightarrow B)[\Gamma,\Delta]}$

$[\longrightarrow_{intro}]\ \ \frac{t:B[\Gamma, v:A]}{\lambda v.F_B : A \longrightarrow B[\Gamma]}$

$[\longrightarrow_{elim}]\ \ \frac{\lambda v.F_B : A \longrightarrow B[\Gamma],\ t:A[\Delta]}{F \bullet t : B[\Gamma,\Delta]}$

$[\wedge_{elim_1}]\ \ \frac{t:B \wedge C[\Gamma]}{t:B[\Gamma]}$

$[\wedge_{elim_2}]\ \ \frac{t:B \wedge C[\Gamma]}{t:C[\Gamma]}$

$[\wedge_{intro}]\ \ \frac{t:B[\Gamma],\ t:C[\Gamma]}{t:B \wedge C[\Gamma]}$

$[\vee_{elim}]\ \ \frac{t:B \vee C[\Gamma],\ t:D[\Delta, t:B],\ t:D[\Delta, t:C]}{t:D[\Gamma,\Delta]}$

$[\vee_{intro_1}]\ \ \frac{t:B[\Gamma]}{t:B \vee C[\Gamma]}$

$[\vee_{intro_2}]\ \ \frac{t:C[\Gamma]}{t:B \vee C[\Gamma]}$

$[\neg_{elim}]\ \ \frac{t:\neg\neg B[\Gamma]}{t:B[\Gamma]}$

$[\neg_{intro}]\ \ \frac{t:B[\Gamma],\ t:\neg B[\Delta]}{t:\neg C[\Gamma, \Delta \setminus \{t:C\}]},\ \{t\colon C\} \subseteq \Gamma \cup \Delta$

Some remarks and explanations to the rules are due.

$concept_{intro}$ and $concept_{elim}$ rules:
As we promised in the previous section, $concept_{intro}$ rule is a rule-form modification of Aristotelian paradeigma designed for formation of new concepts. $a \simeq b\colon B$ means that two objects a and b are identical with respect to type B. This information is not derivable (and thus, there are now intro- or elim-rules for $\simeq$), it is obtained from observations directly in the above form, just like in Arestotel's example. If, in addition, there is information available that one of these objects is of type A, it makes possible a far-reaching conclusion about the functional relationship between these types. It is appropriate to remind here that the functional interpretation of a concept as mappings from A to B implies that all elements of the set B are included in the set A, that is, the proposition 'All B are A' is true, once again supporting Aristotel's idea of paradeigma.

Beyond all doubt, such a generalization is very strong and in most cases incorrect. This means that it is necessary to provide a rule that blocks the appearance of "hasty" concepts. This role is performed by $concept_{elim}$ rule.

$\longrightarrow_{intro}$ and $\longrightarrow_{elim}$ rules:
The rule $\longrightarrow_{elim}$ formalizes the procedure of cate-

gorization. An application of a functional abstract to an object symbolizes its typification, that is, its recognition. In other words, if a concept-as-function ($\lambda v.F_B : A \longrightarrow B$) returns for an object typed as A ($t: A$) the value Truth, then this object can be categorized as B ($t: B$).

Also, both these rules in conjunction permit to introduce and eliminate auxiliary ('secondary') concepts, which is necessary for stating relations between concepts. Thus, a derivation relation between two concepts means that the first of them is the subaltern of the second. For example, the concept $\lambda x.F_{P\wedge Q}$: $\mathbf{A} \longrightarrow (P \wedge Q)$ stands in the subalternation relation with the concept $\lambda x.F_P : \mathbf{A} \longrightarrow P$ just because one can derive $\lambda x.F_P : \mathbf{A} \longrightarrow P$ from $\lambda x.F_{P\wedge Q} : \mathbf{A} \longrightarrow (P \wedge Q)$.

$\wedge_{intro}$, $\vee_{elim}$ and $\neg_{intro}$:
Indirect rule $\vee_{elim}$ as well as restricted rules $\wedge_{intro}$ and $\neg_{intro}$ are triggered by the obvious intention to avoid undesirable consequences, and, first of all, those connected with contradictory types. Without these arrangements it would be possible for a concept with contradictory intension to be the subaltern of arbitrary concept. These peculiarities make **FCC** and calculi of relevant logic akin.

4 Conclusion

The main result of this article is the construction of a natural deduction system **FCC** for the functional calculus of concepts, which provides the formal procedures for introduction and elimination of concepts, as well as for the application of the concept-as-function to the argument that, we believe, can claim to formalize categorization.

Very tentatively, our approach to formal presentation of categorization and concept learning can be classified as a rule-based one. However, there are important differences between them. First of all, rule-based system usually contains a set of implicative rules while in **FCC** we have, so to say, the only one $\longrightarrow_{intro}$ rule and a number of concepts to which it can be applied. Secondly, these systems are often subject for certain weakening aimed at approximation to natural human cognitive procedures, that leads to probabilistic category assignment and often results in models built upon Bayesian inference and inductive learning. On the contrary, our model exploits identity as a criterion for category membership and concept introduction rule based on principle different from inductive generalization.

Undoubtedly, the proposed formalism is, first of all, a logical tool for knowledge acquisition and processing. And in this regard, our logical work has not yet been completed. The calculus needs adequate semantics, it would be useful to consider its quantifier extension — all these are tasks for the future.

Besides, concerning the concept elimination rule, quite natural question arises of what positive information does the conclusion of this rule contain, other than the obvious fact of rejection of a hasty concept? In other words, if the concept was introduced hastily and we have fixed it, does it still give an additional boost to positive knowledge? Above, we have already mentioned the similarity of our calculus with systems of relevant logic. To push analogy further, it seems promising add two extra rules for the introduction and elimination of compatibility connective (fusion, intensinal conjunction) with the corresponding updates of the formal language:

$$[\circ_{intro}] \quad \frac{t{:}B[\Gamma],\ t{:}C[\Delta]}{t{:}B{\circ}C[\Gamma,\Delta]}$$

$$[\circ_{elim}] \quad \frac{t{:}B{\circ}C[\Gamma],\ \lambda v.F_B{:}A{\longrightarrow}(B{\longrightarrow}C)[\Delta]}{t{:}C[\Gamma,\Delta]}$$

These rules make it possible to prove the following 'equivalence', connecting the negation of conceptual implication and compatibility:
$t: \neg(A \longrightarrow B) \dashv\vdash t: A \circ \neg B$.
Now we can claim that rejecting conceptual generalization reports us about the compatibility of two types, one of which assumes negative typification.

Yet one more promising direction of further work is connected with the application of the ideas developed in this article to machine learning and in particular to instance-based learning (for detail, consult [1], [9]). In most cases, instance-based learning algorithms are constructed with regard to such parameters as distance measure, number of neighbors considered, and weighting function values for them, that is, these algorithms again presuppose comparison and degrees of similarity. Our approach employs different methodology, which, we hope, opens up new perspectives for research in the field.

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References:

[1] C. Aggarwal, Instance-Based Learning: A Survey, In: Aggarwal, C. C. (ed.) *Data classification: algorithms and applications*, CRC press, 2014, pp. 157-185.

[2] A. Beavers, Phenomenology and artificial intelligence, *Metaphilosophy*. 33(1-2), 2002, pp. 70-82.

[3] S. Boriah, V. Chandola and V. Kumar, Similarity measures for categorical data: A comparative evaluation. In: *Proceedings of the 2008 SIAM international conference on data mining*, Society for Industrial and Applied Mathematics 2008, pp. 243-254.

[4] F. Botana, A fuzzy measure of similarity for instance-based learning. In: *International Symposium on Methodologies for Intelligent Systems*, Springer, Berlin–Heidelberg 1999, pp. 439-447.

[5] R. Chrisley, Embodied artificial intelligence, *Artificial intelligence* 149(1), 2003, pp. 131-150.

[6] J. Clapper, Graded similarity in free categorization, *Cognition* 190, 2019, pp. 1-19.

[7] H. Dreyfus, *What computers can't do: A Critique of Artificial Reason*, New York: Harper and Row 1972.

[8] T. Froese and T. Ziemke, Enactive artificial intelligence: Investigating the systemic organization of life and mind, *Artificial Intelligence* 173(3-4), 2009, pp. 466-500.

[9] E. Keogh, Instance-Based Learning. In: Sammut C., Webb G.I. (eds.) *Encyclopedia of Machine Learning* Springer, Boston, MA 2011, pp. 549-550.

[10] N. Lachiche and P. Marquise, Scope classification: An instance-based learning algorithm with a rule-based characterisation. In: *European Conference on Machine Learning* Springer, Berlin–Heidelberg 1998, pp. 268-279.

[11] B. Lake, R. Salakhutdinov and J. Tenenbaum, Human-level concept learning through probabilistic program induction, *Science* 350(6266), 2015, pp. 1332-1338.

[12] B. Lake, T. Ullman, J. Tenenbaum and S. Gershman, Building machines that learn and think like people, *Behavioral and brain sciences* 40, 2017, pp. 1-72.

[13] R. Manzotti, Embodied AI beyond Embodied Cognition and Enactivism, *Philosophies* 4(3), 2019, p. 39.

[14] A. Martinho and A. Kacelnik, Ducklings imprint on the relational concept of "same or different", *Science* 353 (6296), 2016, pp. 286-288.

[15] A. Morgan and G. Piccinini,Towards a cognitive neuroscience of intentionality, *Minds and Machines* 28(1), 2018, pp. 119-139.

[16] D. Münch, The Early Work of Husserl and Artificial Intelligence, *Journal of the British Society for Phenomenology* 21(2), 1990, pp. 107-120.

[17] N. Said, M. Engelhart, C. Kirches, S. Körkel and D. Holt, Applying mathematical optimization methods to an ACT-R instance-based learning model, *PloS one* 11(7), 2016, e0158832.

[18] W. Quine, Natural kinds. In: *Essays in honor of Carl G. Hempel* Springer, Dordrecht 1969, pp. 5-23.

[19] L. Rips, Similarity, typicality, and categorization. In: Vosniadou S., Ortony A. (eds.) *Similarity and analogical reasoning* Cambridge University Press 1989, pp. 21-59.

[20] C. ten Cate, The comparative study of grammar learning mechanisms: birds as models, *Current opinion in behavioral sciences* 21, 2018, pp. 13-18.

[21] E. Versace, A. Martinho-Truswell, A. Kacelnik and G. Vallortigara, Priors in animal and artificial intelligence: where does learning begin?, *Trends in cognitive sciences* 22(11), 2018, pp. 963-965.

[22] M. Wrathall and S. Kelly, Existential phenomenology and cognitive science, *The Electronic Journal of Analytic Philosophy* 4 1996.

[23] D. Zaitsev and N. Zaitseva, Categorization in intentional theory of concepts. In: International Symposium on Neural Networks, *Lecture Notes in Computer Science* 9719, 2016, pp. 465–473.

[24] D. Zaitsev and N. Zaitseva,Calculus of concepts, *Tomsk State University Journal of Philosophy, Sociology and Political Science* 49, 2019, pp. 26-33. *In Russian*

[25] N. Zaitseva and D. Zaitsev, Phenomenological perspective in modern neuroscience, *Russian Journal og Philosophical Sciences* 1, 2017, pp. 71–84. *In Russian*