Using Continuous Hopfield Network for Static sectorization of Airspace

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*Abstract:-*As traffic keeps increasing, en route capacity, especially in Europe, becomes a serious problem. According to the European Commission, every year, the number of flights in operation increases by 5%, which is the principal cause of airspace saturation and raise of the controller's workload. Today, the Sectorization of Airspace Problem (SAP) has become one of the most important problems of operational research. The main objective of the SAP is to minimize the total coordination workload between adjacent sectors and to balance the controllers' workload among sectors. To solve this problem, we model the SAP in terms of 0-1 quadratic programming subject to linear constraints. As result, we use the Continuous Hopfield Network CHN to solve the proposed model; in addition, some numerical results are introduced to confirm the most optimal model.

Key-words:- Air Traffic Control ATC, Sectorization of Airspace Problem SAP, Quadratic Programming QP, Continuous Hopfield Network CHN.

1. Introduction

Currently European airports and air routes to reach saturation. According to the European Commission, 1.4 milliard passengers travel each year in 440 airports union. Every day, 26,000 flights cross in the European sky. The increasing number of flights operated every year by 5% [1].

Air traffic controllers are people trained to maintain the safe, orderly and expeditious flow of air traffic in the global air traffic control system.

En route controllers specialise in working airspace outside the main capital city airports, or other regional airports which have control towers. Most of what we do is separating aircraft in the climb, cruise or descent between the departure point and their destination. However we also have to do more than just separate the aircraft. We often need to warn pilots of dangerous weather conditions or changes to weather that haven't been forecast.

Fig.1: En route Air Trafic

Subdivision of airspace into sectors: Figure1, present the Flight Information Region (FIR) controlled by a center may be further administratively subdivided into sectors. Each Area is staffed by a set of controllers trained on all the sectors in that area. Sectors use distinct radio frequencies for communication with aircraft. Each sector also has secure landline communications with adjacent sectors, approach controls, areas, flight service centers, and military aviation control facilities. Aircraft passing from one sector to another shall be handed off and requested to change frequencies to contact the next sector controller. Sector boundaries are specified by an aeronautical chart. The sector number is then determined by the capacity of a controller to manage N aircraft simultaneously. In practice the mean seems to be 10 to 15 aircraft [1][2][3], when this limit is reach we say that the sector is saturated.
As the SAP is a very complex problem of operational research. Several optimization methods by metaheuristic were used to solve this problem [6][9][10][11]. A genetic algorithm has also been tested to solve the sectorization problem [4][5][21]. The G**As** considers a population of chromosomes,

which evolves by crossover, mutation, and selection of the fittest individuals, as described in [16][20][27]. A classical and stochastic method is used to solve the optimal combinations of Air Traffic Control sectors [5][10]. Also in [2] the recruit simule method and Ant Colony Optimization (ACO) are used to solve the SAP problem. In[3] optimization method inspired the fusion and nuclear fission, was created to solve a airspace sectorization problem.

In this paper we provide a global method of airspace sectorization based on mathematical modelling of this problem. The main purpose of this work, is modelling the SAP problem in terms of 0-1 quadratic programming subject to linear constraints. So, in order to solve this latter, we use the Continuous Hopfield Networks (CHN). More recently, airspace has been divided in small volume units and a sector is obtained by joining some of these elementary units. Unfortunately, the most specific constraints cannot be taken into account and for instance, the sectors can be fragmented in the solution.

2. Continuous Hopfield Neural Network

Hopfield neural network was introduced by Hopfield and Tank [17][18][19]. It was first applied to solve combinatorial optimization problems. It has been extensively studied, developed and has found many applications in many areas, such as pattern recognition, design systems and optimization. The Continuous Hopfield Networks CHN consists of interconnected neurons with a smooth sigmoid activation function (usually a hyperbolic tangent function)[21][22]. Dynamics of the CHN as follows:

$$\frac{dy}{dt} = -\frac{y}{\tau} + Tx + I^b \qquad (1)$$

With:

$$y = (y_1, \dots, y_n)^t,$$
$$x = (x_1, \dots, x_n)^t,$$
$$x_i = g(y_i), \forall i, j = 1, \dots, n,$$
$$g(y_i) = \frac{1}{2}(1 + \tanh(y_i / y_0)), u_0 > 0,$$
$$T = (T_{ij}), \forall i, j = 1, \dots, n,$$

Where y, x and I^b are, respectively the vectors of neuron states, the outputs and the bias. The output function $x_i = g(y_i)$ is a hyperbolic tangent, which is bounded below by 0 and above by 1. The real values T_{ij} and I_i^b are, respectively, the weight of the synaptic connection from the neuron i to the neuron j and the offset bias of the neuron i.

For $y_0 \in IR^n$, a vector y^e is called an equilibrium point of the differential equation system (1) if:

$\exists\, t^e \in IR^+, \forall\, t > t^e\; y(t) = y^e.$

Hopfield has introduced the energy function E on $[0, 1]^n$ which is defined by:

$$E(x) = -\frac{1}{2}x^tTx - (I^b)^t + \frac{1}{\tau}\sum_{i=1}^{n}\int_0^{x_i} g^{-1}(z)\,dz \qquad (2)$$

It should be noted that if the energy function (or Layapunov function) exists, the equilibrium point exists too. Hopfield proved that the symmetry of weight matrix is a sufficient condition for the existence of Lyapunov function. In order to solve combinatorial problems using CHN, we will write it in form of the energy function $E(v)$ such that:

$$E(x) = -\frac{1}{2}x^tTx - (I^b)^t x \qquad (3)$$

The extremes of this function are among the corners of the n-dimensional hypercube $H = [0,1]^n$ [7][8].The philosophy of this approach is that the objective function, which characterizes the combinatorial problem, is associated with the energy function of the network when $\tau \to +\infty$. In this way, the output of the CHN can be represented as a solution to combinatorial problem. Unlike a discrete network with the signum (hard-limiter) activation function, an analog neural network (with sigmoid activation function with variable slope) permits to avoid sub-optimal local minima.

Moreover, the analog network is much faster and more reliable than the discrete neural network with an asynchronous update. To ensure the feasibility of the equilibrium points of the CHN, some authors propose two steps (hyperplan method) [12][13][14]. The first one involves the decomposition of the set of the non feasible solutions into appropriate subsets, based on the constraints of the General Knapsack Quadratic Problem. In the second step, the parameters of the function are selected using the analytical conditions of the equilibrium points. The generalization of the energy function and these steps were used to solve the Placement of the Electronic Circuit Problem [7]. Within these papers, the feasibility of the equilibrium points of the CHN is always guaranteed.

Given the combinatorial optimization problem with n variables and m linear constraints:

$$\begin{cases} Min\left(\frac{1}{2}x^t Qx + C^t x\right) \\ \quad Subject\ to \\ \quad\quad Ax = b \\ x_i \in \{0,1\}\ \ i = 1, \dots, n \end{cases}$$

To simplify, the sets are defined as:

- The hamming hypercube
$$H = \{0,1\}^n$$
- The hamming hypercube corners set:
$$H_c = \{0,1\}^n$$
- The feasible solution set:
$$H_f = \{x \in H_c / Ax = b\}$$

The standard form of the energy function is:

$$E(x) = E^c(x) + E^p(x)\ \forall\, x \in H$$

Where:

- $E^c(x)$ is directly proportional to the objective function.
- $E^p(x)$ is a quadratic function that ensures the feasibility of the solution obtained by the CHN, and also penalize the violated constraints of the problem. This function must give the same value for each element x for H_f, and an adequate selection of this function is necessary for a correct mapping.

3. Modeling the SAP Problem

The objective of this part is presenting and modelling the problem of the (SAP) in terms of a quadratic knapsack problem with binary variables.

3.1 Presentation of Airspace

The airspace is made of an airways network and a set of beacons (report point or navigation) installed on the ground.

Fig.2: Air Trafic Flux

Nowadays, air traffic increases throughout the world; thanks to safety, the sectorization of airspace has become one of the most important solutions of Air Traffic Control; in this context, the airspace is divided into a number of sectors; each of them is assigned to a team of controllers. The sector number is then determined by the capacity of a controller to manage N aircraft simultaneously: In practice, the average seems to be 10 to 15 aircrafts, when this limit is attaining we say that the sector is saturated [2][3][4].

The sectorization is a fundamental architectural feature of the Air Traffic Control ATC system.

3.2 Controller workload

Generally, there exist three kinds of workload controllers: [1] [2].

3.2.1. Monitoring workload $\mathrm{M_w}$: This charge is proportional to the time of passage of an aircraft in area along an air route in a Flight Plan. If this route passes through two sectors, the load is distributed across the sectors in proportion to the part of the edge in the sector.

3.2.2. Conflict workload $\mathrm{C_w}$: This charge is proportional to the total number of potential conflicts that can occur on a crossing point.

3.2.3. Coordination $\mathrm{O_w}$: This is proportional to the number of the aircraft which pass along a road section, each of which ends charge are located in different sectors.

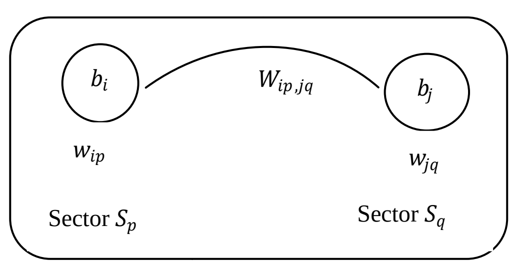

Fig.3: Coordination workload between the sectors $\mathbf{S_p}$ **and** $\mathbf{S_q}$

We define the matrix $\mathbf{W} \in \mathbf{IR^{m\times m}}$ of coordination workload between the sectors as follows:

$$W = \begin{pmatrix} w_{11,11} & w_{12,12} & \cdots\cdots & w_{1k,1k} \\ w_{21,21} & w_{22,22} & \cdots\cdots & w_{2k,2k} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & \cdots\cdots & \vdots \\ w_{n1,n1} & w_{n2,n2} & & w_{nk,nk} \end{pmatrix}$$

Where:

$$\mathrm{W_{ip,jq}} = \begin{cases} \mathrm{O_w} & \text{if beacons } \mathrm{b_1} \text{ and } \mathrm{b_2} \\ & \text{are directly connected} \\ \forall\, \mathrm{i,j} \in \{1,\dots,\mathrm{n}\}\ \mathrm{p,q} \in \{1,\dots,\mathrm{k}\} & \\ 0 & \text{else} \end{cases}$$

We define the vector $w \in IR^m$ of controller workload in each beacon as following:

$$w = (w_{11}w_{21}\dots w_{n1};\dots;w_{1p}w_{2p}\dots w_{np};\dots;w_{1k}w_{2k}\dots w_{nk})$$

Where:

$$w_{ip} = (\mathrm{C_w} + \frac{\mathrm{M_w}}{2})_{ip} \quad \forall\, i \in \{1,\dots,n\}$$

Definition 1:

We define the total workload of controllers in the all airspace by following formula:

$$W_T = \sum_{i=1}^{n} w_i + \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij} \tag{4}$$

Definition 2:

$$k = \frac{\sum_{i=1}^{n} w_i + \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}}{12} = {W_T}/{12} \tag{5}$$

In this work, we suppose that the airspace is formed by airways and $m = n \times k$ beacons $b_1,\dots,b_m$ these beacons will be divided into k sectors$V_1,\dots,V_k$. Each beacon is defined by a positive real number W_i. The airways connecting two beacons $b_1,\dots,b_2$ belonging to two adjacent sectors $V_p,\dots,V_q$is defined by positive real number $W_{ij} = O_w$ represent coordination workload.

The objective of the Sectorization of Airspace Problem (SAP) is: to affect each beacon to only one sector, to minimize the total load of coordination between sectors and to equilibrate the workload between the k sectors.

3.3 Problem formulation

The sectorization of airspace problem consists in finding an assignment of m beacons to k sectors. The main objective is to optimize the total coordination between sectors with controllers workload balanced.

This problem is stated as two sets and two parameters where:

- $S = \{S_1, S_2, \cdots, S_k\}$ a set of k sectors
- $B = \{b_1, b_2, \cdots, b_n\}$ a set of n beacons
- w_{ip}: Controllers workload in beacons i assignment to sector k.
- $W_{ip,jq}$: Coordination workload between beacons b_i and b_j assignment respectively to sectors S_p and S_q.

The main propose of this part is modelling the SAP problem in terms of 0-1quadratic programming problem subject to linear constraints [4][6][7][16].

- ***Binary variables:***

For each beacons$i \in \{1,\dots,n\}$, we introduce p binary variablesx_{ip}, $p \in \{1,\dots,k\}$, such that:

$$x_{ip} = \begin{cases} 1\ if\ b\ i\ is\ assigned\ to\ sector\ p \\ 0\quad \text{Otherwise} \end{cases}$$

We convert these suits of variables to m-vector:

$$x = (x_{11},\dots,x_{1k},\dots,x_{i1},\dots,x_{ik},\dots,x_{n1},\dots,x_{nk})^T$$

With $m = n \times k$

Each beacon should be assigned to exactly one sector:

$$\sum_{p=1}^{k} x_{ip} = 1 \quad \forall\, i \in \{1,\dots,n\} \tag{6}$$

This leaner constraint equivalent to matrix form:

$$Ax = b$$

The matrix $A \in IR^{m\times n}$ with $m = n \times k$ and the vector $b \in IR^n$ of the linear constraints are:

$$A = \begin{pmatrix} 1 & \cdots & \cdots & 1 & \cdots & \cdots & 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & 0 & \cdots & \cdots & 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & 0 & \cdots & \cdots & 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 & \cdots & \cdots & 1 & \cdots & \cdots & 1 \end{pmatrix}$$

$$b = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

- ***Objective function***

The objective function of the mathematical programming model consists to minimize the total coordination workload with balanced workload between the all sectors. Then, we can define the objective function $f(x)$in the following way:

$$f(x) = \sum_{i=1}^{n}\sum_{p=1}^{k}\sum_{j=1}^{n}\sum_{q=1}^{k}(1-\delta_{ij})(1-\delta_{pq})\ W_{ip,jq}\,x_{ip}x_{jq} \qquad (7)$$

The Boolean δ_{ij} is the Kroenecker delta.

$$\delta_{ij} = \begin{cases} 1\ si\ i = j \\ 0\ si\ i \neq j \end{cases}$$

- ***The family of the allocation constraints**:*

Each beacon must be allocated to only one sector; in this context, we obtain the following family constraints:

$$\sum_{p=1}^{k} x_{ip} = 1\ \forall\, i \in \{1, \dots, n\}$$

We remark that we associate one constraint to each beacon.

- ***The balance constraints***

For each vector x, the controller of the sector p has to manage the next workload:

$$\sum_{i=1}^{n} x_{ip}w_{ip} \qquad (8)$$

To make an equitable allocation between the airspace controllers, we impose the next family of constraint:

$$\sum_{i=1}^{n} x_{ip}w_{ip} = C^k\ \ \forall\, p \in \{1, \dots, k\} \qquad (9)$$

Where:

$$C^k = \frac{(\sum_{i=1}^{m} w_i)}{k} = \frac{\sum_{p=1}^{k}\sum_{i=1}^{n} w_{ip}}{k}$$

From now on, we call this quantity the desired workload. Since it is difficult to realize the later family of constraints, we prefer, only, to control the total error produced by an allocation of the beacons to the various sectors. The error square in sector is following:

$$error(sector) = \left(\sum_{i=1}^{n} w_{ip}x_{ip} - C^k\right)^2, \forall\, p \in \{1, \dots, k\}$$

The total error in airspace is following:

$$error(total) = \sum_{p=1}^{k}\left(\sum_{i=1}^{n} w_{ip}x_{ip} - C^k\right)^2$$

To control the total error, we obtain a new form for function objective:

$$f(x) = \sum_{i=1}^{n}\sum_{p=1}^{k}\sum_{j=1}^{n}\sum_{q=1}^{k}(1-\delta_{ij})(1-\delta_{pq})\ W_{ip,jq}\,x_{ip}x_{jq} + \lambda\sum_{p=1}^{k}\left(\sum_{i=1}^{n} w_{ip}x_{ip} - C^k\right)^2 \qquad (10)$$

With $\lambda \in IR$ is a control parameter

In this work the goal idea is to model the SAP in terms of 0-1 quadratic programming subject to linear constraints and we will penalize this constraint in objective function. As result, we use the Continuous Hopfield Network (CHN) to solve the proposed model.

4. Continuous Hopfield Network for the SAP

Hopfield neural network was introduced by Hopfield and Tank [17][18][19]. It was first applied to solve combinatorial optimization problems. As can be noticed, after modeling the sectorization of airspace problem into a 0-1 quadratic programming with a quadratic function subject to linear constraints, we present a general method for solving the SAP problems using the continuous Hopfield networks.

The main purpose of this section is to apply the CHN to solve the SAP. To this end, we define an appropriate energy function that enables us to resolve the SAP by the approach of CHN. To be precise, the choice of the parameters of this function must ensure the feasibility of the CHN equilibrium points.

4.1. Mathematical model

$$SAP:\begin{cases} Min\left(\begin{array}{c}\sum_{i=1}^{n}\sum_{p=1}^{k}\sum_{j=1}^{n}\sum_{q=1}^{k}(1-\delta_{ij})(1-\delta_{pq})W_{ipjq}\,x_{ip}x_{jq} \\ +\lambda\sum_{p=1}^{k}\left(\sum_{i=1}^{n}w_{ip}x_{ip}-C^{k}\right)^{2}\end{array}\right) \\ Subject\ to: \\ \sum_{p=1}^{k}x_{ip}=1,\forall\, i\in\{1,\dots,n\} \\ x_{ip}\in\{0,1\}\end{cases} \quad (11)$$

To solve the SAP via the CHN, we choose In this case two variables take one$\left(x_{ip}=x_{jq}=1\right)$, which is illogic to avoid this situation, the following condition should be imposed.

an energy function which includes the objective function.

4.2. Energy function for SAP

In this problem we consider a matrix array of $m=n\times k$ neurons represent an assignment of the m beacons to the k sectors. The m neurons are grouped into k groups of m neurons. Each group of m neurons is used to represent the position of the beacons on the sectors.

$$E(x)=\frac{\alpha}{2}f(x)+\beta\sum_{i=1}^{n}e_i(x)+\frac{\emptyset}{2}\sum_{i=1}^{n}(e_i(x))^2 + \mu\sum_{j=1}^{n}\sum_{p=1}^{q}x_{ip}\left(1-x_{ip}\right) \quad (12)$$

Where the $e_i(x)=\sum_{p=1}^{k}x_{ip}\ \forall\, i=1,\dots,n$ the real parameters $\alpha,\beta,\emptyset$ and μ are the penalty parameters.

$$f(x)=\sum_{i=1}^{n}\sum_{p=1}^{k}\sum_{j=1}^{n}\sum_{q=1}^{k}\left(1-\delta_{ij}\right)\left(1-\delta_{pq}\right)W_{ipjq}\,x_{ip}x_{jq} \quad (13)$$

Basing on the energy function E, the weights and threshold of the constructed (CHN) are given by the coming system:

$$i,j\in\{1,\dots,n\}\ and\ p,q\in\{1,\dots,k\}$$

$$\begin{cases}T_{ipjq}=-\dfrac{\partial^2E(x)}{\partial x_{ip}\partial x_{jq}} \\ I_{ip}=-\dfrac{\partial E(x=0)}{\partial x_{ip}}\end{cases}$$

4.3. Parameters setting

$$E_{ip}(x)=\frac{dE(x)}{dx_{ip}}=\sum_{j=1}^{n}\sum_{q=1}^{k}w_{ip,jq}\,x_{jq}+\lambda\alpha\sum_{i=1}^{n}\sum_{p=1}^{k}(w_{ip}x_{ip}-C^{k}) + \emptyset\sum_{q=1}^{k}x_{ip}+\beta+\mu(1-2x_{ip}) \quad (14)$$

In order to guarantee the instability of the interior points $x\in H-H_c$, some initial condition is imposed on some parameters[24]:

$W_{ip,jq}=-\emptyset+2\mu\geq 0 \quad -\emptyset+2\mu\geq 0$

The SAP problem has only one family of linear constraints:

$$e_i(x)=\sum_{p=1}^{k}x_{ip} \quad (15)$$

Then the partition of the set H_c-H_F is deined as:

H_c-H_F=$H_{11}\cup H_{12}$

Where:

$$H_{11}=\{\exists\, i: e_i()>0\}\cap\{e_0(x)\geq n\}$$

And $e_0(x)=\sum_{i=1}^{n}e_i\,(x)$

$$E_{ip}(x)\geq\alpha W_{min}+\lambda\alpha C_{min}^{k}\,2\emptyset+\beta-\mu\geq\varepsilon$$

Where:

$$W_{min}=min\left\{\frac{W_{ip,jq}}{(i,j)}\in\{1,\dots,n\}^2\ and\ (p,q)\in\{1,\dots,k\}^2\right\}$$

$$C_{min}^{k}=min\{\left(w_{ip}-C^{k}\right)/i\in(1,\dots,n)\ and\ p\in(1,\dots,k)\}$$

$$H_{12}=\{\exists\, i: e_i(x)<1\}\cap\{e_0(x)<n\}$$

In this case, one variable of decision take zero$\left(x_{ip}=0\right)$, which is tradictory with the linear constraint of our problem. the enforce, the following condition should be imposed :

Where:

$$E_{ip}(x)\leq\alpha W_{max}-\lambda\alpha C^{k}+\beta+\mu\leq-\varepsilon$$

$$W_{max}=max\{W_{ip,jq}/(i,j)\epsilon\{1,\dots,n\}^2\ and\ (p,q)\ \varepsilon\{1,\dots,k\}^2\}$$

Consequently, we can determine the parameters setting by resolving the following system:

$$\begin{cases}\emptyset>0 \\ \emptyset\geq 0 \\ -\emptyset+2\mu\geq 0 \\ \alpha W_{min}+\lambda\alpha C_{min}^{k}+2\emptyset+\beta-\mu \\ \alpha W_{max}-\lambda\alpha C^{k}+\beta+\mu=-\varepsilon\end{cases}$$

These parameters setting are determinate by fixing $(\alpha,\varepsilon,\lambda)$ and compute the rest of parameters$\mu,\beta\ et\ \emptyset$.

$$\begin{cases}\alpha>0,\emptyset\geq 0 \\ \emptyset=2\mu \\ \mu=\varepsilon-\dfrac{\alpha}{2}(W_{min}-W_{max})+\dfrac{\lambda\alpha}{2}\left(C_{min}^{k}-C^{k}\right) \\ \beta=-2\varepsilon-\dfrac{\alpha}{2}(W_{min}+W_{max})-\dfrac{\lambda\alpha}{2}\left(C_{min}^{k}+C^{k}\right)\end{cases}$$

Then the weights and thresholds of the connections between one neurons.

$$\begin{cases}T_{ip,jq}=-\alpha\left(1-\delta_{pq}\right)\left(1-\delta_{ij}\right)w_{ip,jq}-\lambda\alpha w_{ip}w_{jq}+2\delta_{ij}\delta_{pq}\mu \\ I_{ip}=\alpha\lambda\delta_{ip}w_{ip}C^{k}-\beta-\mu\end{cases}$$

To ensure the feasibility of the equilibrium point of the proposed CHN, we use the hyperplan method [4][5][16]. In this context, we propose the following feasible solution:

$$S\begin{cases} \emptyset = 2\mu - \alpha a(n-1)w_{max}^2 \\ \mu = \varepsilon - \frac{\alpha}{2}(W_{min} - W_{max}) - \frac{\lambda\alpha}{2}\left(C_{min}^k - C^k\right) \\ \beta = -2\varepsilon - \frac{\alpha}{2}(W_{max} + W_{min}) - \frac{\lambda\alpha}{2}\left(C^k + C_{min}^k\right) \end{cases} \quad (16)$$

Where:

$$\forall\, p, q \in \{1, \dots, k\}$$

$$w_{max} = Max_{i=1}^{n}(w_{ip})$$

$$w_{min} = Min_{i=1}^{n}(w_{ip})$$

$$W_{max} = Max_{i,j=1}^{n}(w_{ipjq})$$

$$W_{min} = Max_{i,j=1}^{n}(w_{ipjq})$$

5. Computational Experiments

In order to show the practical interest of the proposed approach, we have worked on a series of experimentations to solve the (SAP) problem. Most of the airspaces are generated as follows:

- the number of beacons n=50, 60, 70, 80, 90, 100, 150, 200, 250;
- the number of sectors 5, 6, 7, 8, 9, 10;
- the components of the vector weighs of beacons are generated from the interval [0,1];

Table1

Computational results of airspace sectorization according the number of beacons versus parameter by CHN approach

beacons / Parameter λ	40	50	60	70	80	90	100	150	200	250
0,5	3,52	3,84	3,98	4,5	5,1	6,25	6,98	7,5	8,9	9,96
1	3,012	3,53	3,64	3,96	4,2	5,12	6,32	7,01	8,55	9,023
1,5	2,84	3,12	3,4	3,56	4,023	4,97	5,23	6,5	7,101	8,5
2	2,134	2,98	3,34	3,48	3,86	4,25	4,96	6,03	6,94	8,12
2,5	2,231	2,312	2,86	3,001	3,45	4,001	4,546	5,5	6,45	7,35
3	2,56	2,64	2,902	3,25	3,5	4,34	4,89	5,8	6,25	7,86
3,5	2,82	2,96	3,012	3,56	3,96	4,74	5,12	5,98	6,95	8,15
4	3,25	3,102	3,45	3,74	4,194	4,98	5,801	6,3	7,076	8,91
4,5	3,46	3,54	3,64	3,96	4,9	5,6	6,11	6,96	7,5	9,15

- for two beacons i and j, we choose randomly a real x from the interval [0, 1]:

There is an airways between the beacons i and j if and only if the real x is superior than the value 0.5;

- the components of the coordination workload matrix are generated from the interval [0, 1].

We choose paramete $\lambda = 2.5$, $\alpha = 1.025$ (Fig.4), and $\varepsilon = 10^{-6}$; the parameters $\beta, \emptyset, and\ \mu$ are calculated from the system (S).

The initial states are randomly generated:

$$x_{ip} = 0.888 + \frac{n-i-p}{n}10^{-4}x$$

Where $i = 1, \dots, n$ and $p = 1, \dots, k$, i and x is a random uniform variable in the interval [0.25, 0.5].

The CPU time was recorded using an IBM compatible PC (Pentium IV, 1.82 GHz and 512 MB of RAM) running through Java language.

From the Table1, the proposed approach performs much better than the Constraint Satisfaction Programming (CSP) method with respect to the feasibility of the proposed sectorisation, total gap of

the desired workload, and the CPU time. In fact, the proposed approach allows rapid sectorization.

airspace in comprised with the CSP method. In addition, the proposed method does not violate any constraint assignment

Besides, the total gap of the desired workload tend to zero when the parameter a becomes large. This means that the proposed approach allows sectorization airspace which equilibrates the workload between the k sectors.

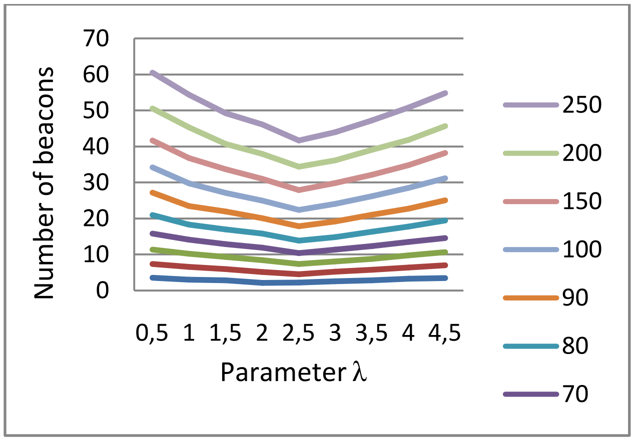

Fig.4: Total gap workload in sector Versus Control Parameter a in objective function

Table 2

Computational results: for execution time of airspace sectorization according the number beacons by CHN approach.

Beacons b / ***Parameter λ***	***40***	***50***	***60***	***70***	***80***	***90***	***100***	***150***	***200***	***250***
0,5	0,01	0,013	0,024	0,03	0,05	0,059	0,082	0,237	0,332	0,694
1	0,034	0,046	0,05	0,056	0,059	0,066	0,134	0,284	0,435	0,73
1,5	0,04	0,05	0,056	0,062	0,068	0,072	0,142	0,296	0,452	0,75
2,5	0,06	0,065	0,069	0,072	0,084	0,098	0,145	0,34	0,49	0,801
4	0,073	0,076	0,082	0,089	0,143	0,152	0,156	0,42	0,52	0,832
4,5	0,082	0,089	0,123	0,143	0,198	0,23	0,286	0,532	0,632	0,898

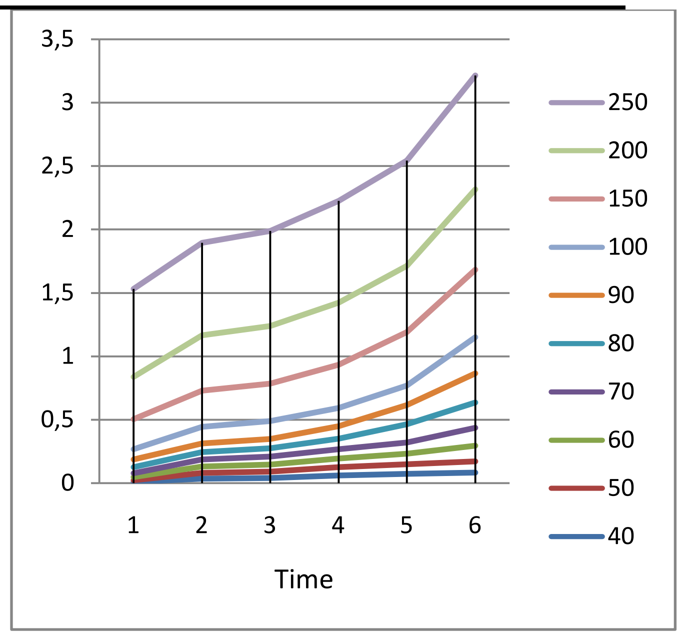

Fig.5: Time according the beacons number

In Fig.4, the graph presents total error of workload for airspace sectorization. This error evolves according control parameter λ, is a convex function the value minimum λ=2,5. In addition this error increases with beacons and sectors numbers in airspace.

In Fig.5, the graph present execution time of airspace sectorization is an exponential function according the beacons numbers in airspace and sectors numbers.

6. CONCLUSION AND FUTURE WORK

In this work, the sectorization of airspace problem is formulated as 0-1 quadratic programming subject to linear constraints. The proposed model minimizes the total load coordination and equilibrates the workload between the proposed sectors. Given its ability to provide a feasible solution in real time, we have used the continuous Hopfield networks to solve the proposed model. In this regard, we have proposed an adequate energy function. To ensure the feasibility of the equilibrium point, we have used the hyper-plan method. Besides, the proposed approach performs much better than the CSP method with respect to the feasibility of the proposed sectorization, total gap of the desired workload, and the CPU time. In the future, we will propose a new model for the sectorization of airspace problem. This latter will improve the model (SAP) by taking into account the geometrical constraints.

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