2D Wireless Sensor Network Deployment Based on Centroidal Voronoi Tessellation

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Abstract: - In recent years, Wireless Sensor Networks (WSNs) have rapidly evolved and now comprise a powerful tool in monitoring and observation of the natural environment, among other fields. The use of WSNs is critical in early warning systems, which are of high importance today. In fact, WSNs are adopted more and more in various applications, e.g. for fire or deformation detection. The optimum deployment of sensors is a multi-dimensional problem, which has two main components; network and positioning approach. Although lots of work has dealt with the issue, most of it emphasizes on mere network approach (communication, energy consumption) and not on the topography (positioning) of the sensors in achieving ideal geometry. In some cases, it is hard or even impossible to achieve perfect geometry in nodes' deployment. The ideal and desirable scenario of nodes arranged in square or hexagonal grid would raise extremely the cost of the network, especially in unfriendly or hostile environments. In such environments the positions of the sensors have to be chosen among a list of possible points, which in most cases are randomly distributed. This constraint has to be taken under consideration during the WSN planning. Full geographical coverage is in some applications of the same, if not of greater, importance than the network coverage. Cost is a crucial factor at network planning and given that resources are often limited, what matters, is to cover the whole area with the minimum number of sensors. This paper suggests a deployment method for nodes, in large scale and high density WSNs, based on Centroidal Voronoi Tessellation (CVT). It approximates the solution through the geometry of the random points and proposes a deployment plan, for the given characteristics of the study area, in order to achieve a deployment as near as possible to the ideal one.

Key-Words: - WSN, Wireless Sensor Network, CVT, Centroidal Voronoi Tessellation, sensors, sensor deployment, spatial covarage

1 Introduction

Recently, Wireless Sensor Networks (WSNs) have rapidly evolved and now are a powerful tool for monitoring and observation of the natural environment, among other fields. The use of WSNs is critical in early warning systems, which are of high importance today. In fact, WSNs are adopted more and more in various applications, e.g. for fire or deformation detection.

Lots of work has been done for the optimization of such networks and concern both geographical and network coverage. Full geographical coverage of an area is meaningless, if communication between the sensors is weak or the energy consumption is extremely high. On the other hand, even if the connection problems are solved, the network fails its mission if geographical coverage is not achieved.

Algorithms for geographical coverage can be found. In most cases, deployment is based on achieving the ideal geometry (i.e. deployment on regular square or hexagonal grid). The ideal geometry offers full coverage, but is not always possible to achieve.

When an application requires thousands of sensors, the deployment in ideal positions, increases significantly both the cost and the deployment time. Thus, the deployment positions must belong to a possible set (e.g. a list of coordinates), derived from the definition of the problem. Such positions could be specific points on a slope, in the case of deformation monitoring or specific trees in a large forest area, in an application for fire alarm.

In this paper a deployment method for nodes, in large scale and high spatial density WSNs is suggested. The proposed method is based on

Centroidal Voronoi Tessellation (CVT). The solution is approximated through the geometry of the random distributed points and a deployment plan is proposed. The plan takes into consideration the given characteristics of the study area, in order to achieve a deployment as near as possible to the ideal one [1].

The rest of this paper is organized as follows: In Section 2, previous works related to the area coverage by sensors are described. In Section 3, the theoretical background for CVT is given. Section 4, the suggested methodology is thoroughly explained. Finally, Section 5 concludes the paper.

2 Related Work

The scientific community has been occupied to a significant extent with finding the optimum solution for deployment of WSN nodes. Algorithms for optimizing network communication can be found in many papers and variations. An indicated work in communication optimization for large-scale WSNs has been produced by Toumpis & Tassiulas [2]. An algorithm based on Centroidal Voronoi Tessellation (CVT) is proposed as a method for optimizing the communication by Zhou, Jin, & Wu [3].

A heuristic algorithm for deployment of WSNs in wide areas and high spatial density is proposed by Kolega [4], [5], [6]. In this model, the final node positions are compared to these of an ideal hexagonal grid. There is the constraint that the nodes must be deployed in certain positions chosen from a l ist of possible ones (the coordinates are given and correspond to tree positions for a fire detection WSN). The area is divided into smaller parts, using Quad-trees algorithm and grouped with the k-means algorithm. The sensor range (that is the length of the ideal hexagonal grid) and a value E0 are defined. The nodes that are within the distance E0 from the ideal grid are found and locked as deployment positions. Obviously as E0 increases, more points are locked, but the geometry deviates from optimum.

A solution, which achieves full geographical coverage while guaranteeing communication for large areas is proposed by Wang, Hu, & Tseng [5]. It compares the sensing range rs with the communication range rc of the sensor and two scenarios are examined, depending on which one is bigger. A similar coverage method based on sensing and communication range are proposed by Zhang & Zhou [6].

An approach that uses Voronoi Diagram (VD) is proposed by Vieira et al [7]. As in the case studied in this paper, the potential deployment positions are a priori known. The sensors are considered to be placed at all points, and the corresponding VD is produced. Then, the point with the smallest polygon is removed, as the area is then supervised by the adjacent sensors. The process is repeated till all Voronoi Polygons reach a specified threshold. For a large number of sensors the algorithm becomes extremely time consuming.

Delaunay Triangulation (DT) is the basis for coverage algorithms. Its advantages compared to the square grid are discussed by Darag & Saroha [8]. Wang & Medidi [9] propose a methodology to minimize energy consumption and achieve complete coverage of the area, but they study the ideal geometry scenario, as well. Vu & Li [10] improve the aforementioned algorithm studying the boundary effect, but they mainly focus on minimizing energy consumption. A different coverage proposal using DT is set by Wu, Lee, & Chung [11]. The idea of the gradual elimination of nodes through the Delaunay Triangulation with constraints (CDT) is proposed by Devaraj [12].

Another study has been conducted by Argany et al [13]. They gather and record different coverage algorithms for WSN. They focus on algorithms based on DT and VD, and propose a solution that uses Voronoi polygons based on spatial information (physical boundaries, DTM etc.).

A solution that uses mobile sensors is proposed by Song et al [14]. The sensors move individually to the deployment positions. Although this solution is ideal for hostile or dangerous to human environments, the development and installation costs of such a mobile WSN are extremely high.

3 Preliminaries

3.1 Centroidal Voronoi Tessellation (CVT)

By the term Voronoi Diagram (VD), in 2D space, is meant a p artitioning of a subset of the plane into convex polygons with specific properties. Each polygon is generated from a point - generator, so that each point inside the polygon, being closest to this generator, than to any other.

In mathematical terms, given a set of points $\{z_i\}_{i=1}^{k}$ belonging to the closed set $\overline{\Omega} \in \mathbb{R}^N$, the Voronoi region $\widehat{V}_i$ corresponding to the point z_i, is defined by [15]:

$$\widehat{V}_i = \{x \in \Omega \mid \|x - z_i\| < \|x - z_j\| \; \forall\, j = 1, \dots, k, \; j \neq i\} \quad (1)$$

Despite the flexibility and the number of fields in which Voronoi Diagrams (VD) can be used, they have characteristics which are undesirable in many other applications. Most important is that the generator of a Voronoi polygon does not coincide with the centroid of the polygon. When the only criterion for creating polygons is the distance of the points from the generator, the edges created depend only from on the relative positions of the generators. Thus, the Voronoi polygons have irregular shape with different edge sizes.

A CVT is a special Voronoi diagram, where the generating point of each Voronoi cell is also its mean (i.e., center of mass) [3]. Substantially, it approximates an ideal partition of the area, through the optimal allocation of the generators. According to Gersho's conjecture, "as the number of generators increases, the optimum CVT will form a uniform partitioning of the space, with shapes that would result from the repetition of a single polytope. The shape of the polytope only depends from the spatial dimension". In 2D the basic polygon is a regular hexagon [16].

3.2 Mathematical Model of CVT

Comparing the mathematical model of the construction of a Voronoi Diagram, the construction of a CVT, has an additional constraint. The generator of the polygon must also be the centroid of each polygon.

In Fig. 1, t he difference between a VD and a CVT is shown. In Fig. 1(a), where a VD is represented, the black dots are the Voronoi generators and the circles are the corresponding centroids. In Fig. 1(b) dots are simultaneously the generators for the Voronoi Tessellation and the centroids of the Voronoi regions [15].

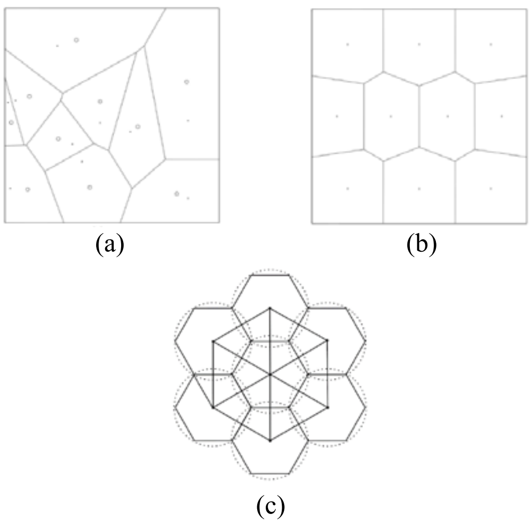

FIG 1. Difference between a VD (a) and CVT (b) and the ideal CVT geometry (c)

Given a region $V \subset \mathbb{R}^N$, and a density function ρ, defined in V, the mass centroid z^* of V is defined by the equation 2.

$$z^* = \frac{\int_V y\,\rho(y)\,dy}{\int_V \rho(y)\,dy} \quad (2)$$

Given k points $z_i,\ i = 1, \ldots, k$, their associated Voronoi regions, $V_i,\ i = 1, \ldots, k$, can be defined. Moreover, given the Voronoi polygons, $V_i,\ i = 1, \ldots, k$ their mass centroids, $z_i^*,\ i = 1, \ldots, k$, can also be defined. What matters is that the generators of the polygons be the mass centroids too: $z_i = z_i^*$, $i = 1, \ldots, k$.

This partition is called Centroidal Voronoi Tessellation (CVT). In the special situation that the density function is constant and uniform, the CVT tends to consists from regular hexagons, as shown in Fig. 1(c).

The construction of Centroidal Voronoi Tessellation can also be extended even if, instead of a region Ω, a discrete set of points $W = \{y_i\}_{i=1}^{m}, W \in \mathbb{R}^N$ is given. In this case the centroids of the polygons are the means of the points that are closer to them. In literature several algorithms for CVT construction are recorded, with most common the Lloyd's algorithm [17]. Other algorithms are the algorithm of MacQueen, the iterative method of Newton, and hybrid approaches of them [15], [17].

4 Sensor Deployment Methodology Based on CVT

In limited field applications where the sensors to be deployed are few, the deployment can be done to the positions arising after the CVT construction. These positions are the center of mass of the polygons and they do not refer to specific points of the original dataset. There are cases that the deployment positions must belong to the original set. Therefore result additional constraints [1].

Thus, the solution will be approached in two phases: a) Determination of the theoretical positions as they arise for the given geometry, i.e. CVT generators, and b) Find the actual deployment position, by moving the theoretical points to the closest real position.

4.1 Phase one: Determination of the theoretical deployment positions

In order to be able to construct CVT polygons, it is necessary to have a large dataset of points, as a discontinuous subset of the study area. Among these, the final deployment position will be selected. It is considered that the initial dataset is quite dense, to obtain a satisfactory geometry.
In the case of subareas without points the entire area could be divided into smaller parts and each one managed separately. An important parameter is the minimum number of sensors (or CVT polygons generators). This has to do with the size of the area and the sensor's observation range. To determine the size of area convex hull is used, as in most computational geometry problems [18], [19]. The sensing range determines the radius Rs of a circle around the sensor, inside which the phenomenon can be supervised. An approximation for the minimum number N is the division of the total surface area to the sensing area corresponds to a specific range.

$$N = \frac{\text{Total\_area}}{\pi \times R_s^2} \tag{3}$$

The required sensors can't be less than that, and probably during the process will come up that this number is insufficient for full coverage and the algorithm should be ran again with a bigger number of sensors.
Last parameter is the number of iterations till the algorithm stops. The process terminates when none of the N sensors (polygons generators), moves more than a value (threshold) during the Lloyd's algorithm. This threshold depends on the needs of each application.
Summing, the problem can be described as follows:

Given:

- The coordinates (x,y) of the candidate points inside the study area
- The sensing range Rs, of the sensor to be used
- The termination condition for the Lloyd's algorithm.

Find:

- The theoretical deployment positions, that correspond to the CVT generators.
- The corresponding CVT diagram

The result of the iterative process is the coordinates of the CVT generators. Moreover, the CVT polygons plot visualizes how well the desired geometry is achieved. Useful are some additional statistical indicators such as the number of iterations until termination of the algorithm.

4.2 Phase two: Determination of the actual deployment positions

Up to this point, the solution would be acceptable and the methodology would be useful for limited-range applications. But, as mentioned, a basic constraint is that the final deployment positions must belong to the original dataset. Initially, it is checked if any of the points of the theoretical solution are identical to the actual/original points. The user defines the tolerance range (the maximum distance between a theoretical and an actual point, within it is considered that the two points are the same). This tolerance is defined according to the needs of each application. The more severe the requirements of the application are, the lower the tolerance will be, and therefore likely less of theoretical and actual points to be matched.
For the not-matched theoretical points, the nearest neighbors that belong to the initial dataset are found. A radius is defined. Within this radius the nearest neighbor is searched and the point is moved. It is obvious, that the smaller this radius is, the more the final solution will resemble the theoretical CVT geometry. But it is harder to find a real point within this radius. Conversely, as the radius increases, the more likely is to find an actual point therein, but the final solution will deviate from the theoretical CVT.
At the end of the procedure the final deployment positions (coordinates) are extracted. Moreover, one can derive useful statistics such as the percentage of points that are not in any sensor's observation area or the percentage of all points that are supervised by two or more sensors (network redundancy). From these statistics, the user can feedback the system by increasing (or decreasing) the number of sensors in order to achieve the desired coverage or redundancy.
There were scenarios with different number of points, areas of different dimensions and various sensing range Rs in order to test the proposed methodology. In the images that follow an area with dimensions 500mx500m is presented and two basic scenarios were created. One with 2000 points inside the area (fig.2) and one with 5000 points inside the area (fig. 3).
For the first basic scenario are presented the CVT and the final solution for Rs=50 m & N=31 sensors (minimum needed) (fig. 2a, 2b), Rs=50 m & N=50 sensors (fig. 2c, 2d), Rs=25 m & N=123 sensors

(minimum needed) (fig. 2e, 2f), Rs=25 m & N=150 sensors (fig. 2g, 2h).

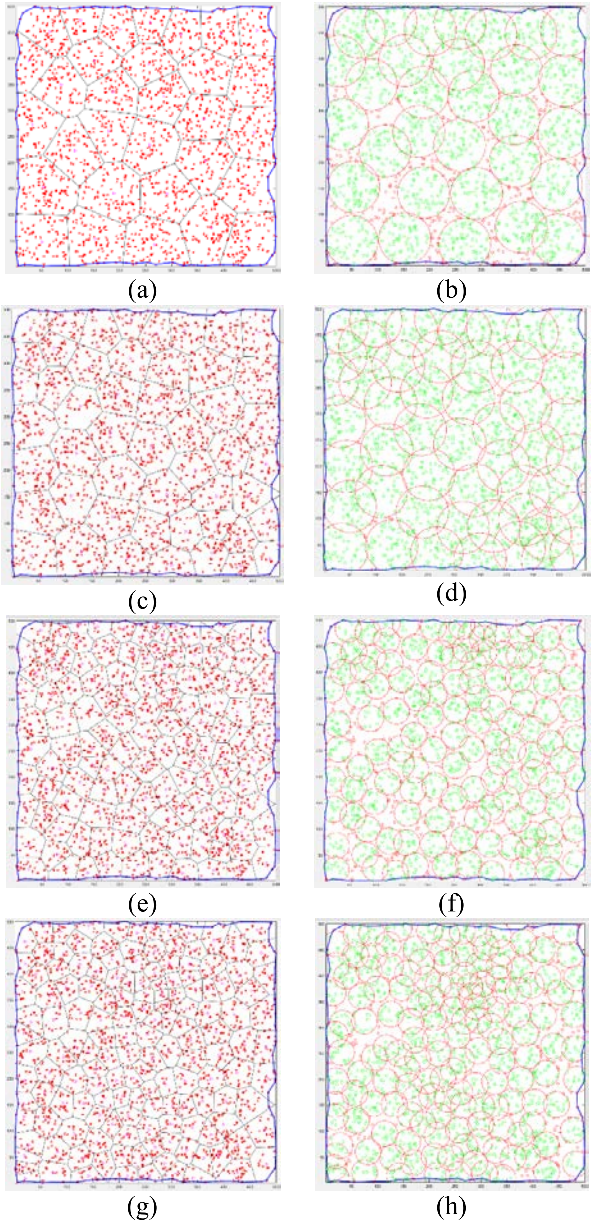

FIG. 2. The different scenarios that were created for 2000 points

Respectively, for the second basic scenario (5000 points inside a 500m x 500m area) are presented the CVT and the final solution for Rs=50 m & N=31 sensors (minimum needed) (fig. 3a, 3b), Rs=50 m & N=50 sensors (fig. 3c, 3d), Rs=25 m & N=123 sensors (minimum needed) (fig. 3e, 3f), Rs=25 m & N=150 sensors (fig. 3g, 3h).

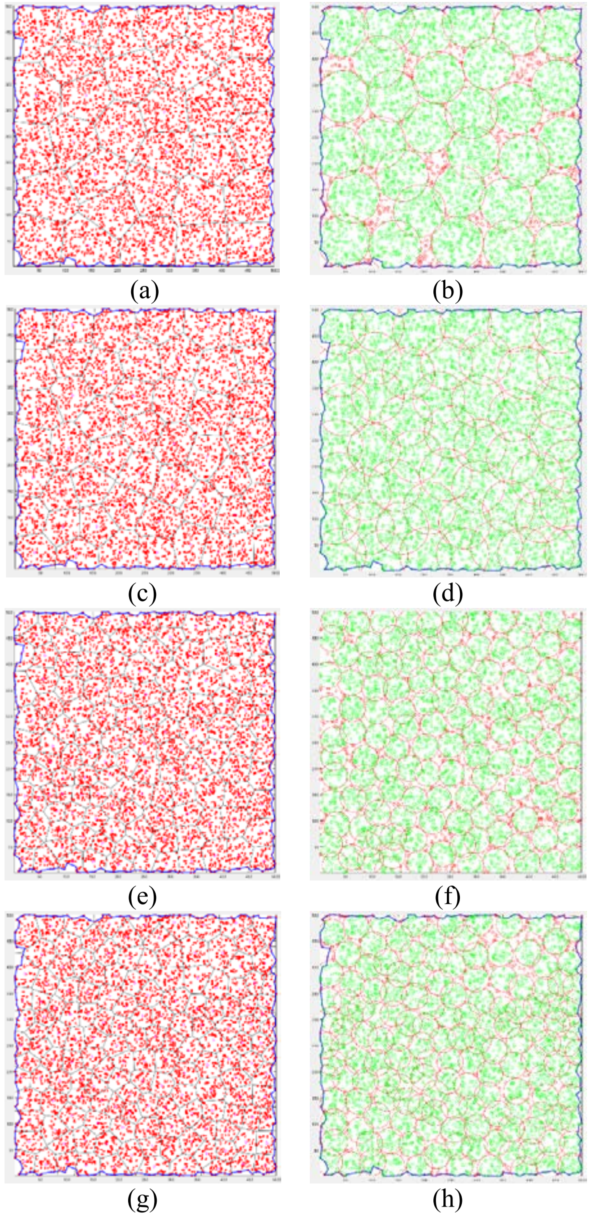

FIG. 3. The different scenarios that were created for 5000 points

5 Conclusions

Wireless Sensor Networks (WSNs) are increasingly used the last few years to support a wide variety of applications, such as environmental monitoring, structural monitoring, security detection, just to name a few. In all cases apart from the need for network operation, full geographical coverage is required, taking into consideration the constraints of each case (given the current, at each case constraints). One important constraint is to deploy the sensors in a m anner that will achieve full coverage with the minimum number of nodes.

This paper proposes an algorithm for sensors' deployment using Centroidal Voronoi Tessellation (CVT). The sensing area is modeled as a p olygon and the possible deployment positions appear as points inside it. The deployment benefits from the properties of CVT and ensures that each sensor is placed as far away as possible from its neighbors. Thus, it guarantees the minimum sensor nodes for the given geometry of the area and the sensing range.

For the CVT construction from discrete points, the Lloyd's algorithm is used. So the proposed methodology is easy to be programmed.

The final solution consists of close-to-equilateral triangles (or, alternatively, close-to-hexagonal grid), which has many advantages not only in geographical coverage, but also in a variety of other applications, one of which is the desired network topology. Beside this, the more the points density increases, the closest to the ideal geometry is achieved. In the current paper 2D convex hull was examined. Many simulations have been carried out under various network shapes, number of sensors and sensor radii to evaluate the effectiveness and efficiency of the proposed algorithm. For future work concave polygons and polygons with holes will be examined.

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