Multiobjective Optimization of Construction Project Time-Cost-Quality Trade-off Using a Genetic Algorithm

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***Abstract*—** In construction projects, time, cost and quality are the most important factors to be considered. In the present study a hybrid genetic algorithm is used to solve this multiobjective time-cost-quality optimization problem. The chromosome representation of the problem is based on random keys. The schedules are constructed using a priority rule in which the priorities are defined by the genetic algorithm. The execution mode of each activity is selected by the genetic algorithm. Schedules are constructed using a procedure that generates parameterized active schedules. The results indicate that this approach could assist decision-makers to obtain good solutions for project duration, cost and incorporating quality with minimum function evaluation.

***Keywords*—** Construction Management, Project management, Genetic algorithms, Multiobjective optimization, Time-cost-quality trade-off.

1 Introduction and Background

Construction projects are found throughout business and areas such as manufacturing facilities, infrastructure development and improvement, and residential and commercial building.

As projects are unique in nature, the creation of a schedule for construction tasks by a planner, for example, should consider an array of conditions such as technological and organizational methods and constraints, as well as the availability of resource to ensure that a client's needs and requirements in terms of time, cost and quality are met (Jaskowski and Sobotka [44]).

In a construction project, there are two main factors, such as project duration and project cost. The activity duration is a function of resources (i.e. crew size, equipments and materials) availability. On the other hand, resources demand direct costs. Therefore, the relationship between project time and direct cost of each activity is a monotonously decreasing curve. It means if activity duration is compressed then that leads to an increase in resources and so that direct costs. But, project indirect costs increase with the project duration. In general, for a project, the total cost is the sum of direct and indirect costs and exists an optimum duration for the least cost, see Fig.1. Hence, relationship between project time and cost is trade-off [36].

Since the cost and time are two of the most important objectives which are easily quantified in a construction project, time-cost tradeoff problem has been researched for a long time. Several approaches have been used to solve the construction scheduling problem and they can be classified as mathematical, heuristic and search methods.

Several mathematical models such as linear programming (Kelley [12]; Hendrickson and Au [4]; Pagnoni [2]), integer programming, or dynamic programming (Butcher [33]; Robinson [8]; Elmaghraby [27]; De et al. [25]) and LP/IP hybrid (Liu et al. [21]; Burns et al. [29]), Meyer and Shaffer [31] and Patterson and Huber [14] use mixed integer programming. However, for large number of activity in network and complex problem, integer programming needs a lot of computation effort (Feng et al. [6]).

Heuristic algorithms are not considered to be in the category of optimization methods. They are algorithms developed to find an acceptable near optimum solution. Heuristic methods are usually algorithms easy to understand which can be applied to larger problems and typically provide acceptable solutions (Hegazy [30]). However, they have lack mathematical consistency and accuracy and are specific to certain instances of the problem (Fondahl [19]; Prager [32]; Siemens [23] and Moselhi [24]) are some of the research studies that have utilized heuristic methods for solving TCO problems.

Some researchers have tried to introduce evolutionary algorithms to find global optima such as genetic algorithm (GA) (Feng et al. [6]; Gen and Cheng [22]; Zheng et al. [10]; Zheng and Ng [9]; Mendes [39]); the particle swarm optimization algorithm (Yang [11]), ant colony optimization (ACO) (Xiong and Kuang [34]; Ng and Zhang [29]; Afshar et al. [1]) and harmony search (HS) (Geem [36]).

Quality is an important parameter correlating highly with time and cost parameters. But it is not a quantitative parameter in nature, practical time-cost-quality tradeoff models are seldom developed from previous research works of the literature [38].

Babu and Suresh [40] proposed a framework to study the tradeoff among time, cost, and quality using three interrelated linear programming models. Khang and Myint [41] applied the linear programming models in an actual cement factory construction project, which was depicted by a 52-activity CPM incorporated with their time, cost, and quality. Tareghian and Taheri [42] developed a solution procedure to study the tradeoffs among time, cost and quality in the management of a project. This problem assumes the duration and quality of project activities to be discrete, non-increasing functions of a single non-renewable resource. Three interrelated integer programming models are developed such that each model optimizes one of the given entities by assigning desired bounds on the other two.

Hu and He (37) presented a time-cost-quality optimization model that enables managers to optimize multiobjectives. The model is from the project breakdown structure method where task resources in a construction project are divided into a series of activities and further into construction labors, materials, equipment, and administration. The resources utilized in a construction activity would eventually determine its construction time, cost, and quality, and a complex time-cost-quality trade-off model is finally generated based on correlations between construction activities. A genetic algorithm tool is applied in the model to solve the comprehensive nonlinear time-cost-quality problems.

Narayanan and Suribabu [38] developed a differential evolution algorithm to solve the multiobjective time-cost-quality optimization problem.

El-Rayes and Kandil [43] presented a multiobjective model to transform the traditional two-dimensional time-cost tradeoff analysis to an advanced three-dimensional time-cost-quality trade-off analysis. The model is developed as a multiobjective genetic algorithm to provide the capability of quantifying and considering quality in construction optimization. An application example is analyzed to illustrate the use of the model and demonstrate its capabilities in generating and visualizing optimal tradeoffs among construction time, cost, and quality.

Although the objectives of cost and time might be mentioned frequently by natural numbers, the objective of quality is seldom described in quantities, which worsens numerical tradeoff among project time, cost, and quality [38]. This paper will present a new solution for solving the time-cost-quality tradeoff problem using a hybrid genetic algorithm with an evaluation function based on the work of Mendes [39].

2 Problem description and Formulation

Project time-cost-quality tradeoff problem (PTCQTP) can be defined as follows: a project is represented by an activity-on-node network with n+2 activities that is an acyclic digraph G = (A), where A = {0, 1,. . . , n+1} is the set of nodes (construction activities). In the network both node (0) and node (n+1) are dummy activities. P is the set of all paths in the activity-on-node network, starting from activity (0) and ending at activity ($n+1$) and P_l is the set of activities contained in path $l \in P$.

Each activity $i \in$ A is associated with its time T_i, cost C_i, and quality Q_i. The project time-cost-quality performance is essentially formed from each activity's time, cost, and quality, respectively [37].

To model the multiobjective time-cost-quality optimization problem an evolutionary technique is used incorporating a genetic algorithm.

With evolutionary techniques being used for single-objective optimization for over two decades, the incorporation of more than one objective in the fitness function has finally gained popularity in the research [3].

In principle, there is no clear definition of an ''optimum'' in multiobjective optimization (MOP) as in the case of single-objective issues; and there even does not necessarily have to be an absolutely superior solution corresponding to all objectives due to the incommensurability and conflict among objectives. Since the solutions cannot be simply compared with each other, the ''best'' solution generated from optimization would correspond to human decision-makers subjective selection from a potential solution pool, in terms of their particulars [10].

The classical methods reduce the MOP to a scalar optimization optimization by using multiobjective weighting (MOW) or a utility function (multiobjective utility analysis). Multiobjective weighting allows decisions makers to incorporate the priority of each objective into decision making. Mathematically, the solutions obtained by equally weighting all objectives may provide the least objective conflicts, but in most cases, each objective is first optimized separately and the overall objective value is evaluated depending on the weighting factors. The weakness of MOW is that the overall optimum is usually at the dominating objective only [6].

In a certain way we can say that the work of Zadeh [20] is the first to advocate the assignment of weights to each objective function and combined them into a single-object function.

In this paper the objective function expressing the cost, time and quality of the project can be expressed in the following way:

$$Min\,Z = W_t \frac{(T - T_{\min})}{(T_{\max} - T_{\min})} + W_c \frac{(C - C_{\min})}{(C_{\max} - C_{\min})}) + W_q \frac{(Q_{\max} - Q)}{(Q_{\max} - Q_{\min})} \qquad (1)$$

where,

W_t, W_c and Wq are the new adaptive weights for time, cost and quality given by:

$W_t = gene_t;\ W_c = gene_c;\ W_q = gene_q.$

$C_{\max}$ = maximal value for total cost in the current chromosome;
$T_{\max}$ = maximal value for time in the current chromosome;
$C_{\min}$ = minimal value for total cost in the initial population;
$T_{\min}$ = minimal value for time in the initial population;
C = represents the total cost of the xth solution in current chromosome;
T = represents the time of the xth solution in current chromosome;
$Q_{\max}$ = maximal value for quality in the current chromosome;
$Q_{\min}$ = minimal value for quality in the current chromosome;
Q = represents the quality of the xth solution in current chromosome.

3 The approach

The approach presented in this paper is based on a genetic algorithm to perform its optimization process. Fig. 1 shows the architecture of approach.

The approach combines a genetic algorithm, a schedule generation scheme and a local search procedure. The genetic algorithm is responsible for evolving the chromosomes which represent the priorities of the activities.

For each chromosome the following four phases are applied:

1) *Transition parameters* - this phase is responsible for the process transition between first level and second level;
2) *Schedule parameters* - this phase is responsible for transforming the chromosome supplied by the genetic algorithm into the priorities of the activities and delay time;
3) *Schedule generation* - this phase makes use of the priorities and the delay time and constructs schedules;
4) *Schedule improvement* - this phase makes use of a local search procedure to improve the solution obtained in the schedule generation phase.

After a schedule is obtained, the quality is processed feedback to the genetic algorithm. Fig. 1 illustrates the sequence of phases applied to each chromosome. Details about each of these phases will be presented in the next sections.

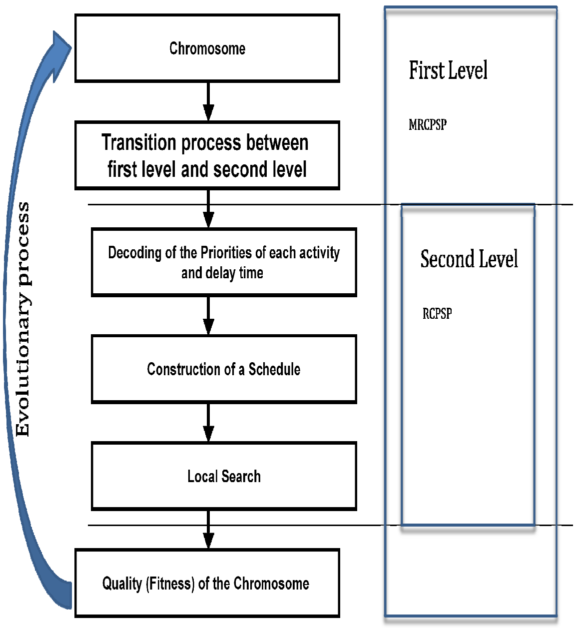

Fig. 1. Architecture of the approach.

3.1 GA Transition Process

The Genetic Algorithms (GAs) are search algorithms which are based on the mechanics of natural selection and genetics to search through decision space for optimal solutions. One fundamental advantaged of GAs from traditional methods is described by Goldberg [7]: in many optimization methods, we move gingerly from a single solution in the decision space to the next using some transition rule to determine the next solution.

First of all, an initial population of potential solutions (individual) is generated randomly. A selection procedure based on a fitness function enables to choose the individual candidate for reproduction. The reproduction consists in recombining two individuals by the crossover operator, possibly followed by a mutation of the offspring. Therefore, from the initial population a new generation is obtained. From this new generation, a second new generation is produced by the same process and so on. The stop criterion is normally based on the number of generations.

The GA based-approach uses a random key alphabet U (0, 1) and an evolutionary strategy identical to the one proposed by Goldberg [7].

Each chromosome represents a solution to the problem and it is encoded as a vector of random keys (random numbers). Each solution encoded as initial chromosome (first level) is made of $mn+n$ genes where n is the number of activities and m is the number of execution modes, see Fig. 2.

The called first level as the capacity to solving the multi-mode resource constrained project scheduling problem (MRCPSP) [16, 18].

In this case of study we do not consider the requirements to the type and number of resources needed for construction mode for each activity as well as the maximum number of available resources.

The transition process between first level and second level consists in choosing the option or construction mode m_j for each activity j. Using this process we obtain the solution chromosome (second level) composed by $2n$ genes+3.

The called second level as the capacity to solving the resource constrained project scheduling problem (RCPSP) [16, 18].

In this case of study we do not consider the requirements to the type and number of resources needed for each activity as well as the maximum number of available resources.

Genes for Activity 1	Activity 1	Mode 1	Gene $_{11}$
		Mode 2	Gene $_{12}$
		…	…
		Mode m	Gene $_{1m}$
		Delay 1	Gene $_{1m+1}$
Genes for Activity 2	Activity 2	Mode 1	Gene $_{21}$
		Mode 2	Gene $_{22}$
		…	…
		Mode m	Gene $_{2m}$
		Delay 2	Gene $_{2m+1}$
….	…		…
Genes for Activity n	Activity n	Mode 1	Gene $_{n1}$
		Mode 2	Gene $_{n2}$
		…	…
		Mode m	Gene $_{nm}$
		Delay n	Gene $_{nm+1}$
Genes for Objective Function	OF	Time	Gene $_{t}$
		Cost	Gene $_{c}$
		Quality	Gene $_{q}$

Fig. 2. Chromosome structure.

After, we evaluate the quality (fitness) of the solution chromosome.

3.2 GA Decoding

A real-coded GA is adopted in this paper. Compared with the binary-code GA, the real-coded GA has several distinct advantages, which can be summarized as follows (Y.-Z. Luo et al. [35]):

- It is more convenient for the real-coded GA to denote large scale numbers and search in large scope, and thus the computation complexity is amended and the computation efficiency is improved;
- The solution precision of the real-coded GA is much higher than that of the binary-coded GA;
- As the design variables are coded by floating numbers in classical optimization algorithms, the real-coded GA is more convenient for combination with classical optimization algorithms.

The priority decoding expression uses the following expression:

$$PRIORITY_j = \frac{LLP_j}{LCP} \times \left[\frac{1 + gene_{mj}}{2} \right] \quad j = 1, \ldots, n \qquad (2)$$

where,

[1] LLP_j is the longest length path from the beginning of the activity j to the end of the project;
[2] LCP is the length along the critical path of the project [15];
[3] m_j is the gene of the selected mode for activity j.

The gene jm+1 is used to determine the delay time when scheduling the activities. The delay time used by each activity is given by the following expression:

$$Delay\,time = gene_{jm+1} \times 1.5 \times MaxDur \qquad (3)$$

where *MaxDur* is the maximum duration of all activities. The factor 1.5 is obtained after some experimental tuning.

A maximum delay time equal to zero is equivalent to restricting the solution space to non-delay schedules and a maximum delay time equal to infinity is equivalent to allowing active schedules. To reduce the solution space is used the value given by formula (3), see Gonçalves et al. [13].

3.3 Construction of a Schedule

Schedule generation schemes (SGS) are the core of most heuristic solution procedures for project scheduling. SGS start from scratch and build a feasible schedule by stepwise extension of a partial schedule.

There are two different classics methods SGS available. They can be distinguished into activity and time incrementation. The so called serial SGS performs activity-incrementation and the so called parallel SGS performs time-incrementation.

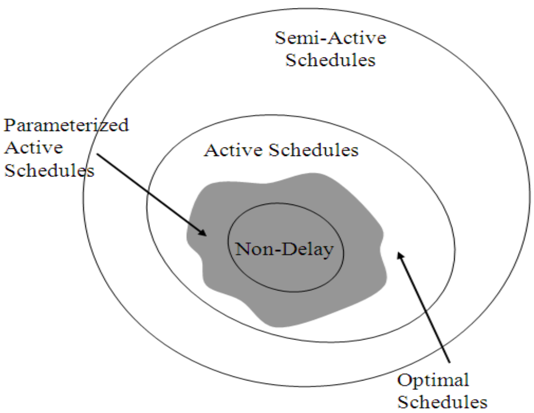

Fig. 3. Types of schedules (adapted from Mendes [18]).

A third method for schedule generating can be applied: the parameterized active schedules. This type of schedule consists of schedules in which no resource is kept idle for more than a predefined period if it could start processing some activity. If the predefined period is set to zero, then we obtain a non-delay schedule. This type of SGS is used on this work.

Fig. 3 presents the relationship diagram of various schedules with regard to optimal schedules.

3.4 Local Search

Local search algorithms move from solution to solution in the space of candidate solutions (the search space) until a solution optimal or a stopping criterion is found. In this paper it is applied backward and forward improvement based on Klein [27].

Initially it is constructed a schedule by planning in a forward direction starting from the project's beginning. After it is applied backward and forward improvement trying to get a better solution. The backward planning consists in reversing the project network and applying the scheduling generator scheme. A detailed example is described by Mendes [15].

3.5Evolutionary Strategy

There are many variations of genetic algorithms obtained by altering the reproduction, crossover, and mutation operators. Reproduction is a process in which individual (chromosome) is copied according to their fitness values (makespan). Reproduction is accomplished by first copying some of the best individuals from one generation to the next, in what is called an elitist strategy.

In this paper the fitness proportionate selection, also known as roulette-wheel selection, is the genetic operator for selecting potentially useful solutions for reproduction. The characteristic of the roulette wheel selection is stochastic sampling.

The fitness value is used to associate a probability of selection with each individual chromosome. If f_i is the fitness of individual i in the population, its probability of being selected is,

$$p_i = \frac{f_i}{\sum_{i=1}^{N} f_i} \quad , \qquad i = 1, \ldots, n \qquad (4)$$

A roulette wheel model is established to represent the survival probabilities for all the individuals in the population. Then the roulette wheel is rotated for several times [7].

After selecting, crossover may proceed in two steps. First, members of the newly selected (reproduced) chromosomes in the mating pool are mated at random. Second, each pair of chromosomes undergoes crossover as follows: an integer position k along the chromosome is selected uniformly at random between 1 and the chromosome length l. Two new chromosomes are created swapping all the genes between k+1 and l, see Mendes [16].

The mutation operator preserves diversification in the search. This operator is applied to each offspring in the population with a predetermined probability. We assume that the probability of the mutation in this paper is 5%.

3.6 GA Configuration
Though there is no straightforward way to configure the parameters of a genetic algorithm, we obtained good results with values: population size of 5 × number of activities in the problem; mutation probability of 0.05; top (best) 1% from the previous population chromosomes are copied to the next generation; stopping criterion of 30 generations.

4 Case Study
In order to compare the proposed RKV-TCQ (Random Key Variant for Time-Cost-Quality) approach, a case study of seven activities proposed originally by Feng al. [6] was used.

The same example was later investigated by Zheng et al. [9], Afshar et al. [45], Lakshminaryana et al. [46] and Narayanan and Suribabu [38] using different optimization approaches. The data presented in Table 1 is obtained from Afshar et al. [46]. For comparison, the indirect cost is assumed to be zero.

Table 1 Detailed data of the example.

Activity description	Activity number	Precedent activity	Option/ Mode	Duration (days)	Direct cost ($)	Weight (%)	Quality (%)
Site preparation	1	-	1	14	23,000	8	98
			2	20	18,000		89
			3	24	12,000		84
Forms and rebar	2	1	1	15	3,000	6	99
			2	18	2,400		95
			3	20	1,800		85
			4	23	1,500		70
			5	25	1,000		59
Excavation	3	1	1	15	4,500	14	98
			2	22	4,000		81
			3	33	3,200		63
Precast concrete girder	4	1	1	12	45,000	19	94
			2	16	35,000		76
			3	20	30,000		64
Pour foundation and piers	5	2, 3	1	22	20,000	17	99
			2	24	17,500		89
			3	28	15,000		72
			4	30	10,000		61
Deliver PC girders	6	4	1	14	40,000	19	100
			2	18	32,000		79
			3	24	18,000		68
Erect girders	7	5, 6	1	9	30,000	17	93
			2	15	24,000		71
			3	18	22,000		67

The robustness of the new proposed model RKV-TCQ in the deterministic situation was compared with three other previous models:

1) Afshar et al. [45];
2) Lakshminaryana et al. [46];
3) Narayanan and Suribabu [38].

The Table 2 shows the results of the present approach and other methods. Direct comparison shows that RKV-TCQ provided the best time, cost and quality when compared with the best approaches.

Additionally we can also state that the RKV-TCQ approach produces high-quality solutions quickly once needed only 1 second to complete 30 generations.

The Table 3 shows the results of the present approach and other methods for an optimization problem with time and cost. Direct comparison shows that RKV-TCQ provided the best time and cost when compared with the best approach.

Table 2 Comparison of approaches.

Approaches	Models	Time (days)	Cost ($)	Quality (%)	Resource Option
Lakshminaryana et al. [46]	MOOM	60	165500	97	1/1/1/1/1/1/1
Afshar et al. [45]	*50, 30 - MOACO	60	155500	92	1/1/1/2/1/1/1
Narayanan and Suribabu [38]	*30, 30 DE APPROACH (Method 1)	60	165500	97	1/1/1/1/1/1/1
Narayanan and Suribabu [38]	*30, 30 DE APPROACH (Method 2)	60	165500	97	1/1/1/1/1/1/1
This paper	*30, 35 RKV-TCQ	60	165500	97	1/1/1/1/1/1/1

*50, 30 – the first number is the number of iterations and the second is the population size.

Table 3 Comparison of approaches for the results obtained to the time-cost optimization problem.

Approaches	Models	Time (days)	Cost ($)
Lakshminaryana et al. [46]	MOOM	60	165500
Afshar et al. [45]	*30, 30 - MOACO	60	155500
Narayanan and Suribabu [38]	*30, 30 DE APPROACH (Method 1)	68	118500
Narayanan and Suribabu [38]	*30, 30 DE APPROACH (Method 2)	60	143500
This paper	RKV-TCQ	60	143500

*50, 30 – the first number is the number of iterations and the second is the population size.

Additionally we can also state that the RKV-TCQ approach produces high-quality solutions quickly once needed only 2 seconds to complete 30 generations.

Figs. 4 and 5 show the average value of objective time and cost for RKV-TCQ.

This computational experience has been performed on a computer with an Intel Core 2 Duo CPU T7250 @2.33 GHz and 1,95 GB of RAM. The algorithm proposed in this work has been coded in VBA under Microsoft Windows NT.

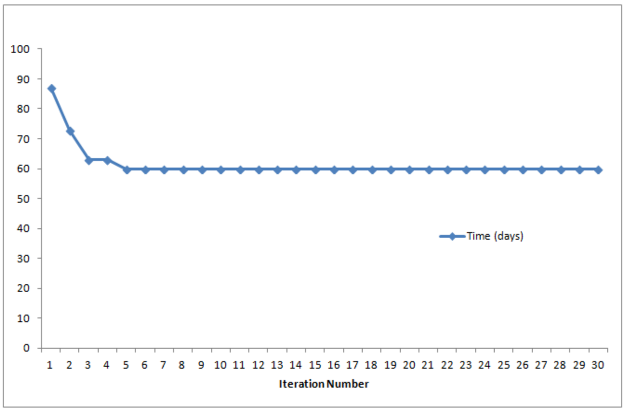

Fig. 4. Average objective time value.

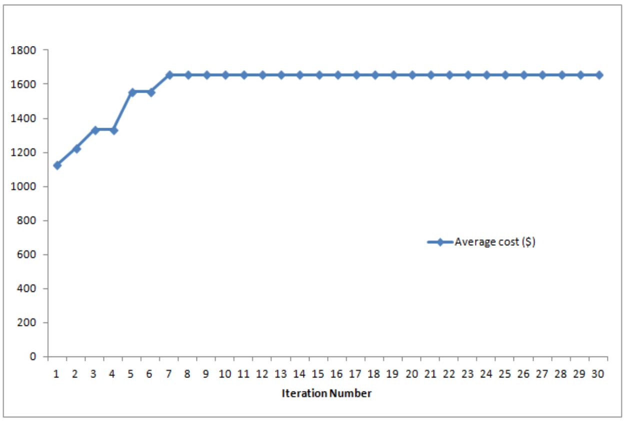

Fig. 5. Average objective cost value.

5 Conclusions and further research

A GA based-approach to solving the time-cost-quality optimization problem has been proposed. The project activities have various construction modes, which reflect different ways of performing the activity, each mode having a different impact on the duration and cost of the project. The chromosome representation of the problem is based on random keys. The schedules are constructed using a priority rule in which the priorities are defined by the genetic algorithm. The present approach provides an interesting alternative for the solution of construction multiobjective optimization problems.

Future studies are to assume more sophisticated relationship among the cost, time and quality of different project resources and test more projects.

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