# Formal Cognitive Models of Data, Information, Knowledge, and Intelligence

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***Abstract:*** - It is recognized that data, information, knowledge, and intelligence are the fundamental cognitive objects in the brain and cognitive systems. However, there is a lack of formal studies and rigorous models towards them. This paper explores the cognitive and mathematical models of the cognitive objects. The taxonomy and cognitive foundations of abstract mental objects are explored. A set of mathematical models of data, information, knowledge, and intelligence is formally created. On the basis of the cognitive and mathematical models of the cognitive objects, formal properties and relationship of contemporary data, information, knowledge, and intelligence are rigorously explained.

***Key-Words:*** - Cognitive informatics, brain science, mathematical models, formal theories, data science, information science, knowledge science, intelligence science, and system science

## 1. Introduction

A general worldview of cognitive informatics reveals that the natural world is a dual encompassing both the physical world and the abstract world as shown in Fig. 1 [43, 45, 48, 74, 76, 78]. There are four essences in modeling the natural world known as *matter* and *energy* for the physical world, as well as *information* and *intelligence* for the abstract world.

**Definition 1.** The *universe of discourse of the mankind* is a dual that can be denoted by the *information-matter-energy-intelligence* (IME-I) model of the natural world (*NW*). One facet of it is the *physical* world (*PW*), and the other is the *abstract* world (*AW*), where *intelligence* ($\mathbb{I}$) plays a central role in the transformation between information (*I*), *matter* (*M*), and *energy* (*E*).

In the IME-I model, the double arrows denote bi-directional relations between the essences in the dual universe of discourse, where known relations are denoted by solid lines, and relations yet to be discovered are denoted by dashed lines. According to the IME-I model, information is the generic model for representing the abstract world perceived by human beings. It is noteworthy that intelligence ($\mathbb{I}$) plays an irreplaceable role in the transformation between information, matter, and energy according to the IME-I model.

All cognitive objects in the forms of data, information, knowledge, and intelligence are a result of abstraction as a gifted ability of human brain [2, 6, 8, 25, 27, 28, 32, 34, 39, 42, 48, 58, 68, 76]. Abstraction is a basic cognitive process of the brain at the cognitive layer according to the *Layered Reference Model of the Brain* (LRMB) [89].

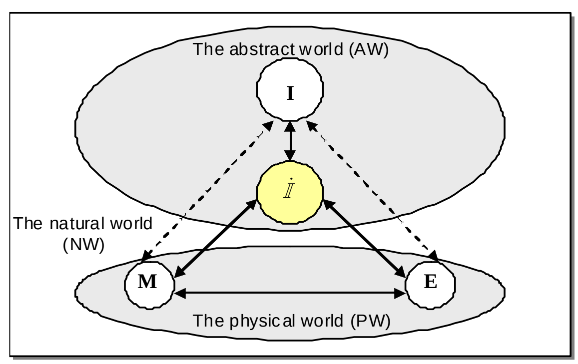


Fig. 1. The IME-I model of the worldview

**Definition 2.** *Abstraction* is a cognitive process to elicit a target subset of objects in a given discourse that shares a common property as an identity of the subset from the whole in order to facilitate denotation and reasoning.

Abstraction is not only a cognitive process of knowledge acquisition and learning, but also a powerful means of philosophy and mathematics. Abstraction plays a centric role in human cognition, thinking, and reasoning because all types of cognitive objects represented in the brain are in the abstract form. No formal inference and thinking may be conducted without the cognitive objects yield by abstraction.

Mathematics is the abstract science of numbers, quantity, and space as well as their applications in all other disciplines of sciences, engineering, society, and humanities. In order to efficiently and rigorously deal with the complex problems in abstract intelligence, brain science, cognitive informatics, knowledge science, and system science, a set of contemporary mathematics has been developed collectively known as denotational mathematics [48, 52, 60, 66, 70, 76, 79, 81, 82, 93].

**Definition 3.** *Denotational mathematics* (DM) is a category of mathematical structures that deals with complex mathematical entities in the domain of hyperstructures ($\mathbb{H}$) beyond those of real numbers ($\mathbb{R}$) and bits ($\mathbb{B}$), by series of embedded functions and processes in order to formalize rigorous expressions and inferences.

Denotational mathematics as function of functions on hyperstructures deals with high-level mathematical entities beyond numbers and sets, such as abstract objects, complex relations, big data, information, concepts, knowledge, processes, inferences, decisions, intelligence, and systems [67, 70].

This paper is a basic study that explores and contrasts the cognitive and mathematical models of cognitive objects in the brain such as data, information, knowledge, and intelligence. In the remainder of this paper, the taxonomy and cognitive foundations of cognitive objects is explored in Section 2. A set of mathematical models of data, information, knowledge, and intelligence is created, respectively, in Sections 3 through 6. On the basis of the conceptual and formal models of the cognitive objects, formal principles and properties of data, information, knowledge, and intelligence is explained and their relationship is clarified.

## 2. Taxonomy of Cognitive Objects in the Brain

Although it is well recognized that data, information, knowledge, and intelligence are the fundamental cognitive objects in the brain and cognitive systems, there is a lack of formal studies on them. The contemporary and traditional perceptions, metaphors, and relationships of cognitive objects are contrasted and elaborated in this section.

The taxonomy of cognitive objects represented in the brain can be classified into four forms [6, 7, 11, 25, 34, 39, 42, 48, 49, 58, 61, 68, 76] as illustrated in Fig. 1. It is perceived that *data* are acquired raw information which are usually a quantitative abstraction of external entities and their relations. *Information* is meaningful data or an interpretation of data. *Knowledge* is consumed information related to existing knowledge in the brain. Intelligence is a collection of cognitive abilities of humans or systems that transforms information into behaviors [68, 74, 76, 78, 80].

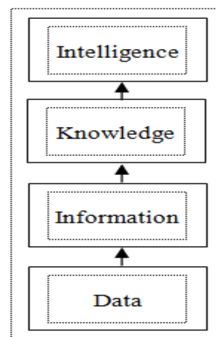


Fig. 2. The hierarchical framework of cognitive objects

**Definition 4.** The *hierarchy of cognitive objects*, $\mathfrak{CO}$, represented in human brain is a 4-tuple in the categories of *data* ($\mathbb{D}$), *information* ($\mathbb{I}$), *knowledge* ($\mathbb{K}$), and *intelligence* ($\dot{\mathbb{I}}$) from the bottom up according to their levels of abstraction, i.e.:

$$\mathfrak{CO} \triangleq (\mathbb{D}, \mathbb{I}, \mathbb{K}, \dot{\mathbb{I}}) = \begin{cases} \mathbb{D} = f_d : O \to Q \\ \mathbb{I} = f_i : D \to S \\ \mathbb{K} = f_k : I \to C \\ \dot{\mathbb{I}} = f_{\dot{i}} : I \to B \end{cases} \quad (1)$$

where the symbols denote *object* ($O$), *quantity* ($Q$), *semantics* ($S$), *concept* ($C$), and *behavior* ($B$), respectively.

**Definition 5.** The *hierarchical abstraction model* (HAM) *of knowledge* states that the extent of abstraction of cognitive information can be classified at five levels such as those of *analogue objects, diagrams, natural languages, professional notations,* and *mathematics.*

The HAM model is illustrated in Fig. 3 where each level is corresponding to a certain descriptive means. The higher the level of abstraction, the higher the efficiency in reasoning. Inversely, the lower the level of abstraction, the easier the intuition in comprehension. According to the HAM model, there are two approaches to system modeling and description known as *abstraction* and *explanation*. The former enables the enhancement of the descriptive power in terms of expressiveness, precise, and rigor; while the latter enables the improvement of intuitiveness in understanding and comprehension using a means much closer to real-world images and analogue objects directly acquired by sensations of the brain.

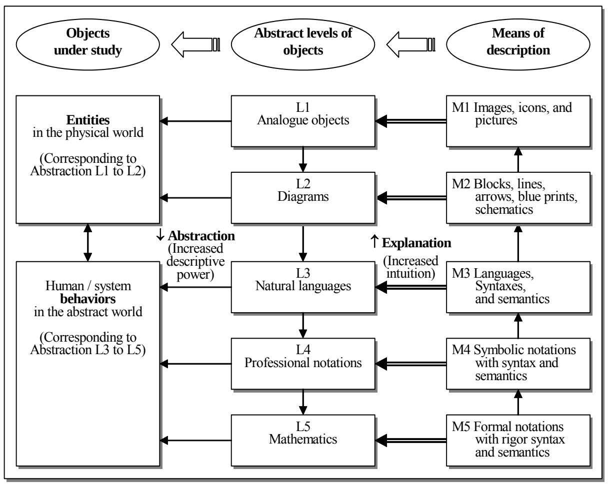


Fig. 3. The Hierarchical Abstraction Model (HAM) of knowledge and information

It is noteworthy in HAM that the core scientific knowledge of humans is mainly archived in mathematical forms [48, 61]. That is, any other form of knowledge would be merely data, factor, and instances towards principles expressed in mathematical forms in the widest and most general domain.

## 3. Formal Models of Data

Data are the most fundamental cognitive objects in the brain that link the real-world entities and their attributes to mental abstractions via sensor and quantities [9, 15, 21, 38, 40, 41, 48, 81].

**Definition 6.** *Data* are an abstract representation of the quantity of real-world entities or abstract objects according to specific quantification scales.

*Big data* are extremely large-scaled heterogeneous data in terms of quantity, complexity, semantics, distribution, and processing costs in computer science, information science, cognitive informatics, web-based computing, cloud computing, and computational intelligence [15, 81].

**Corollary 1.** Data is generated by human cognitive processes and formal inferences, as well as system quantifications.

In Corollary 1, the cognitive processes refer to observation, sensor, abstraction, and reasoning; while formal inferences refer to qualification, quantification, mathematical operations, statistics, information processing, and computing. A special form of data is known as *signals* directly yielded by sensors and communication systems, which link data to information.

The domain of number theories for quantification in mathematics has been continuously expanding from binary numbers ($\mathbb{B}$), natural numbers ($\mathbb{N}$), integers ($\mathbb{Z}$), and real numbers ($\mathbb{R}$) to fuzzy numbers ($\mathbb{F}$) and hyper numbers ($\mathbb{H}$) [18, 35, 37, 48, 52, 53, 60, 66, 70, 82, 96]. It demonstrates an interesting course of advances in human ability of abstraction and quantification in order to deal with the real-world entities and their perceptive representations in the brain. The characteristics of the domain of fuzzy numbers $\mathbb{F}$ is a 2-demential hyperstructure $\mathbb{F} = \mathbb{R} \times \mathbb{R} = \{(\mathbb{R}, \mathbb{R})\}$, with a crisp set of member elements in [-∞, +∞] and an associate crisp set of degrees of membership in [0, 1] for each of the members. Hyper numbers, $\mathbb{H}$, is a contemporary extension of the domain of numbers to a typed *n*-tuple [48, 76, 81].

**Definition 7.** *Data* with respect to a quantity $X$ against a measure scale $\mathcal{S}$, $D(X, \mathcal{S})$, is yielded via a quantification $\Gamma_{\mathcal{S}}(X)$ that results in a real number, $I_X^{\mathcal{S}}.R_X^{\mathcal{S}}$, in unit $[\mathcal{S}]$ denoted by the integer part $I_X^{\mathcal{S}}$ and the decimal part $R_X^{\mathcal{S}}$ known as the remainder, i.e.:

$$\begin{aligned} D(X,\mathcal{S}) &\triangleq \Gamma_{\mathcal{S}}(X) = \frac{X}{\mathcal{S}} \\ &= I_X^{\mathcal{S}}.R_X^{\mathcal{S}}\ [\mathcal{S}],\ \mathcal{S} \in \mathbb{R}, I_X^{\mathcal{S}} \in \mathbb{Z}, 0 \le R_X^{\mathcal{S}} < \mathcal{S} \\ &= \begin{cases} I_X^{\mathcal{S}} = mod_{\mathcal{S}}(X) \\ R_X^{\mathcal{S}} = rem_{\mathcal{S}}(X) \end{cases} \end{aligned} \quad (2)$$

**Example 2.** Given the length of an object as estimated as $L$ = 1682 *mm*, corresponding data may be generated based on the given measure scale(s) $\mathcal{S}$ according to Definition 7 as follows:

$\mathcal{S}_1 = 1m = 1000mm$:

$$D_1(L, \mathcal{S}_1) = \frac{L}{\mathcal{S}_1} = \frac{L}{m} = \frac{1682}{1000} = 1.682\ m$$

$\mathcal{S}_2 = 1cm = 10mm$:

$$D_2(L, \mathcal{S}_2) = \frac{L}{\mathcal{S}_2} = \frac{L}{cm} = \frac{1682}{10} = 168.2\ cm$$

$\mathcal{S}_3 = 1\mu m = 0.001mm$:

$$D_3(L, \mathcal{S}_3) = \frac{L}{\mathcal{S}_3} = \frac{L}{\mu m} = \frac{1682}{0.001} = 1682000\ \mu m$$

**Definition 8.** *Big data* are extremely large-scaled heterogeneous data in terms of quantity, complexity, semantics, distribution, and processing costs in computer science, information science, cognitive informatics, web-based computing, cloud computing, and computational intelligence.

**Definition 9.** The *mathematical model* of the *general big data structure (BDS)*, $\Theta$, in the discourse of abstract data is a two dimensional $n \times m$ matrix where each row $\mathop{R}\limits_{i=0}^{n} r_i|\text{SM}$ denoted by a structure model (|SM) is called a *typed tuple*, each column $\mathop{R}\limits_{j=0}^{m} e_j|\mathbb{T}_j$ is called a *field* with a certain type (|𝕋j), and each cell $\mathop{R}\limits_{i=0}^{n}\mathop{R}\limits_{j=1}^{m} d_{ij}|\mathbb{T}_j$ is a data object that can be a meta data element or structured element $\tau|\mathbb{T}$, i.e.:

$$\Theta \triangleq \mathop{R}\limits_{i=0}^{n}\mathop{R}\limits_{j=0}^{m} d_{ij}|\mathbb{T}_j = \left[\begin{array}{c|cccc} r_0 & e_0|\mathbb{T}_0 & e_1|\mathbb{T}_1 & \cdots & e_m|\mathbb{T}_m \\ \hline r_1 & d_{10}|\mathbb{T}_0 & d_{11}|\mathbb{T}_1 & \cdots & d_{1m}|\mathbb{T}_m \\ r_2 & d_{20}|\mathbb{T}_0 & d_{21}|\mathbb{T}_1 & \cdots & d_{2m}|\mathbb{T}_m \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_n & d_{n0}|\mathbb{T}_0 & d_{n1}|\mathbb{T}_1 & \cdots & d_{nm}|\mathbb{T}_m \end{array}\right] \quad (3)$$

where the set of types allows arbitrary forms and medias of data are represented.

The general abstract model of big data systems as given in Definition 9 can be used to represent arbitrary multi-dimensional big data structures by modeling the structured element $\tau$|SM as an *n*-D structure. This methodology provides a general and flexible approach to model any complex big data structure in the real world known as the *hierarchical refinement model of big data.*

## 4. Formal Models of Information

Information is the second level of cognitive objects that represents or embodies the semantics of data and facts collected from the real-world or yielded by mental processes. Information is the third essence for modeling the natural world in addition to matter and energy as described in the IME-I model in Definition 1.

**Definition 10.** *Information* is a general form of abstract objects perceived by human brains and represented by symbolical, mathematical, communication, computing, and cognitive systems.

According to Definition 10, anything intangible, which the brain may acquire and process or which a computing/communication system may manipulate and convey, is a kind of information. However, anything tangible that cannot enter the brain is not information, although its attributes, quantity, and properties are information. Any product and/or process of human mental activities result in the generation of information.

In classic information theory [3, 5, 16, 33, 98], information is treated as a probabilistic measure of properties of message in signal transmission. The early notion of information is highly influenced by the thermal dynamic concept known as entropy, which denotes the extent of the trend of a system towards disorder or randomization. Classic information theories focused on information transmission rather than information itself. The measure of the quantity of information is highly depended on the receiver's subjective judgment on the probability distributions of signals in the message.

**Definition 11.** The content of information, $I_i$, of the *i*th sign in a message is determined by its unexpected probability $p_i^{-1}$ in a 2-based logarithm scale given its average probability is $p_i$ in a two sign system, i.e.:

$$I_i = \log_2 \frac{1}{p_i} \quad [bit] \quad (4)$$

where the unit of information is a *bit* shortened from a *binary digit*. Any other base *k, k* > 1, can be transformed into the binary base.

**Definition 12.** The *information of* an *n-sign system*, *I*, is determined by a weighted sum of the probabilities $p_i$ and unexpectedness $I_i$ of each sign in the information system, i.e.:

$$\begin{aligned} I &\triangleq \sum_{i=1}^{n} p_i \bullet I_i \\ &= \sum_{i=1}^{n} p_i \bullet \log_2 \frac{1}{p_i} \\ &= \sum_{i=1}^{n} -p_i \bullet \log_2 p_i \quad [bit] \end{aligned} \quad (5)$$

**Example 3.** For a binary signal source, $S_1$, that adopts two equal-probability signs, i.e., $p_1 = p_2 = 0.5$, its total information, $I_{S_1}$, according to Definition 12 is:

$$\begin{aligned} I_{S_1} &= \sum_{i=1}^{2} p_i \bullet \log_2 \frac{1}{p_i} \\ &= 2(0.5 \bullet \log_2 2) = 2 \bullet 0.5 \\ &= 1\ bit \end{aligned}$$

**Example 4.** Given a natural-language-based communication system in English, $S_2$, assume the probability of each of the 27 alphabets is the same, i.e., $p_1 = p_2 = \ldots = p_{27} = 1/27$, the total information of the system $I_{S_2}$ normalized to bit according to Definition 12 is:

$$\begin{aligned} I_{S_1} &= \sum_{i=1}^{27} p_i \bullet I_i \\ &= \sum_{i=1}^{27} \frac{1}{27} \bullet \log_2 27 \\ &= 27 \bullet \frac{1}{27} \bullet 4.7549 = 4.7549 \ \ [bit] \end{aligned}$$

It is noteworthy that in the bivalent systems, the content of information is always with a certain bits as the inherent property of signals. In other words, $I$ is not proportional to the size of messages. That is, on the basis of the classical information measurement, no matter how many bits of message are transmitted, the value of total information is merely a probable property of the given system rather than the size of the transmitted information. Classic information theories focused on information transmission rather than information itself. Further, the property of classic information, $p_i$, is determined by the receiver's subjective expectation. Thus, there is no information when a message received had already been known due to $p_i \equiv 1$.

However, modern computational informatics tends to model information as an abstract entity for data, memory, messages, signals, and knowledge representation rather than a probable property of communication system as in the classic information theory. This notion reflexes the contemporary theories and practices across computer science, software engineering, the IT industry, and everyday lives [45, 48, 76, 87, 88].

**Definition 13.** *Information, I*, in computational informatics is the normalized 2-based size of any abstract object $O_k$ arbitrarily represented in a $k$-based measure *scale* $S_k$, i.e.:

$$\begin{aligned} I &\triangleq f_k : O_k \to S_k,\ k \in \mathbb{R} \\ &= k^{\log_k O_k} \log_2 k \\ &= O_k \log_2 k \ \ [bit] \end{aligned} \tag{6}$$

Note that the unit of bit in Eq. 6 is concrete and deterministic, which is no longer a scalar quantity as that of Shannon informatics. In other words, the unit *bit* has been extended to a general quantity of information size in computer science and modern information science.

**Example 5.** For a natural-language-based communication system in English, the information transformed by 100 letters, i.e., $O_k = O_{27} = 100$, can be equivalently normalized in bit according to Definition 13 as follows:

$$\begin{aligned} I &= O_k \log_2 k \\ &= O_{27} \log_2 27 \\ &= 100 \bullet 4.7549 \\ &= 475.4900\ bit \end{aligned}$$

The result indicates that the minimum information required to transmit $O_{27}$ = 100 letters is 476 bits without redundancy.

The contemporary information theory is merged in cognitive informatics [43, 45, 46, 49, 52, 58, 59, 62, 63, 66-68, 71, 72, 74, 75, 82, 84-92]. *Cognitive informatics* (CI) is a transdisciplinary enquiry on the internal information processing mechanisms and processes of the brain and minds in order to reveal the principles of natural intelligence and engineering applications [43, 45].

It is discovered in cognitive informatics that the computational information as given in Definition 13 [45, 48] may still not represent the entire properties of information. In other words, the third generation of information may be described by a more general model towards the abstract artefact or its symbolic representation that can be modeled, acquired, memorized, and processed by human brains.

**Definition 14.** *Contemporary information*, $I = (I_\kappa, I_\Omega)$, is a tuple of general two-dimensional properties inherent in any system called the *characteristic information* $I_\kappa$ and the *denotational information* $I_\Omega$, generated by the combinatorial mechanism of system variables $V = \{v_1, v_2, ..., v_\beta\}$ on a given *base* β of representation, $\beta \in \mathbb{R}$, i.e.:

$$\begin{aligned} I &\triangleq (I_\kappa, I_\Omega) \\ &= \begin{cases} I_\kappa = |V| \log_2 \beta = \log_2 I_\Omega & [bit] \\ I_\Omega = \beta^{|V|} = 2^{I_\kappa} = 2^{|V| \log_2 \beta} & [bit] \end{cases} \end{aligned} \tag{7}$$

where $I_\kappa$ is determined by the size of the problem dimensions β, $I_\Omega$ is determined by the entire state space of the system as a combinatorial function of $I_\kappa$ and $\beta$. Both $I_\kappa$ and $I_\Omega$ are normalized to the unit *bit* with respect to $\beta_0 = 2$.

The third generation of information as formally modeled in Definition 14 reveals that information is an inherent property of any system where the normalized denotational information is determined by its characteristic information, and vice versa. The measure of cognitive information is compatible to but more general than the classic and computational information. For instance, many problems that Shannon information theory cannot deal with as given in Examples 6 through 8 can be solved by the contemporary information measure.

**Example 6.** For a natural-language-based communication system in English ($\beta_1$ = 27), the information for transmitting $|V_1| = 100$ letters can be equivalently normalized in bit according to Definition 14 as follows:

$$I_{\kappa_1} = |V_1|\log_2\beta_1 = 100 \bullet 4.7549 = 475.4900\ bit$$
$$I_{\Omega_1} = \beta_1^{|V_1|} = 2^{|V_1|\log_2\beta_1} = 2^{100 \bullet 4.7549} = 2^{475.4900}\ bit$$

where, it is noteworthy, that the classic information measure yields 4.7549 bit as obtained in Example 4 ignoring $|V_1|$. However, the computational information measure as shown in Example 5 does not cover the other important facet of information known as the denotational information.

**Example 7.** The big data represented in the state space of a logical AND-gate with 1,000 input pins can be determined according to Definition 14 where $|V_2| = 1000\ bit$ and $\beta_2$ = 2 as follows:

$$I_{\kappa_2} = |V_2|\log_2\beta_2 = |V_2| = 1000\ bit$$
$$I_{\Omega_2} = \beta_2^{|V_2|} = 2^{I_{\kappa_2}} = 2^{1000}\ bit$$

where note that the classic information measure yields 1 bit.

**Example 8.** Given a communication system specified as $|V_3| = 70$ decimal digits on base $\beta_3$ = 10, what is its capacity for information representation and what is its equivalent characteristic information normalized to base 2?

$$\begin{aligned}I_{\kappa_3} &= |V_3|\log_2\beta_3 = 70 \bullet \log_2 10 \\ &= 70 \bullet 3.3220 = 232.5400\ bit \\ I_{\Omega_3} &= \beta_3^{|V_3|} = 10^{70} \\ &= 2^{I_{\kappa_3}} = 2^{232.5400} = 1.0\ (Zb)^{3.3220} \\ &= (1,180,591,620,717,411,303,424)^{3.3220}\ bit\end{aligned}$$

That is, the given decimal system possesses a 233 bit characteristic information and a $2^{233}$ bit denotational information space in the normalized scale. However, the classic information measure yields only 3.3220 bit for the given system.

**Corollary 2.** The computational information $I^{G_2} = I^c$ is a special case of cognitive information $I^{G_3} = I^{CI} = (I_\kappa, I_\Omega)$ where only characteristic information is considered, i.e.:

$$I^{G_2} = I^c = I^{G_3}.I_\kappa,\ \ I_\kappa \sqsubset I^{G_3} = I^{CI} = (I_\kappa, I_\Omega) \quad (8)$$

where $\sqsubset$ denotes a subdimension in a hyperstructure.

**Corollary 3.** The classic Shannon information $I^{G_1} = I^s$ is a special case of the characteristic information $I_\kappa$ where the size of information is not considered, i.e.:

$$I^{G_1} = I^s = \frac{I^{G_2}}{|I_\kappa^{G_2}|} = \frac{I^{G_3}.I_\kappa}{|I^{G_3}.I_\kappa|},\ I_\kappa \sqsubset I^{G_3} = I^{CI} = (I_\kappa, I_\Omega) \quad (9)$$

# 5. Formal Models of Knowledge

Knowledge is the third level of cognitive objects as learnt and comprehended information. Knowledge is creative products generated by the brain embodied by concept networks and behavioral processes. The former represent the form of to-be knowledge; while the latter embody the form of to-do knowledge, which is more precisely classified as intelligence as elaborated in Section 6.

In traditional epistemology, knowledge is perceived as a justified true belief [10, 11, 32, 83] expressed as follows.

**Definition 15.** The *tripartite form of knowledge* perceives knowledge *k* as a subjective proposition of a person *H* that is *p iff H* believes that *p* and *H*'s belief that *p* is justified, i.e.:

$$\begin{aligned}&\forall p, \theta, \text{and } H,\ \exists k, \\ &\quad k : p \vdash \mathrm{T}\ \wedge \mu_H(p \vdash \mathrm{T}) \geq \theta\end{aligned} \quad (10)$$

where $\vdash$ denotes an inference, $\mu_H$ a justification of the inference by *H*, *p* a proposition, $\theta$ a certain threshold of confidence, and *T* the logical constant of true.

The neurological foundation of knowledge can be explained by synaptic connections among neurons representing individual objects and attributes as shown in Fig. 4. The *dynamic neural cluster* (DNC) indicates that knowledge is not only retained in neurons as individual objects or attributes, but also dynamically represented by newly created synaptic connections. This leads to the development of the formal object-attribute-relation model of knowledge [50].

**Definition 16.** The *object-attribute-relation* (*OAR*) *model* of memory in the brain is a triple, i.e.:

$$OAR \triangleq (O, A, R) \quad (11)$$

where *O* is a set of objects identified by unique symbolic names, *A* a set of attributes for characterizing an object, and *R* a set of relations between the objects and attributes, i.e., $R = O \times A$.

The OAR model reveals the nature and essence of knowledge and its neurological foundations. The OAR model can be adopted to explain a wide range of human information processing mechanisms and cognitive processes.

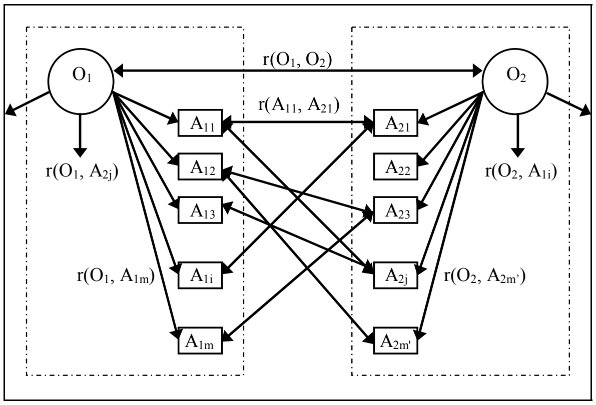


Fig. 4. The OAR model of knowledge as a dynamic neural cluster (DNC)

**Definition 17.** *Knowledge*, *K*, is acquired and comprehended information generated by the brain embodied as a concept, i.e.:

$$K \triangleq f_k: I \rightarrow C \tag{12}$$

where *C* represents a formal concepts as modeled in the PAR model and concept algebra [50, 57].

Investigation into the cognitive models of information and knowledge representation in the brain is perceived to be one of the fundamental research areas that help to unveil the mechanisms of the brain. The OAR model describes human memory, particularly long-term memory, using the *relational* metaphor, rather than the traditional *container* metaphor that was used to be adopted in psychology, cognitive science, computing, and information science. The OAR model shows that human memory and knowledge are represented by relations, i.e., connections of synapses between neurons, rather than by the neurons themselves as the traditional container metaphor perceived.

**Corollary 4.** *Knowledge* is a *synaptic connection* among neurons in the brain.

According to cognitive informatics and neuroinformatics [13, 72, 87], the structures and configurations of the human memory system can be logically described by a cognitive memory model as follows.

**Definition 18.** The *cognitive memory model* (CMM) is a functional partition of the human memory system in a parallel configuration (||) with five-type memories according to their functions, i.e.:

$$\begin{aligned} CMM \triangleq (\ & LTM \\ & \| \ STM \\ & \| \ SBM \\ & \| \ ABM \\ & \| \ CSM \\ ) & \end{aligned} \tag{13}$$

where memories stand for long-term memory (LTM), short-term memory (STM), sensory buffer memory (SBM), action buffer memory (ABM), and conscious status memory (CSM), respectively.

Knowledge is acquired via learning and inference that is retained in different forms/areas of memories. According to CMM [72], the forms of abstract, behavioral, experienced, and skilled knowledge are retained in the temporal lobe of LTM, the motor cortex of ABM, the parietal lobe of LTM, and the cerebellum of CSM, respectively. Although knowledge and experience are memorized as abstract relations in LTM, behaviors and skills are embodied as wired neural connections in ABM. However, all contingent manipulation of knowledge and thinking threads are processed in STM [Wang, 2012c].

The identification and allocation of internal knowledge and information in cognitive informatics can be used to explain a wide range of phenomena of learning and practices. For instance, it explains why people have to make the same mistakes in order to gain empirical experiences and skills; and why skills and experience transformation would be so hard and could not be gained by indirect.

# 6. Formal Models of Intelligence

Intelligence is the fourth level of cognitive objects in the brain. Intelligence may be classified in the categories of reflexive, perceptive, cognitive, and instructive intelligence [1, 17, 22, 24, 26, 29, 36, 39, 43, 45, 53, 58, 67, 68, 74, 80, 82, 84, 85, 93-95]. Although all animal species possess reflexive and perceptive intelligence, only humans and a few advanced species developed the ability of cognitive and instructive intelligence.

**Definition 19.** *Intelligence* is a human or a system's ability that transforms information into behaviors.

Paradigms of intelligence are such as natural intelligence, artificial intelligence, machinable intelligence, and computational intelligence. The development of cognitive robots, cognitive computers, intelligent systems, and software agents indicates that intelligence may also be created or implemented by machines and man-made systems.

**Definition 20.** *Cognitive informatics* (CI) is a transdisciplinary enquiry of intelligence science, information science, computer science, cognitive science, and brain science, that studies the internal information processing mechanisms and processes of the brain, the underlying abstract intelligence (αI) theories and denotational mathematics (DM), and engineering applications in cognitive computing, computational intelligence, cognitive robotics, and cognitive systems [43, 45].

The layered reference model of the brain as shown in Fig. 5 [89] provides an overarching logical configuration of the mechanisms of the brain as an advanced natural intelligence system. The four lower layers encompassing those of sensation, action, memory, and perception are classified as subconscious mental functions of the brain equivalent to the *mental operating system* (MOS). However, the three higher layers encompassing those of cognition, inference, and intelligence are classified as conscious mental functions of the brain equivalent to the mental applications (MApps).

**Definition 21.** The *layered reference model of the brain* (LRMB) is a hierarchical layout of mechanisms and relations of the brain formally embodied by 52 cognitive processes at seven layers as follows:

$$\begin{aligned}
\text{LRMB} &\triangleq (\text{L}_1, \text{L}_2, \text{L}_3, \text{L}_4, \text{L}_5, \text{L}_6, \text{L}_7) \\
&= (\textit{Sensation}, \textit{Action}, \textit{Memory}, \textit{Perception}, \textit{Cognition}, \textit{Inference}, \textit{Intelligence}) \\
&= \text{L}_1\text{-}\textit{Sensation} = \{\text{vision, hearing, smell, taste, touch, spaciality, time, motion}\} \\
&\quad \| \ \text{L}_2\text{-}\textit{Action} = \{\text{reflex, recurent, temporary, complex}\} \\
&\quad \| \ \text{L}_3\text{-}\textit{Memory} = \{\text{SBM, STM, LTM, CSM, ABM}\} \\
&\quad \| \ \text{L}_4\text{-}\textit{Perception} = \{\text{attention, consciousness, motivation, emotion,} \\
&\qquad\qquad \text{attitude, imagination, posture, equilibrium}\} \\
&\quad \| \ \text{L}_5\text{-}\textit{Cognition} = \{\text{object_identitfy, abstraction, concept_establishment,} \\
&\qquad\qquad \text{catergirization, comparison, memorirization,} \\
&\qquad\qquad \text{qualification, quantification, selection, search}\} \\
&\quad \| \ \text{L}_6\text{-}\textit{Inference} = \{\text{deduction, induction, abduction, analogy,} \\
&\qquad\qquad \text{causation, analysis, aynthesis, recursion}\} \\
&\quad \| \ \text{L}_7\text{-}\textit{Intelligence} = \{\text{comprehension, learning, problem_solving,} \\
&\qquad\qquad \text{decision_making, creation, modeling, planning,} \\
&\qquad\qquad \text{information_fusion, pattern_recognition}\}
\end{aligned} \tag{14}$$

LRMB reveals that the brain can be formally embodied by 52 cognitive processes at seven layers known as the *sensation*, *action*, *memory*, *perception*, *cognitive*, *inference*, and *intelligence* layers from the bottom up. In this view, any complex mental process or behavior is a temporary composition of the fundamental processes of LRMB at run-time. Further details on formal descriptions of the cognitive processes of LRMB may refer to [51, 86].

*Abstract intelligence*, αI, is a human enquiry of the core and commonly shared properties of natural and artificial intelligence at the embody levels of neural, physiological, cognitive, functional, and logical models from the bottom up.

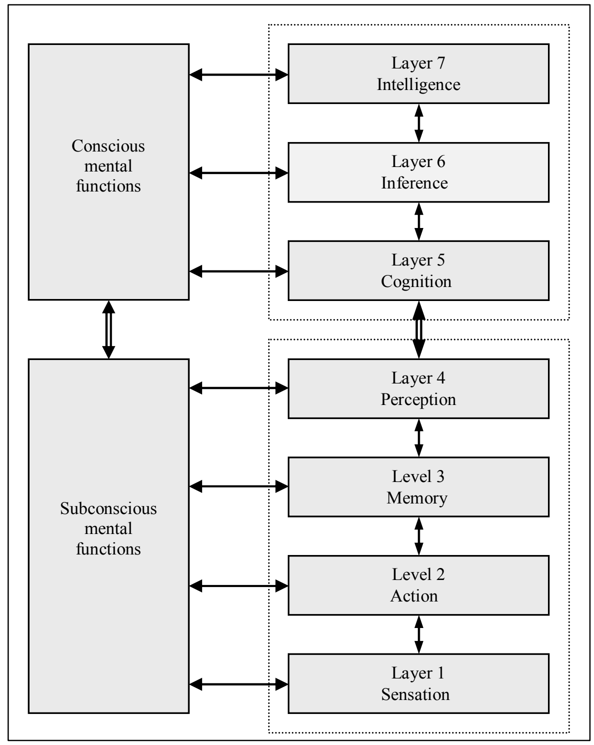


Fig. 5. The layered reference model of the brain (LRMB)

**Definition 22.** The *mathematical model of abstract intelligence* (αI), embodies the brain in four forms known as the *perceptive intelligence* $\dot{I}_p$, *cognitive intelligence* $\dot{I}_c$, *instructive intelligence* $\dot{I}_i$, and *reflexive intelligence* $\dot{I}_r$, corresponding to the specific forms of cognitive objects and their associate memories in the brain, i.e.:

$$\begin{aligned}
\alpha\text{I} &\triangleq (\dot{I}_p, \dot{I}_c, \dot{I}_i, \dot{I}_r) \\
&= \begin{cases}
\dot{I}_p = f_p : D \to I & \text{// Perceptive} \\
\dot{I}_c = f_c : I \to K & \text{// Cognitive} \\
\dot{I}_i = f_i : I \to B & \text{// Instructive} \\
\dot{I}_r = f_r : D \to B & \text{// Reflexive}
\end{cases}
\end{aligned} \tag{15}$$

where a behavior is a type of cognitive objects which embodies an abstract input to an observable action.

**Corollary 5.** All forms and paradigms of intelligence in the αI model share the same cognitive informatics foundations as described in Definition 21.

It is recognized that studies on the natural intelligence must be carried out across the neurological, physiological, cognitive, and logical levels from bottom-up aggregations and top-down reductions. A single layer perception would not explain the complex nature of the brain. Therefore, the creation of a coherent mathematical model of the brain will play the overarching role towards modeling the brain and minds as well as the hyperstructures of cognitive objects in them.

# 7. Conclusion

A set of fundamental cognitive objects in brain science and computational intelligence, such as data, information, knowledge, and intelligence, has been formally described. This basic research has explored the cognitive foundations of key cognitive objects and their relations. As a result, a set of mathematical models of the cognitive objects has been rigorously created. This work has led to a set of interesting findings on principles and properties of cognitive objects towards a coherent theory for transdisciplinary cognitive informatics across system science, brain science, and cognitive science in general, and contemporary data, information, knowledge, and intelligence sciences in particular.

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