control the identification algorithm. This input provides possibility of start, stop and restart the identification algorithm in selected instant of time. Outputs of the block are estimate of parameter vector, one-step prediction of output of model, covariance matrix and data vector. The inputs and outputs of the block are shown in Fig.3.

4.1 Examples

Using the recursive identification algorithms library is illustrated in two examples. The first example shows the simple application of the identification block in the model. Simulink diagram and the results are shown in Fig4.

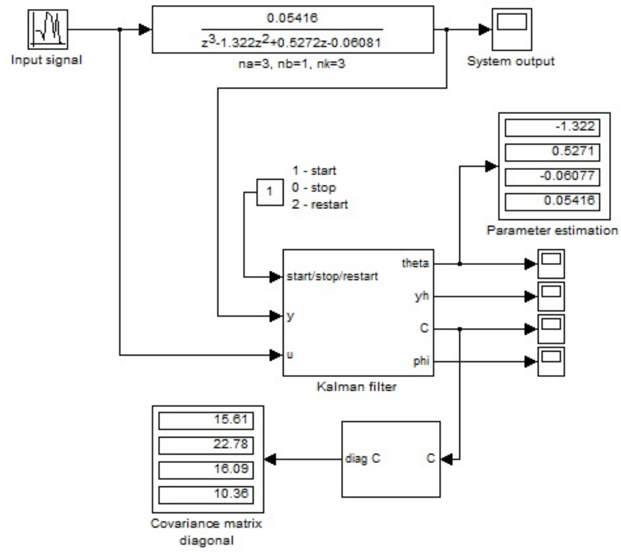

Fig.4 Example of application of identification block

Second example shows application of recursive identification algorithms library in identification part of self-tuning controller.

Several recursive identification algorithms mentioned above were tested in closed loop on system given by transfer function (50) with self-tuning LQ controller. The controller is based on minimization of quadratic criterion with controller output signal penalization. The minimization of quadratic criterion is realized by spectral factorization.

Continuous-time transfer function of the controlled system is

$$G(s)=\frac{s+1}{50s^2+15s+1}, \quad for\ t\le 1000s$$
$$G(s)=\frac{s+1{,}5}{50s^2+15s+1}, \quad for\ t\ge 1000s \tag{50}$$

The block diagram of controlled system is shown in Fig.5. It can be seen that the system output is directly influenced by non-measurable disturbance. This case is commonly fulfilled in practice.

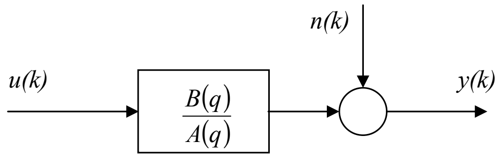

Fig.5 Block diagram of controlled system

Discrete transfer function is then

$$G\left(z^{-1}\right)=\frac{0.1529z^{-1}+0.0287z^{-2}}{1-1.1196\,z^{-1}+0.3012z^{-2}}, \quad for\ k\le 250$$
$$G\left(z^{-1}\right)=\frac{0.2072z^{-1}+0.0651z^{-2}}{1-1.1196\,z^{-1}+0.3012z^{-2}2}, \quad for\ k\ge 250 \tag{51}$$

The sampling period was chosen $T_0 = 4s$.

The same initial conditions for system identification were used for all the types of recursive algorithms we tested. The initial parameter estimates were chosen to be

for ARX, OE model

$$\hat{\boldsymbol{\Theta}}(k)=[0,0,0,0]^T \tag{52}$$

for ARMAX model

$$\hat{\boldsymbol{\Theta}}(k)=[0,0,0,0,0,0]^T \tag{53}$$

Estimation algorithms with fixed and variable exponential forgetting were applied using a forgetting factor $\lambda=\lambda(0)=0.985$. Forgetting factor for fixed directional forgetting was set to $\lambda'=0.985$. Initial values for adaptive forgetting were chosen to be $\varphi(0)=1,\ \rho(0)=0.99,\ \upsilon(0)=10^{-6},\ \lambda(0)=0.001$

System dynamics were described by ARX, OE, ARMAX model, respectively. Parameters of ARX model were identified by RLS, RIV and ERIV methods, RELS and RPEM methods were used to parameter estimation of ARMAX model and parameters of OE model were estimated by RPEM method. To assure parameters tracking the forgetting factors were used. Only parameters of deterministic part of the estimated models were utilized for controller synthesis.

4.1.1 Estimation of ARX model

RLS with variable exponential forgetting

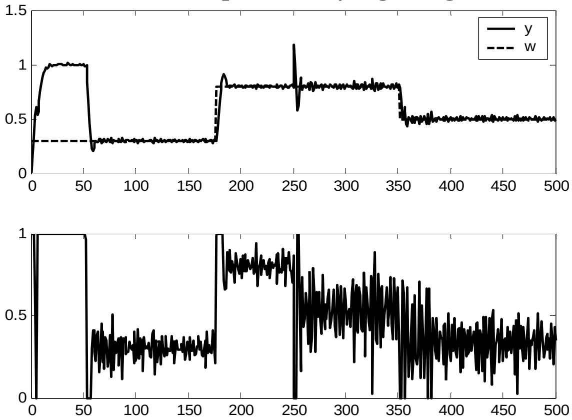

Fig.6 The adaptive control with LQ controller

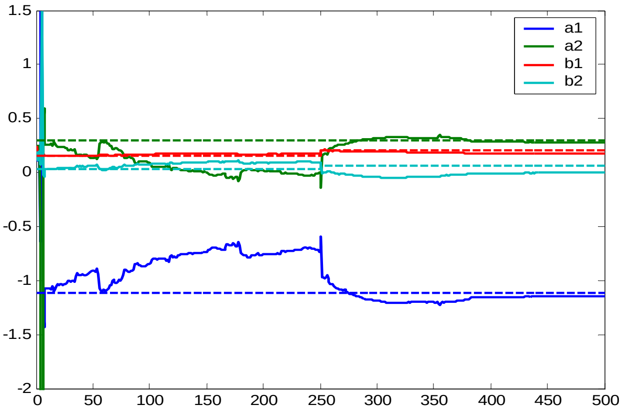

Fig.7 Parameter estimates (solid line) computed with the RLS with variable exponential forgetting – true parameters (dashed line)

RLS with adaptive directional forgetting

From Fig.8 and Fig.9 can be seen that adaptive directional forgetting can improve control quality but there is fluctuation in parameters.

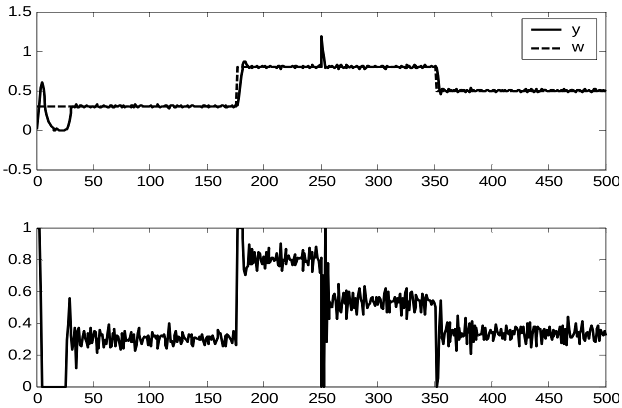

Fig.8 The adaptive control with LQ controller

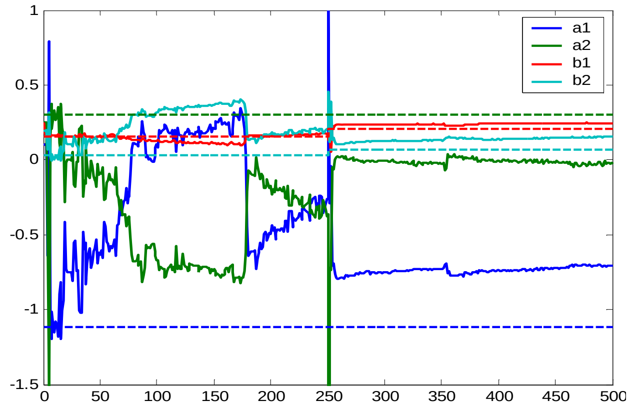

Fig.9. Parameter estimates (solid line) computed with the RLS with adaptive directional forgetting – true parameters (dashed line)

RIV with adaptive directional forgetting

Fig.10 shows that the adaptive control with RIV method provide better results from system output point of view than adaptive control with RLS method.

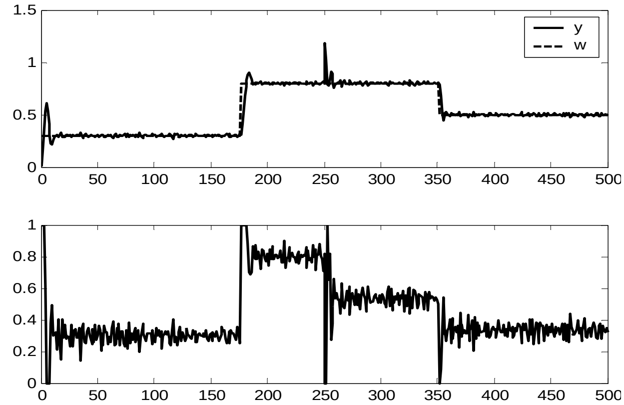

Fig.10. The adaptive control with LQ controller

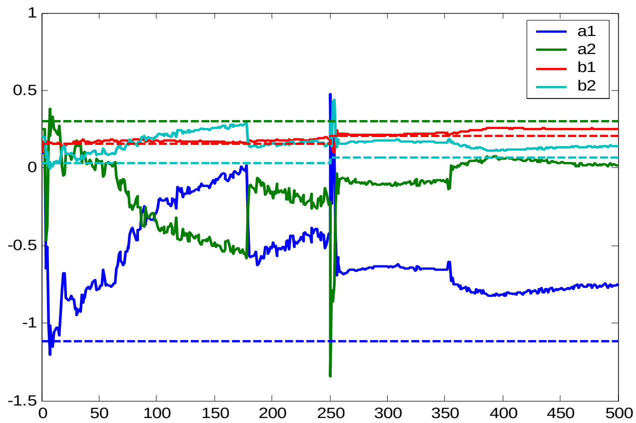

Fig.11. Parameter estimates (solid line) computed with the RIV with adaptive directional forgetting – true parameters (dashed line)

ERIV with fixed exponential forgetting

Fig.13 shows that despite the fact that parameters are estimated correctly the adaptive controller does not provide appropriate output signal. From Fig.14 can be seen that the speed of convergence is faster that in RLS but the estimator is not able to track changes in parameters.

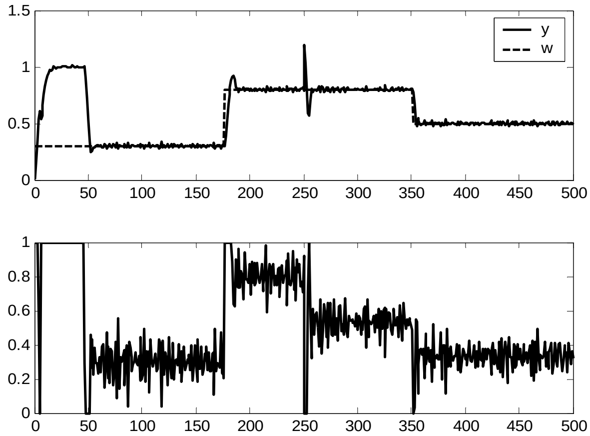

Fig.12 The adaptive control with LQ controller

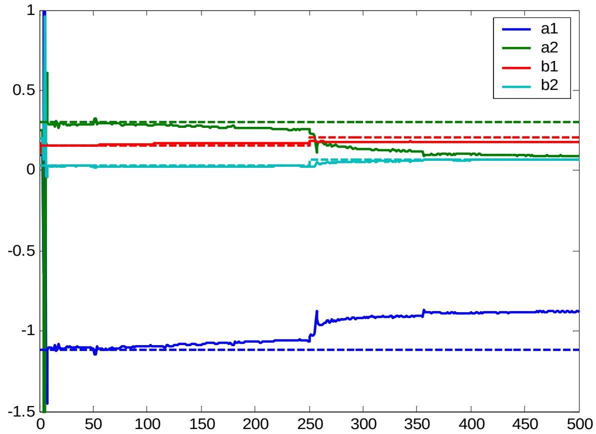

Fig.13 Parameter estimates (solid line) computed with the ERIV with fixed exponential forgetting - true parameters (dashed line)

4.1.2 Estimation of ARMAX model

RPEM-ARMAX with fixed exponential forgetting

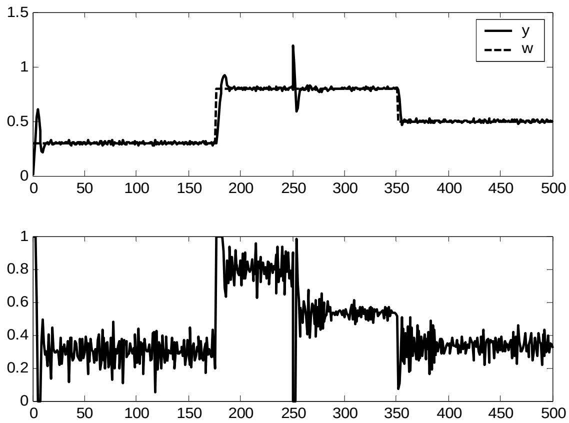

Fig.14 The adaptive control with LQ controller

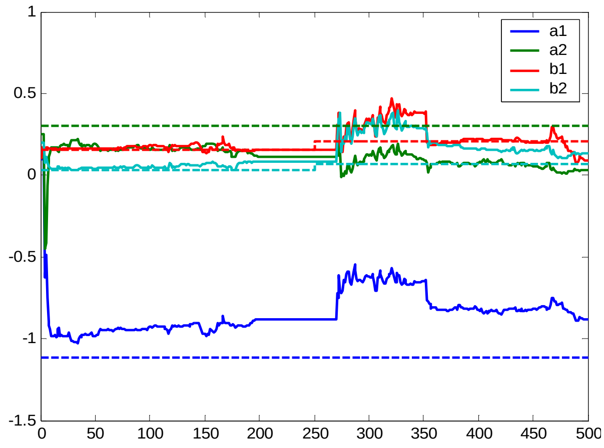

Fig. 15 Parameter estimates (solid line) computed with the RPEM-ARMAX with fixed exponential forgetting – true parameters (dashed line)

From step k=200 to k=270 the parameters estimates is maintained constant and in step k=270 the restart of covariance matrix is made. This setting improves the adaptive controller behaviour.

RELS with adaptive directional forgetting

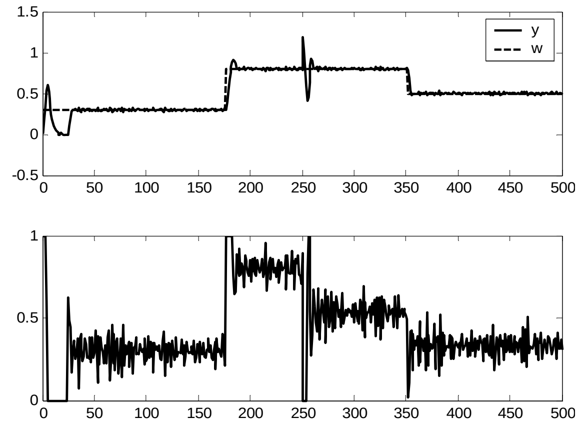

Fig.16 The adaptive control with LQ controller

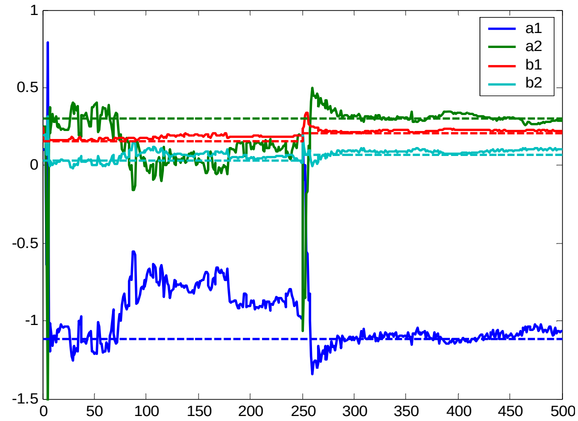

Fig.17 Parameter estimates (solid line) computed with the RELS with adaptive directional forgetting – true parameters (dashed line)

4.1.2 Estimation of OE model

RPEM-OE with fixed exponential forgetting

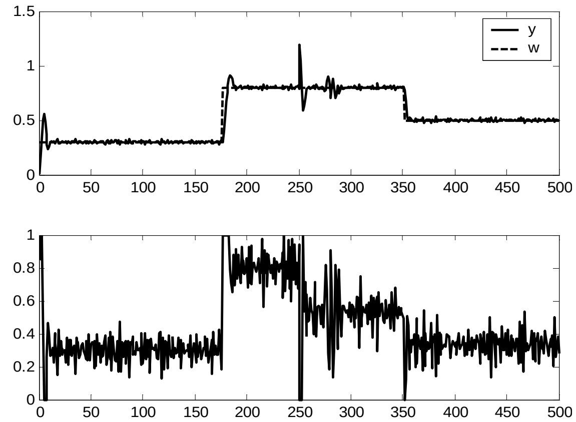

Fig.18 Simulation results: the adaptive control with LQ controller

From step k=200 to k=270 the parameters estimates is maintained constant and in step k=270 the restart of covariance matrix is made. This setting improves the adaptive controller behaviour.

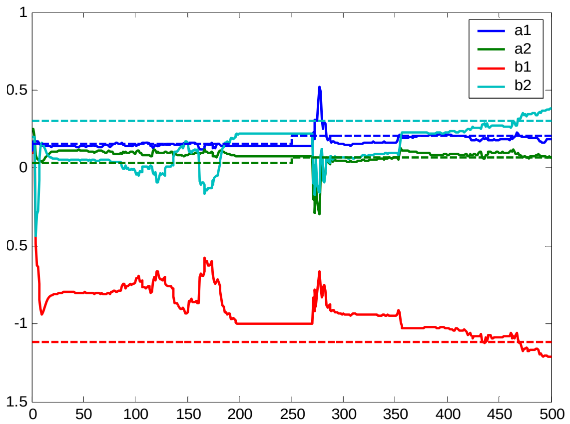

Fig.19 Parameter estimates (solid line) computed with the RPEM-OE with fixed exponential forgetting – true parameters (dashed line)

Influence of each recursive algorithms on adaptive control performance were evaluated from quality control point of view. The results can be seen in Table 1.

The criteria were defined

$$S_y = \frac{1}{k_2 - k_1 + 1} \sum_{k=k_1}^{k_2} e^2(k); \qquad (54)$$

$$S_u = \frac{1}{k_2 - k_1 + 1} \sum_{k=k_1}^{k_2} \Delta u^2(k) \qquad (55)$$

where $e(k)$ denotes control error, $u(k)$ is controller output and $k_1 = 1, \quad k_2 = 500$.

Methods	S_y	S_y
RLS with VEF	0.0460	0.0429
RLS with ADF	0.0054	0.0146
ERIV with FEF	0.0404	0.0276
RIV with ADF	0.0025	0.0126
RPEM-ARMAX with FEF	0.0027	0.0205
RELS with ADF	0.0058	0.0207
RPEM-OE with FEF	0.0026	0.0294

Table 1 Influence of recursive algorithm on adaptive control performance

From Table 1 can be seen that the minimum of sum of squared control error and minimum of sum of squared difference of controller output signal were achieved by RIV method with adaptive directional forgetting for parameter estimate of ARX model. Other methods listed in Table 1 also provide satisfactory results except RLS with variable exponential forgetting.

The results in Table 1 can be expressed in more transparent form. Fig.21 and Fig.22 show the control quality results in graphic form.

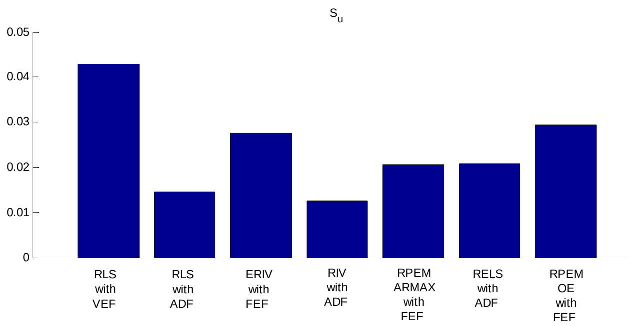

Fig.20 The criterion control quality S_u in graphic representation

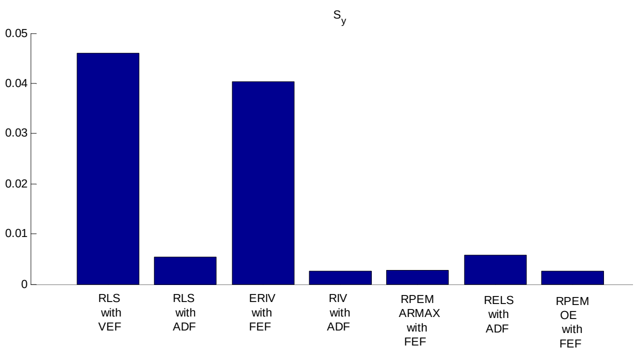

Fig.21 The criterion control quality S_y in graphic representation

5 Conclusion

The Recursive Identification Algorithm Library is designed for recursive parameter estimation of linear dynamic model ARX, ARMAX, OE using recursive identification methods. The simple library can be used e.g. in identification part of self-tuning controller or in educational process when it is possible to demonstrate the properties and behavior of the recursive identification algorithms and forgetting factors under various conditions. Proposed library can be used not only in educational process to demonstrate the behavior and properties of recursive identification algorithms, but for example in connection with such Real Time Toolbox to identify in real time the parameters of the model of real systems.

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