Simultaneous Multi-Start Simulated Annealing for Capacitated Vehicle Routing Problem

ARMAN DAVTYAN, SUREN KHACHATRYAN
College of Science and Engineering
American University of Armenia
40 Baghramian avenue, 0019 Yerevan
ARMENIA
arman_davtyan19@alumni.aua.am, skhachat@aua.am

Abstract: - A new metaheuristic algorithm is proposed for Capacitated Vehicle Routing Problem. CVRP is one of the fundamental problems in combinatorial optimization that deals with transport route minimization. The algorithm combines Simulated Annealing, multi-start and simultaneous computing techniques. A series of computational tests are conducted on several CVRP benchmarks and near-optimal solutions are obtained. The results indicate superior performance compared with Simulated Annealing.

Key-Words: - vehicle routing problem, combinatorial optimization, metaheuristics, simulated annealing, multi-start.

Received: August 29, 2019. Revised: February 2, 2020. Accepted: February 18, 2020. Published: February 29, 2020.

1 Introduction

The product distribution management is one of the most important areas of optimization and has a major role in the effectiveness of transport management. Several mathematical models are constructed in combinatorial optimization to solve this problem. Capacitated Vehicle Routing Problem (CVRP) is one of the fundamental problems that addresses transport management optimization. The objective is to determine the sequence of customers visited by each vehicle and satisfy constraints such as distance, capacity, and cost of the vehicle for those customers. Vehicle Routing Problem (VRP) is a general version of CVRP that was first introduced in 1959 by Dantzig and Ramser [1].

The formulation of the model led to the development of various algorithms, currently grouped into three types: exact, approximation and heuristic. Exact algorithms were developed right after the formulation of the problem. These algorithms include the Branch and Bound Principle, Dynamic Programming, Lagrangian Relaxation, etc. [2]. However, VRPs are NP-hard and can be exactly solved only for small-scale instances. The inefficiency of exact algorithms for large-scale solutions led to the development of heuristic algorithms, which do not guarantee an optimal solution, but produce good suboptimal solutions in reasonable time. Problem-independent metaheuristic algorithms appeared better solvers for NP-hard problems and searched more thoroughly in the solution space. The well- known metaheuristics for VRP are Simulated Annealing, Tabu Search, Genetic Algorithms, Ant Colony Optimization, etc. During the early research there was a notable bias towards Tabu Search-based approaches [2].

Simulated Annealing (SA) follows a local neighborhood search by picking random feasible solution from its neighborhood [3]. If the newly obtained solution is better than the previous one, it is accepted, otherwise it is accepted with a certain probability. The main shortcoming of the method is that the solution can be trapped in a local minimum. Various modifications were suggested to solve the problem. One of them is Multi-Start Simulated Annealing (MSSA), which incorporates a multi-start diversification mechanism into SA [4].

The remainder of the paper is organized as follows. Section 2 states the problem statement and the mathematical model. Section 3 describes the proposed algorithm. Section 4 discusses the testing results and shows the efficiency of the proposed algorithm over SA. Finally, Section 5 summarizes the results and concludes the paper.

2 Problem Statement

The current paper presents an approach to handle CVRP problems as stated above. Each node is a customer than needs to be visited, while every route should start and end at a depot. The objective is to minimize the overall distance. Capacity is hard constraint and any violation of that constraint implies infeasible solution. We propose to run simultaneously several instances of Multi-Start Simulated Annealing (SMSSA). After certain iterations, the solutions of the instances are compared and the parameters of all but the best suboptimal instance are altered. This method aims at improvement of the vulnerability of being trapped in a local minimum by searching more thoroughly in the solution space.

Let us consider the CVRP problem as a complete graph $G = (V, E)$, where V is the set of all vertices and E is the set of all edges. The zeroth vertex represents the depot, while the remaining n vertices represent the customers. The depot vertex contains a group of k vehicles each having capacity Q_k. Each customer from the set $V\backslash\{0\}$ possesses a certain positive demand with capacity q_i. The vertex set can be described as $V = \{0, 1, \dots, n\}$, $Q = \{Q_1, \dots, Q_k\}$, $E = \{(i, j);\ \forall\ i, j \in V,\ i \neq j\}$, $q = \{q_1, \dots, q_n\}$. Meanwhile, each edge $(i, j) \in E$ has its cost parameter c_{ij}. The problem is to construct the routes in a way that the cost of the total route is minimized and every vertex is visited. Also, the solution must fulfil the following restrictions: every vertex is visited only once, the capacity of the vehicle is never violated, and each route starts and ends at the depot.

The Integer Linear programming model of the CVRP can be described by a binary variable x_{ij}^k that indicates whether the route from location i to j is active or not. The model can be mathematically formulated as to minimize the target function

$$\sum_{i\in N}\sum_{j\in N}\sum_{k\in K} c_{ij}x_{ij}^k,$$

subject to constraints

$$\sum_{i\in V}\sum_{\substack{k\in V\\ i\neq j}} x_{ij}^k = 1, \forall\, j \in V \qquad (1)$$

$$\sum_{j\in V}\sum_{\substack{k\in V\\ i\neq j}} x_{ij}^k = 1, \forall\, i \in V \qquad (2)$$

$$\sum_{i\in V}\sum_{k\in V} x_{i0}^k = \sum_{i\in V}\sum_{k\in V} x_{0i}^k, i \neq 0 \qquad (3)$$

$$\sum_{i\in V}\sum_{j\in V} q_j x_{ij}^k \leq Q_k, \forall\, k \in K \qquad (4)$$

$$\sum_{i\in V}\sum_{\substack{k\in V\\ i\neq j}} x_{ij}^k \subseteq S, \forall\, S \subseteq \{1, \dots, n\} \qquad (5)$$

$$x_{ij}^k \in \{0,1\}, \forall\, i, j \in V, \forall\, k \in K \qquad (6)$$

The objective function is to minimize the sum of the travelled cost. Equations (1) and (2) ensure that only singular visits are made to each location except the depot. Equation (3) guarantees that each route must start and end at the depot. Equation (4) is for the car capacity constraints. Equation (5) is the sub-tour elimination constraint. It ensures that there are no cycles included in the routes. The last mandatory constraint simply defines the domain of the variables. This is the classical formulation of VRP proposed by Dantzig [5].

3 Proposed Algorithm

We propose a solution based on simultaneous and repetitive implementation of MSSA. Unlike the standard approach, however, we will not lower the temperature, but constantly compare the solutions and dynamically rise the temperature of all solutions except the best known one.

3.1 Insertion algorithm

The algorithm requires an initial solution. We choose a simple insertion algorithm as a satisfactory initial approximation. The closest unassigned point is added to the current rout that satisfies the constraints (1) – (6). The rout returns to the depot, if no such additions are possible, and a new rout starts from the depot. Note that the number of routs is not predefined.

3.2 Simulated Annealing

Simulated Annealing (SA) is a local neighbourhood search that explores the search space and accepts solutions with some probability. The SA can be described in the following steps:

1. Start with the initial solution $d = d_0$ and identify the minimization function $f(d)$;
2. Initialize the temperature T_0 and a cooling factor $\alpha \in (0, 1)$;
3. If the stopping condition is not met, take a random feasible neighbor d_n of d and check, if $d_n \leq d$ then $d = d_n$;
4. Otherwise, assign $d = d_n$ with probability $e^{-\frac{|f(d)-f(d_n)|}{T_0}}$;
5. After each k iterations update the temperature $T_0 = \alpha\, T_0$.

3.3 Simultaneous Multi Start SA

Simulated Annealing is a one-time-search metaheuristics. Even though it is possible to avoid local minima at low temperatures, SA may still be trapped around them. We propose to run several instances of SA memorizing the best known solution. Given the best known solution, M instances are initialized and launched in cycles of N iterations. After each cycle the best known solution is compared with the instance solution and updated, if outperformed by the latter. Otherwise, the parameters of the latter are adjusted. The cycles are repeated until the overall number of iterations is exhausted.

4 Testing Results

The performance and efficiency of the proposed SMSSA algorithm have been tested on the CMT benchmarks developed by Christofides, Mingozzi and Toth [6]. The tests were run 3 times on each benchmark dataset. Because the algorithm is the upgraded version of the classical SA, the benchmarks were tested with both algorithms to reveal the range of improvement. There are two types of instances among 14 datasets. The instances CMT1, CMT2, CMT3, CMT4, CMT5, CMT11 and CMT12 satisfy the classical constraints (1) – (6), while the rest were generated under the additional constraints of maximal route duration and service time. Vehicles are assumed to travel at unitary speed. The benchmarks also differ by number of customers, their distribution and the depot location.

Bench.	SMSSA		SA		Optimal	
	Dist	V	Dist	V	Dist	V
CMT1	533	5	540	5	525	5
CMT2	871	10	882	10	835	10
CMT3	834	8	856	8	826	8
CMT4	1085	12	1104	12	1028	12
CMT5	1380	17	1441	17	1291	17
CMT6	568	6	573	6	555	6
CMT7	924	12	942	12	910	11
CMT8	932	10	960	11	866	9
CMT9	1212	15	1242	15	1163	14
CMT10	1478	20	1507	20	1396	18
CMT11	1091	7	1150	7	1042	7
CMT12	852	10	852	10	820	10
CMT13	1598	12	1645	12	1541	11
CMT14	872	11	888	11	866	11

Table 1. CMT benchmark instances

Table 1 summarizes the numerical experiments and compares them with the optimal solution in the last two columns. The "Dist" columns show the computed overall route distance and "V"– the number of used vehicles. Note that SMSSA does not practically improve the number of used vehicles compared with the optimal one.

Figure 1 depicts the relative deviations from the optimal CMT solutions defined as

$$\delta_{alg} = \frac{Dist_{alg} - Dist_{opt}}{Dist_{opt}} \quad (7)$$

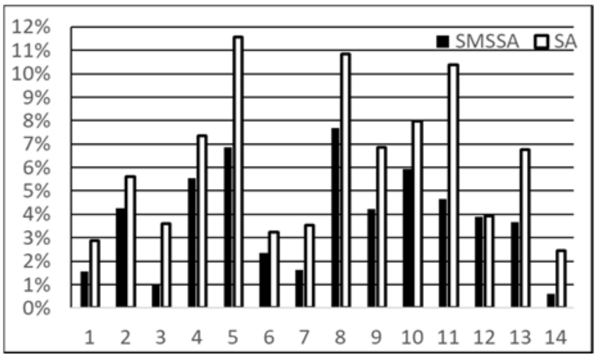

Figure 1. Relative deviation from the optimal CMT solutions: black – SMSSA, white – SA.

It can be seen that the proposed SMSSA algorithm results in reasonably sub-optimal solutions and systematically outperforms the classical SA by around 2.5% on average.

5 Conclusions

The Capacitated Vehicle routing problem is one of the most known and common problem in combinatorial optimization because of its considerably significant and important applications. Being *NP*-hard, it requires approximations and heuristics for most of the practical cases. Simultaneous Multi-Start Simulated Annealing algorithm has been presented in the current paper and tested on the CMT benchmarks. The obtained results indicate that the algorithm produces near optimal solutions and systematically outperforms the classical SA method. It permits natural parallelization and, thus, minimizes computational overhead. However, SMSSA is based on local search neighborhood and is still vulnerable to local optima and large solution spaces. The ways of further performance enhancement will be investigated in the next paper [7].

References:

[1] Dantzig G. B., and Ramser J. H., The Truck Dispatching Problem, *Management Science*, Vol.6, No.1, 1959, pp. 80-91.

[2] Laporte G., Toth, P. and Vigo D., Vehicle Routing: Historical Perspective and Recent Contributions, *EURO Journal on Transportation and Logistics*, Vol.2, 2013, pp. 1-4.

[3] Kirkpatrick S., Gelatt C., and Vecchi M., Simulated Annealing Methods, *J. Stat. Phys*, Vol.34, 1984, pp. 975-986.

[4] García-Villoria A., Corominas A., and Pastor R., Pure and Hybrid Metaheuristics for the

Response Time Variability Problem, *Meta-Heuristics Optimization Algorithms in Engineering, Business, Economics, and Finance*, pp. 275-311, IGI Global, 2013.

[5] Chong E. K. P, and Zak S. H., *An Introduction to Optimization*, John Wiley & Sons, 2004.

[6] Uchoa E., Pecin D., Pessoa A., Poggi M., Vidal T., and Subramanian A., New Benchmark Instances for the Capacitated Vehicle Routing Problem, *European Journal of Operational Research*, Vol.257, No.3, 2017, pp. 845-858.

[7] Feige U., Rigorous Analysis of Heuristics for NP-hard Problems, *Proceedings of the 16-th Annual ACM-SIAM Symposium on Discrete Algorithms*, 2005, p. 927.