Mathematical and Computer RAI Models of False Information Dissemination

NUGZAR KERESELIDZE
Department of Natural Science, Mathematical, Technology and Pharmacy
Sukhumi State University
61 Politkovskaya street, 0186, Tbilisi
GEORGIA
nkereselidze@sou.edu.ge http://www.kereseli.sou.edu.ge

Abstract: - New mathematical and computer models of information warfare are built and researched in the given work. One of the forms of information warfare: dissemination of false information is considered. Risk groups are defined: prone to the perception of misinformation; adepts - who have received false information and with immunity - reject false information from the beginning or future adeptors. RAI models of numerical change in the composition of these groups are considered. An expanded RAI model of dissemination of false and objective information has been built, mathematical conditions for prevention of mass misinformation of society have been defined..

Key-Words: - mathematical, computer, model, information, warfare, flows, groups, dissemination, false.

1 Introduction

As among the various forms of introducing the Information Warfare, of particular interest is the type of information warfare, the purpose of which is to influence on the psyche of people, manipulate the consciousness of the masses, disorientate individuals, impose strange stereotypes of behavior on them, etc. Such an effect is sometimes achieved by specific information flows that intensively fall on members of society. A feature of such information flows is the dissemination of false information, misinformation, etc. More details about the goals and means of introducing the Information Warfare through the "word" can be found in [1], [2].

The relevance and significance of the problem associated with organized campaigns for the dissemination of knowingly false information is adequately evaluated in the modern world and certain steps have been taken in this direction. For example, by the decision of the Heads of State and Government of the Council of Europe, a special group “Strategic Communication with the East” was established - **EastStratCom Task Force**, https://euvsdisinfo.eu, for the informational “counteraction to Russian disinformation campaigns” aimed at discrediting the EU’s Eastern Neighborhood Policy. As stated in the report of the special group - quote: “in four years of its existence, the EU campaign against disinformation has issued more than 140 "Review of Disinformation" newsletters containing more than 5,000 cases of reports of disinformation in 18 different languages. The product is regularly used and cited by various governments, ministries, government agencies, secret services, researchers, analytic centers and journalists across Europe and beyond” – end of quote [3].

It is logical that a campaign against misinformation can be planned and carried out more effectively if there is a clear idea and quantitative characteristics of the false information dissemination. Naturally, the study of false information dissemination using mathematical and computer modeling, conducting computer experiments along with other methods of researching the task, allows us to effectively describe the process under study and plan actions against the disinformation campaign. When modeling the dissemination of false information, we will assume that false information negatively affects the health of human psyche and nervous system. Thus, we have the right to identify false information as an infection, a virus causing painful changes in a person and the right to apply the achievements of modeling the spread of epidemics by modifying them for the task.

2 Problem Formulation

Suppose that in a society with a number of people N at each time t there is a so-called Risk Group (RG) in the number of people $R(t)$ who are exposed to false information. People from the RG, who received false information and thought it was true move to the Group of Adepts (AG). Let us say, that at the moment of time t the number of people in AG is $A(t)$. Suppose that a person from the AG is aware for a time μ that false information has been imposed on him, after which he leaves this group and replenishes the Group with Immunity (IG). Let us denote the number of people in the IG at time point t through $I(t)$. Noting that the IG member is already insured against repeated exposure to false information, i.e. gains certain immunity. The transition of an individual from one group to another can be represented by the following scheme: RG→AG→IG, Fig. 1.

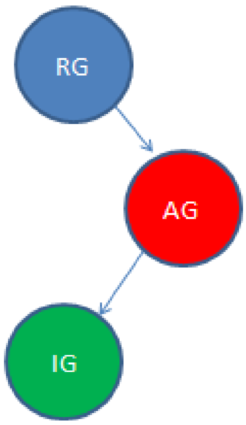

Fig. 1 Transition Scheme of an individual from one group to another in RAI model

If we assume that false information can be disseminated by directly contacting members of the Risk and Adept groups, then we can mathematically describe the process of transition from RG to AG. The change rate in the number risk group members at a time t is equal to $-\alpha R(t)A(t)$, where α is the coefficient of contact between group members with the subsequent dissemination of information. The sign "-" means that the number of the risk group members is decreasing. Note that the members who left the risk group, in the same amount are added to the adapter group, so the change rate in the number of adapters at a time t is equal to $\alpha R(t)A(t)$, but at the same time, the member number of the Adepts decreases, because after some time μ each member of the Adept group understands that he was influenced by false information and leaves this group. Thus, the change rate in the number of adapters in a given period of time t also depends on $-\gamma A(t)$, where γ is the coefficient of exemption intensity from false information for members of the adapters and $\gamma = 1/\mu$. The sign "-" means that the member number of the group of Adepts is reduced. It is also not difficult to describe the transition from AG to IG. As a result, we obtain, as a first approximation, the following mathematical model for the dissemination of false information:

$$\begin{cases} \dfrac{dR(t)}{dt} = -\alpha R(t)A(t), \\ \dfrac{dA(t)}{dt} = \alpha R(t)A(t) - \gamma A(t), \\ \dfrac{dI(t)}{dt} = \gamma A(t). \end{cases} \quad (1)$$

Where $\alpha, \gamma > 0$. To the ODE system (1) we add the initial conditions:

$$R(0) = R_0 > 0, \quad A(0) = A_0 > 0, \quad I(0) = I_0 \geq 0, \quad (2)$$

We obtain a mathematical model **RAI** of false information dissemination, which is analogous to the well-known classical **Kermak-Mackendrick SIR** model [4]. If we add all three equations of the (1) system, we get:

$$\frac{d\left(R(t) + A(t) + I(t)\right)}{dt} = 0.$$

From the last equality it follows that

$$R(t) + A(t) + I(t) = N = const. \quad (3)$$

The last equation (3) indicates that we are considering a "closed" society, the number of which does not change over time - is constant. With such an idealization of society, birth, immigration, death, emigration and other factors are not taken into account.

The mathematical **RAI** model (1), (2), like the **SIR** model, is non-linear, since in the **RAI** model the system of ordinary differential equations (1) is nonlinear and despite the apparent simplicity of the **RAI** and **SIR** models, they cannot be solved explicitly, i.e. get an exact analytical solution [5]. All recent reports on the exact analytical solution found for the Cauchy problem (1), (2) in fact turn out to be an asymptotic analytical solution. With this approximate analytical solution, the authors (Alexsandro M. Carvalho, Sebastian Gonçalves) proudly report: "We verified that the present general solution is in excellent agreement with numerical solutions of the same equations over the entire dynamic". Since the numerical solution of model (1),(2) becomes the reference accuracy for an approximate analytical solution, we will use the computer experiment method from the very beginning to study the RAI model. The numerical solution method is simply indispensable for an extended RAI model, which is non-linear and more complex than the RAI model.

3 Problem Solution

From the second equation of system (1), the conditions are determined under which the number of adherents will increase or not. Meanwhile, the number of adherents indicates in general, how much the society is exposed to false information i.e. how misinformed it is. It is advisable to introduce a threshold effect, in which it is determined whether the society is massively affected by false information or not. At

$$R_0 > \gamma/\alpha, \tag{4}$$

The number of adherents is increasing and complete misinformation risk of society is high. Similar inequality (4), which is obtained in SIR models, shows that in society there is an increase in the disease and there is a high probability of an epidemic. To prevent an epidemic, models like SIR give recommendations on how to obtain the reverse inequality (4). In particular, it is proposed to reduce the initial number of susceptible people. In model RAI, the initial number of people at risk is - RG. In order to reduce the initial number of susceptible people in SIR models, it is proposed to conduct vaccination in a susceptible group. As a result of this vaccination, on the one hand, the initial number of susceptible people decreases and on the other, the initial number of people with immunity increases. A computer experiment conducted for (1) and (2) under condition (4) confirms the growth of adept number and the danger of mass misinformation of society. For example, for the initial conditions - $R(0) = R_0 = 54$, $A(0) = A_0 = 2$, $I(0) = I_0 = 1$ and the value of the coefficients of the system (1) - $\alpha = 0.09$, $\gamma = 0.2$, the condition (4) is fulfilled - 54 > 2.22, and the numerical solution of the Cauchy problem (1), (2) given in the form of graphs in Fig. 2 shows the growth of the number of adepts.

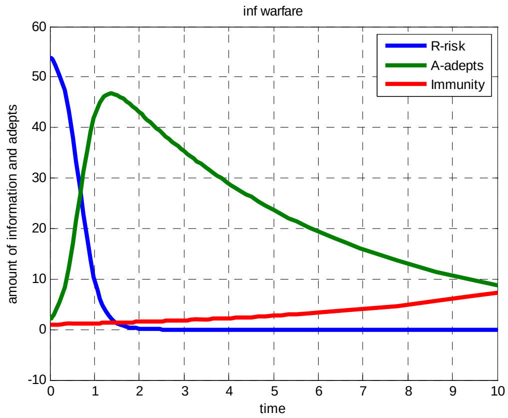

Fig. 2 Growth of the number of adepts, $R_0 = 54$, $\alpha = 0.09$, $\gamma = 0.2$

A computational experiment conducted in the MATLAB environment. Listing 1 shows the code that was used for the computer experiment.

Listing 1. Growth of the number of adepts.

```
rai_1.m
n0=[54 2 1];
[T,Y]=ode15s('rai_1_pr',[0
40],n0);plot(T,Y,'LineWidth',3)
title('inf warfare')
xlabel('time')
ylabel('amount of information and adepts')
legend('R','A','I')
grid on

rai_1_pr.m
%ode- right side of the system i
function dndt=rai_1_pr(t,n)
dndt=zeros(3,1);
a=0.09; g=0.2;
dndt(1)=-a*n(1)*n(2);
dndt(2)= a*n(1)*n(2)-g*n(2);
dndt(3)=g*n(3);
end
```

For model (1), (2), in order to prevent mass misinformation of society, it is necessary to achieve the reverse inequality in (4). Select such values of the coefficients and the initial values in (1) and (2) so that (4) is not performed. Let's say we have $R_0 = 54$, $\alpha = 0.025$, $\gamma = 2$, then $R_0 < \gamma/\alpha$, and, as shown in Fig. 3, the number of adepts does not increase and there is no danger of mass misinformation of society.

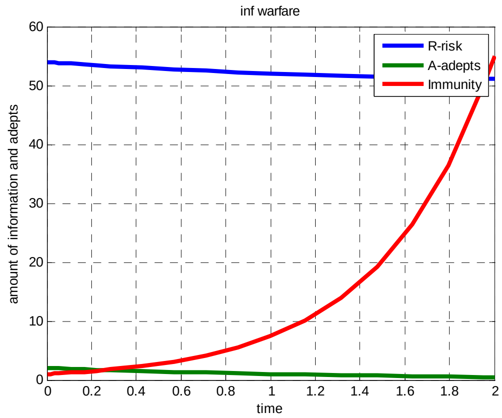

Fig. 3 The number of adepts does not increase, $R_0 = 54$, $\alpha = 0.025$, $\gamma = 2$

What actions should be taken for the RAI model to have the inverse inequality (4)? What should be the equivalent of vaccinating susceptible groups in the SIR model for the RAI model? To solve the problems, it is advisable to build an extended RAI model.

But first, we carry out a qualitative analysis of system (1). Note that in this system on the right-hand side only the first two equations contain the derivative of the desired functions. Therefore, we will investigate them and from the system (1) we select the following system:

$$\begin{cases} \dfrac{dR(t)}{dt} = -\alpha R(t)A(t), \\ \dfrac{dA(t)}{dt} = \alpha R(t)A(t) - \gamma A(t), \end{cases}$$

This system has a zero solution $R(t) \equiv 0$, $A(t) \equiv 0$. It is also a point of equilibrium and we study the stability of this solution. From the Jacobian of system at zero we obtain the characteristic equation $\lambda(\lambda + \gamma) = 0$. Jacobian eigenvalues take on values: $\lambda = 0$, $\lambda = -\gamma < 0$. Thus, we obtain a critical case, and therefore the zero solution can be both stable and unstable. We restrict ourselves to stating this fact and will not conduct further research on the stability of the zero solution. When there is an opportunity to receive full solutions of a system, by using their analysis you can get sufficient information about the system. We give only the phase trajectory for R and A, the number of risk groups and adherents, respectively Fig. 4.

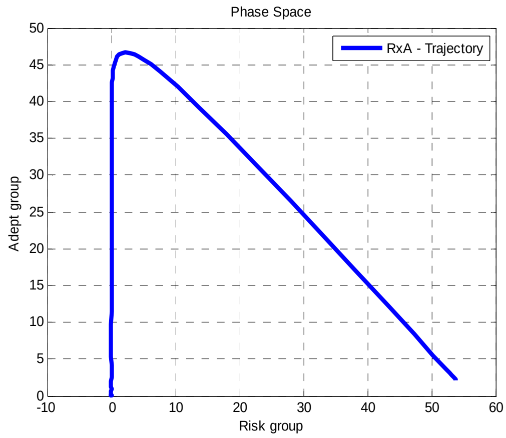

Fig. 4 Phase trajectory R-risk and A-adept

3.1. Extended RAI model

In order to prevent complete misinformation of society in the RAI model, one should introduce control parameters, for which it is necessary to supplement model (1), (2) with new parameters. We will assume that in society a certain source disseminates false information in volume $F(t)$ at any given time t. Similarly, at every moment in time t, the opposite false information in volume $N(t)$ is distributed in society. In fact, with the help of $N(t)$ there is a disavowal of false information $F(t)$. The flows of information $F(t)$ and $N(t)$ are aimed at transferring members of the RG to the AG and IG, respectively, while the flow of information $N(t)$ is also aimed at the transfer of members of the AG to the IG. Thus, in the new RAI model, the spread of false information will occur not only through interpersonal contacts of RG and AG group members, but also through the impact of the false information flow $F(t)$ on RG. On the other hand, the disavowal of false information will occur through the influence of the information flow $N(t)$ on RG and AG, and also through interpersonal contacts between RG and AG with IG. Thus, we get the following scheme of transition of an individual from one group to another, Fig. 5.

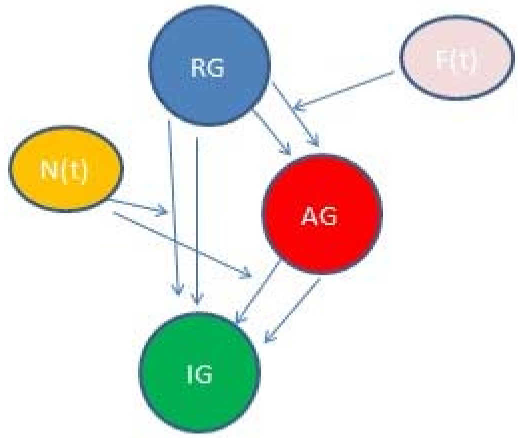

Fig. 5 Transition Scheme of an individual from one group to another in extended RAI model

Note, that $N(t) \geq 0$, $F(t) \geq 0$ are distributed using IT. When constructing a new, extended RAI model, we also used the building model ideas of Samarskiy-Mikhailov [6], information flows of T. Chilachava [1], [2] and the integrated model [7]. The reduction speed of risk group members depends on $\lambda N(t)R(t)$, which means the impact of the flow $N(t)$ on the Risk Group with λ coefficient. Similarly, the reduction rate of risk group members also depends on $\kappa F(t)R(t)$, which means the impact of the flow $F(t)$ on the Risk Group with κ coefficient. The reduction rate of risk group members also depends on interpersonal contacts with groups of adherents and immunity and are accordingly equal to: $-\alpha R(t)A(t)$, $-\beta_1 R(t)I(t)$, where α, β_1 are the effectiveness coefficients of interpersonal contacts of the marked groups. Note that in the right part of the first equation of the system (1) new members are added: $-\lambda N(t)R(t)$, $-\kappa F(t)R(t)$, $-\beta_1 R(t)I(t)$. The change rate in adept numbers for the extended RAI model depends on $\alpha R(t)A(t)$, $\kappa F(t)R(t)$, $-\lambda_1 N(t)A(t)$, $-\gamma A(t)$, $-\beta_2 A(t)I(t)$. Thus, the change rate in the number of adepts for the extended model differs from the same parameter of the original RAI model by new members: $\kappa F(t)R(t)$, $-\lambda_1 N(t)A(t)$, $-\beta_2 A(t)I(t)$. We believe that the dissemination of false information is aimed at the number of risk groups and is determined by the possibility of IT. Similarly, the dissemination of anti-false information is focused on the numbers of the group of adherents, and is determined by the capability of IT. Thus, the new extended RAI mathematical model has the following form:

$$\begin{cases} \dfrac{dR(t)}{dt} = -\lambda N(t)R(t) - \kappa F(t)R(t) - \\ \qquad -\alpha R(t)A(t) - \beta_1 R(t)I(t), \\ \dfrac{dA(t)}{dt} = \alpha R(t)A(t) + \kappa F(t)R(t) - \\ \qquad -\lambda_1 N(t)A(t) - \gamma A(t) - \beta_2 A(t)I(t), \\ \dfrac{dI(t)}{dt} = \gamma A(t) + \beta_1 R(t)I(t) + \beta_2 A(t)I(t) + \\ \qquad +\lambda_1 N(t)A(t) + \lambda N(t)R(t), \\ \dfrac{dN(t)}{dt} = \omega_1 A(t)\left(1 - {N(t)}/{M_1}\right), \\ \dfrac{dF(t)}{dt} = \omega_2 R(t)\left(1 - {F(t)}/{M_2}\right). \end{cases} \tag{5}$$

$$\begin{cases} R(0) = R_0 > 0, A(0) = A_0 > 0, I(0) = I_0 \geq 0, \\ N(0) = N_0, \;\; F(0) = F_0. \end{cases} \tag{6}$$

In system (5) all coefficients are positive, M_1 and M_2 mean the levels of those Internet Technologies with the help of which information flows $N(t)$, $F(t)$, respectively, are distributed; ω_1 and ω_2 are dissemination intensity coefficients of truth and false information accordingly. Thus, a combined mathematical model (5), (6) is constructed in which information flow is included, as well as persons receiving and distributing information. At the same time there are restrictions on flows and number of persons. This type of combined mathematical models of the Information Warfare was first proposed in [6].

In the extended RAI model (5), (6) the possibilities of preventing mass misinformation of society are investigated not by reducing the number of Risk Groups, but by means of variations ω_1 - intensity coefficient and M_1 - technological level of dissemination of objective information. The program code presented in Listing 2 has been compiled for the computer experiment. A numerical experiment was conducted for various values of the initial conditions (6) and system coefficients (5).

Listing 2. Growth of adept number for extended RAI model.

```
rai_2.m
n0=[763 2 1 40 50];
```

```
[T,Y]=ode15s('rai_2_pr',[0 10],n0);plot(T,Y,'LineWidth',3)
title('inf warfare')
xlabel('time')
ylabel('amount of groups and information flows')
legend('R','A','I','N','F')
grid on

rai_2_pr.m
%ode- right side of the system i
function dndt=rai_2_pr(t,n)
dndt=zeros(5,1);
a=0.9; g=0.1428; l1=0.2; l=0.3; k=0.7; b1=0.4; b2=0.6;
o1=70; o2=0.9; m1=900; m2=250;
dndt(1)=-l*n(4)*n(1)-k*n(5)*n(1)-a*n(1)*n(2)-b1*n(1)*n(3);
dndt(2)=a*n(1)*n(2)+k*n(5)*n(1)-l1*    n(4)*n(2)-g*n(2)-b2*n(2)*n(3);
dndt(3)=g*n(3)+b1*n(1)*n(3)+b2*n(2)*n(3)+l*n(4)*n(1)+l1* n(4)*n(2)+ l*n(4)*n(1);
dndt(4)=o1*n(2)*(1-n(4)/m1);
dndt(5)=o2*n(1)*(1-n(5)/m2);
end
```

Let be $\alpha = 0.9$ $\gamma = 0.1428$, $\lambda_1 = 0.2$, $\lambda = 0.3$, $\kappa = 0.7$, $\beta_1 = 0.4$, $\beta_2 = 0.6$, $\omega_1 = 0.7$, $\omega_2 = 0.9$, $M_1 = 300$, $M_2 = 250$. The result of a computer experiment conducted for this data is visualized in Fig. 6.

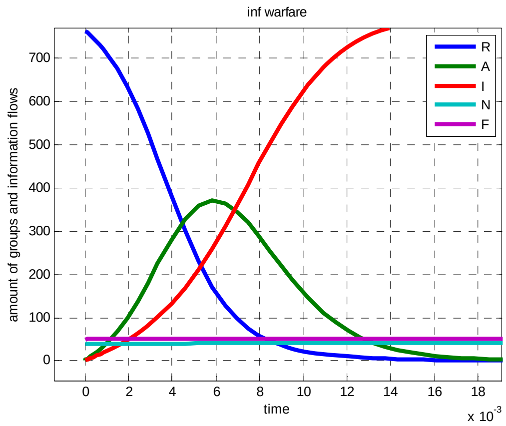

Fig.6 Growth of adept number in extended RAI model

The results of the calculations show that at about 5.8 units of time the maximum number of adepts reaches 370, which is slightly less than half (48.3%) of the total population - 766. Note that the total population is determined by the formula (3), $R(0) = 763$, $A(0) = 2$, $I(0) = 1$. This is perhaps a high indicator for holders of false information in society. For comparison, the spread of the disease among 5% of the total population is considered the epidemic threshold in healthcare organizations.

We will try to reduce the number of adepts by varying the ω_1 and M_1 parameters, which we will call the control parameters. By constantly increasing the values of the control parameters, we can achieve a reduction in the number of adepts. For example, for $\omega_1 = 300$ and $M_1 = 1500$, the visualization of calculations is given in Figure 7.

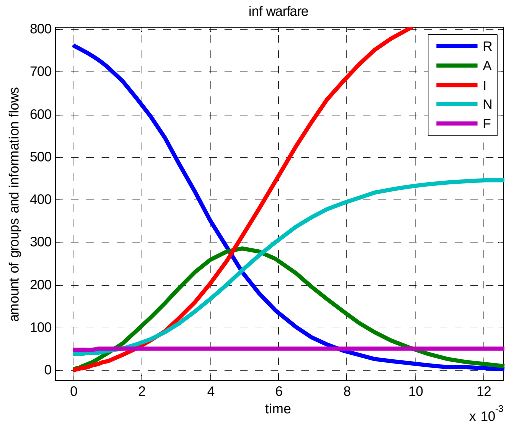

Fig. 7 Reduction of adept number

In these conditions, the results of the calculations show that at about 4.9 units of time the maximum number of adepts is 286, which is 37.3% of the total population - 766. A computer experiment shows that by varying parameters ω_1 and M_1 it is possible to reduce the number of adepts. However, as can be seen, to reduce the maximum number of adherents by ten percent, it was necessary to increase the values of the control parameters by several orders of magnitude. Is it possible to increase the efficiency of control parameters? One approach to solving this problem is to increase the number of control parameters. In the control parameters we can add λ, α, λ_1, β_2. Parameters λ, α, that will increase the transition of people from the risk group to the group with immunity. Parameters λ_1, β_2 will increase the transition of people from the group of adherents to the group with immunity. Gradually increasing the values of the new control parameters by not more than twenty units, the number of adepts can be reduced to 9.3%. This result is achieved by $\alpha = 10.9$, $\lambda = 10.3$, $\lambda_1 = 10.2$, $\beta_2 = 10.6$, $\gamma = 0.1428$, $\kappa = 0.7$, $\beta_1 = 0.4$, $\omega_1 = 300$, $\omega_2 = 0.9$, $M_1 = 1500$, $M_2 = 250$, $R_0 = 763$, $A_0 = 2$, $I_0 = 1$, $N_0 = 40$, $F_0 = 50$, see Fig. 8.

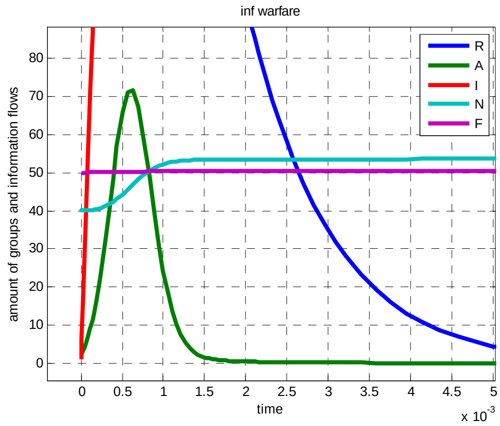

Fig. 8 Reduction of adept number in extended RAI model by new control parameters

In all the numerical experiments we have conducted so far, there was initially an increase in the number of adepts. The number of adepts reached a maximum at some point in time and then decreased. Can the initial growth of adepts be prevented and how can this be achieved?

Because in the initial moment of time $t=0$ the number of adherents did not increase, it is necessary to fulfill the condition $\left.\frac{dA(t)}{dt}\right|_{t=0}<0$. Therefore, the inequality should be satisfied:

$\alpha R_0A_0+\kappa F_0R_0-\lambda_1N_0A_0-\gamma A_0-\beta_2A_0I_0<0$.

When $A_0>0$, we can write:

$$\alpha R_0+\kappa \frac{F_0R_0}{A_0}<\lambda_1N_0+\gamma+\beta_2I_0. \qquad (7)$$

Thus, inequality (7) must be met in order to avoid an increase in the number of adepts. For the computer experiment in which the visualization is shown in Figure 7, the inequality (7) is not performed. Specifically we have 21669.2 > 418.7428.

In the general case, if the values α, κ, F_0 decrease, and the values λ_1, γ, β_2, N_0 increase, it is possible to achieve the fulfillment of inequality (7). Then the number of adherents will not grow and there is no threat of mass misinformation of society.

Inequality (7) shows the significant role of the amount of initial objective information N_0 for fulfilling this inequality. In fact, value N_0 can lead to prevention in the company against misinformation. From (7) we have that for

$$\frac{\alpha R_0+\kappa \frac{F_0R_0}{A_0}-\gamma-\beta_2I_0}{\lambda_1}<N_0, \qquad (8)$$

There will be no increase in the number of adherents.

For the computer experiment in which the visualization is shown in Figure 7, select the value N_0 to be performed (8). The minimum integer N_0 at which inequality (8) is performed is 2124. The result of the computer experiment at is shown in Fig. 9.

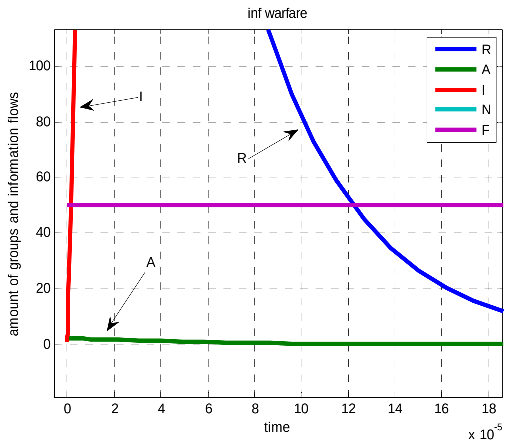

Fig. 9 The number of adepts does not increase in extended RAI model by new control parameters

4 Conclusion

Mathematical model RAI of false information dissemination in society (1), (2) is proposed. SIR models of epidemic spread are used as the basis. Mathematical conditions of mass misinformation of society are defined (4). An expanded RAI (5), (6) model is built for disseminating false information in society, which is opposed to disseminating objective information. A computational experiment conducted in the MATLAB environment on a computer model, based on the mathematical model (5), (6), makes it possible to conclude that by selecting the value of the control parameters ω_1 and M_1 it is possible to select such $N(t)$ value in which the disavowing of false information is sufficient to prevent complete or significant misinformation of the society. It has also been established that disavowing false information occurs more efficiently if the range of control parameters is expanded with the parameters - λ, α, λ_1, β_2, N_0. Condition (7) is obtained in which the number of adepts will not grow. The special role of the initial condition of objective information N_0 in preventing the growth of the number of adepts is stressed.

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [YS17_78].

References:

[1] T. Chilachava and N. Kereselidze, Continuous Linear Mathematical Model of Preventive Information Warfare, *Sokhumi State University Proceedings, Mathematics and Computer Sciences* vol. 7, 2009, pp. 91-112. http://sou.edu.ge/files/samecniero%20mushaobis%20koordinacia/ssu_shromebi-VII-2009-saboloo.pdf#page=113.

[2] T. Chilachava and N. Kereselidze, Non-Preventive Continuous Linear Mathematical Model of Information Warfare, *Sokhumi State University Proceedings, Mathematics and Computer Sciences* vol. 7, 2009, pp. 113-141. http://sou.edu.ge/files/samecniero%20mushaobis%20koordinacia/ssu_shromebi-VII-2009-saboloo.pdf#page=113.

[3] EU vs Disinfo, Disinformation Review, https://euvsdisinfo.eu/disinfo-review/. [Accessed Oct. 01, 2019].

[4] W. O. Kermack and A. G. McKendrick, Contributions to the mathematical theory of epidemics, *Proc. R. Soc. bond. A,* 15:700-721, 1927.

[5] Matt J. Keeling and Pejman Rohani, *Modeling Infectious Diseases in Humans and Animals*, Published by Princeton University Press, 2018, p. 368.

[6] A.A. Samarskiy and A.P. Mikhailov, *Mathematical modeling: Ideas. Models. Examples*, M. Fizmatlit, In Russian, First edition -1997, second revised edition - 2005 -. p. 320.

[7] N. Kereselidze, Combined continuous nonlinear mathematical and computer models of the Information Warfare, *International journal of circuits, systems and signal processing*, Volume 12, 2018, pp. 220-228. http://www.naun.org/main/NAUN/circuitssystemssignal/2018/a682005-aep.pdf.