Reconstruction of Tomographic Images From Limited Projections Using TVcim-p Algorithm

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Abstract: - Computed tomography (CT) has great impact in many fields such as medical applications, industrial inspection, etc... Low dose constraints and Limited projection are common problems in a variety of tomographic reconstruction examples which lead to wrong data. In this work, we propose a method of CT reconstruction based on the simultaneous iterative reconstruction techniques SIRT improved by imposing positivity constraint in the total variation (TVcim-p). We test our method with on Shepp-Logan phantom and different reconstruction methods. The results show that the proposed algorithm can gives images with quality comparable to other algorithms.

Key-Words: - Image Reconstruction; Total Variation Minimization; SIRT, Cimmino method

1 Introduction

X-ray computed tomography (CT) is a technique has developed primarily in the context of medical applications, nowadays is widely used in many applications such as industrial inspection and to the systems for transportation security. This technique enable to representing interior features within solid objects from a series of x-ray measurements taken from different angles around the object.

For the CT method a large number of projections are necessary in order to ensure the accurate reconstruction of density distribution and to obtain a best reconstruction of the inspected object. However, to improve the safety (a lower dose) and the productivity (faster acquisition) of the X-ray tomography (CT) system, we seek to reconstruct a high-quality image with a low number of projections. This usually results in poor signal-to-noise (SNR) ratio of reconstructed images. Reducing the total number of projection images degrades the quality of reconstruction and increases reconstruction errors [1].

Generally, for few projections the associated image quality of results reconstructed by the conventional analytical reconstruction methods is degraded [2].

The reconstruction algorithms in CT fall into two categories. On the one hand, the direct methods as the FBP (filtered back-projection) most commonly used. On the other hand, the iterative methods for example the algebraic reconstruction techniques (ART), statistical image reconstruction techniques (SIRT) and the Expectation Maximization (EM). [3] Various CT reconstruction algorithms using few projections have been proposed recently with varying degrees of success. [4]The iterative reconstruction methods are computationally intensive because the estimated projections must be performed repeatedly. This is in addition to the updates required of the reconstructed pixels based on the difference between the measured projection and the calculated projection. All of the iterative reconstruction algorithms require several iterations before they converge to the desired results. The aim of this work is therefore to evaluating the fidelity of Landweber method, cimmino method, and the combination cimmino method with Total variation in case of limited and noised projections.

The paper is organized as follows. In the section ‘‘problem formulation” we present the mathematical model of CT image formation. In the section ‘‘Materials and methods’’ we present the numerical optimization problem for the reconstruction methods such as, Landweber method, Cimmino’s method, Total Variation Regularization (TV) and the proposed TVcim-p method; in the section ‘‘Results and discussion’’ the numerical results on an image the Shepp-Logan phantom are presented and lastly in the section ‘‘Conclusions’’ we make some final observations.

2 Problem Formulation

The principle of tomography is based on the hypothesis of Radon which states that we can reconstruct the image of an object from all its projections at different angles. [5] It is a linear transformation in other words, it is transforming a 2D function defined by *f(x,y)* into a 1D projection at an angle θ and a given module t it can be expressed by:[6,7]

$$P_\theta(t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\delta(x\cos\theta + y\sin\theta - t)dxdy \quad (1)$$

Where θ and t are respectively the coordinates angular and radial of the projection $P_\theta(t)$. The object $f(x,y)$ can be represented by $f'(t,s)$ in the rotated coordinate system, where the two coordinate systems are related by the following set of equations:

$$\begin{cases} t = x\cos\theta + y\sin\theta \\ s = -x\cos\theta + y\sin\theta \end{cases} \quad (2)$$

The visual representation of Radon Transform is called sinogram. In reconstructions, the concept of sinograms is often used however it is different from the notion of projection. Projections and sinograms contain the same information, they differ only in the organization according to which the information is represented. [7]

CT reconstruction problem can be formulate in the following model:

$$Af = P \quad (3)$$
$$P = P^* + e \quad and \quad f^* = exact\ image$$

where P represents the projection data, f is the reconstructed image, e accounts for any measurement bias and additive noise, and $A = \{a_{ij}\}$ is the projection operator matrix. The element a_{ij}, of the matrix A represents the weight (or contribution) of pixel j to the ith sinogram value.

It is assumed that system (3) is consistent and underdetermined (m < n).

So it has infinitely many solutions. We seek for a solution such that it recovers the original image as good as possible. It is an ill-posed problem. In general, the dimension of f is very large, thus the conventional direct methods are not appropriate.

3 Materials and methods

In this section, we first briefly introduce the problem formulation, the reconstruction methods such as the Landweber method, the Cimmino's method, Total Variation Regularization and then present the proposed TVcim-p method.

3.1 Reconstruction methods

Several algorithms have been developed for calculating the image from a set of projection data. These include back projection, iterative methods and analytic methods. We focus our study in this work on some of the Simultaneous Iterative Reconstruction Techniques (SIRT). These methods make it possible to correct the error induced by one or a set of projections as a function of the image being reconstructed and the data present on the other projections. [8] The general form SIRT methods can express as follows:

$$f^{k+1} = f^k + \lambda_k T A^T M\left(P - Af^k\right), k = 0,1,2,\dots \quad (4)$$

where A is the matrix with the various projections, λ_k is a relaxation parameter and the matrices M and T are symmetric positive definite.

Landweber method

The Landweber method is described by the following form: [8]

$$f^{k+1} = f^k + \lambda_k A^T\left(P - Af^k\right), k = 0,1,2,\dots \quad (5)$$

which corresponds to replacing M = T = 1 in (4).

Cimmino's method

This method it's often presented in a variant based on projections. Using the matrix notation this method uses the equation (4) with M = D and T = I, where D is defined as: [8]

$$D = \frac{1}{m} diag\left(\frac{1}{\|A(i,:)\|_2^2}\right) \quad (6)$$

$$f^{k+1} = f^k + \lambda_k A^T D\left(P - Af^k\right), k = 0,1,2,\dots \quad (7)$$

Total Variation Regularization (TV)

The discrete form of TV can expressed as [9]:

$$J_{TV}(p) = \sum_{i,j}\left\|(\nabla p)_{i,j}\right\|, \quad (8)$$

∇p indicates the gradient of μ. It's expressed as:

$$(\nabla p)_{i,j} = \begin{pmatrix} p_{i+1,j} - p_{i,j} \\ p_{i,j+1} - p_{i,j} \end{pmatrix}. \quad (9)$$

So, TV norm is written as follows:

$$J_{TV}^{reg}(p) = \sum_{i,j}\sqrt{\left\|(\nabla p)_{i,j}\right\|^2 + \varepsilon^2}, \quad (10)$$

where ε is a positive smoothing factor.

3.2 Proposed method

The proposed algorithm is based on the simultaneous iterative reconstruction techniques.

We propose an improvement of cimmino algorithm by imposing positivity constraint in the total variation (TVcim-p).

Algorithm (TVcim-p): $\tilde{f}_0 \in \mathbb{R}^n, \tilde{f} > 0$

set parameters $D = \frac{1}{m} diag\left(\frac{1}{\|A(i,:)\|_2^2}\right)$, $\tau = 0.005$

For each k = 2, 3 ....

$\lambda_k = \frac{D(P - A\tilde{f}^{k-1})}{\|A^T D(P - A\tilde{f}^{k-1})\|^2}$

$\tilde{f}^k = \tilde{f}^{k-1} + \lambda_k A^T D(P - A\tilde{f}^{k-1}) - \tau * GradJ(\tilde{f}^{k-1})$ (11)

$\tilde{f}^k = pos(\tilde{f}^k)$

End

Where *pos* is the value of positive part of the solution. The gradient of the regularized TV is expressed by:

$$GradJ_{TV}(f) = -divJ\left(\frac{\nabla f}{\sqrt{\varepsilon^2 + \|\nabla f\|^2}}\right) \quad (12)$$

3.3 Performance evaluations

For the quantitative evaluation of the TVcim-p algorithm, the mean-square error (MSE), peak signal-to-noise ratio (PSNR), and signal-to-noise ratio (SNR) are used as measures of the reconstruction quality. The MSE, PSNR and SNR are defined as follows:

$$MSE = \frac{1}{N}\sum_{i,j}\left(f_{ij}^* - f_{ij}\right)^2 \quad (13)$$

$$SNR = 10log_{10}\left(\frac{\sum_{ij}\left(f_{ij}^*\right)^2}{\sum_{ij}\left(f_{ij}^* - f_{ij}\right)^2}\right) \quad (14)$$

$$PSNR = 10log_{10}\left(\frac{Max^2(f_{ij})}{MSE}\right) \quad (15)$$

where f_{ij} represents the corrected image, and f_{ij}^* is the ideal image.

Fig. 1: Shepp-Logan phantom.

4 Results

To simulate classic situations of local tomography, we have used as image of test the Shepp-Logan phantom which is very used to perform performance tests in tomographic reconstruction algorithm. The size of phantom image tested in this paper is 256 × 256 as shown in Figure 1.

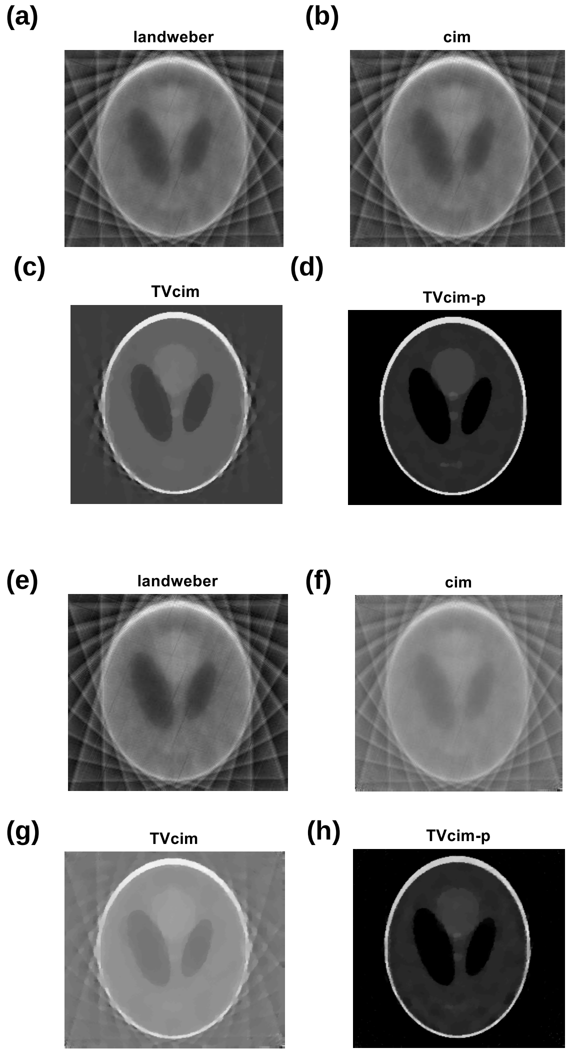


Fig. 2 : The images reconstructed by different reconstruction algorithms from the noise-free and noisy projection (Number of projection = 12, Iteration = 1000). ((a)-(d)) Reconstructed images from noise-free projection by : (a) Landweber, (b) Cimmino, (c) TVcim, and (d) proposed TVcim-p; ((e)-(h)) reconstructed images from noisy projection : (e) Landweber, (f) Cimmino, (g) TVcim, and (h) proposed TVcim-p.

The number of projections is a very important parameter for tomography. We know that with more projections, the reconstruction is better, but the scan time is longer. In this work we compare the different proposed methods to assess the overall performance in cases of reconstructing images containing few projections. The simulated projections were generated from Shepp-Logan phantom with parallel projections geometry. In the experiments below, the sinogram is generated by multiplying the matrix $A = \{a_{ij}\}$ with the phantom image. The images were reconstructed using 12, 18, 36, and 45 projection views selected. In order to compare the different algorithms used, we have added 0.15% Gaussian noise to noise-free projection data.

Figure 2 shows the images reconstructed by different reconstruction algorithms from the noise-free and noisy projection with number of projection equal to 12 and a number of iteration equal to 1000 for all algorithms. In Figure 2(b) and 2(h), it is clear that the proposed TVcim-p gives best reconstruction in cases of noise-free projection and noisy projection. Figure 3 shows the results for the line intensity profile 128th reconstructed by different algorithms of 12 projections. Figure 3a and 3c show the results in case of noise free projections and noisy projections respectively. As observed the prposed TVcim-p gives best reconstruction compared to landweber, cimmino and TVcim.

Figure 4 shows the images reconstruction of 36 projections and 1000 iteration for all reconstruction algorithms. Figure 5 shows the results for the line intensity profile 128th reconstructed by different algorithms of 36 noisy projections.

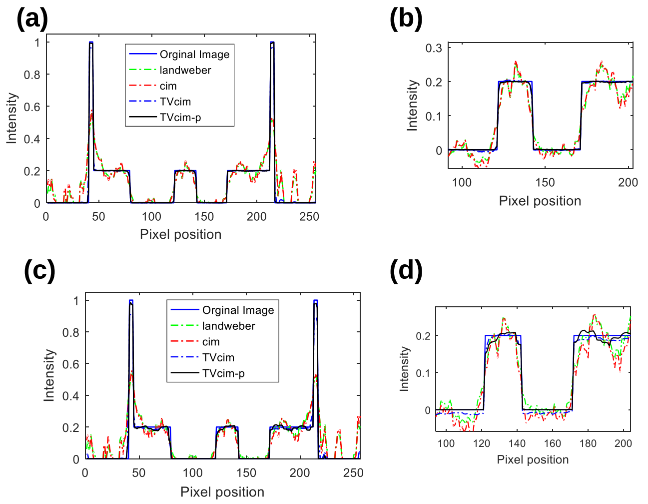


Fig. 3: (a) Line intensity profile (128th row) reconstructed by different algorithms of noise free projection (Number of projection = 12), (b) zoomed image of (a), (c) Line intensity profile (128th row) reconstructed by different algorithms of noisy projection, (d) zoomed image of (c).

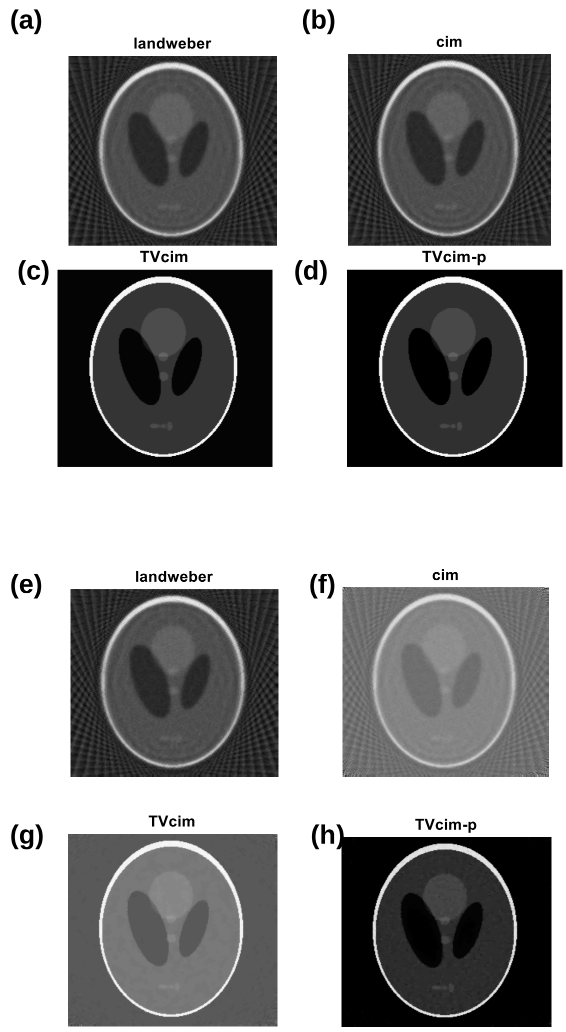


Figure 4: The images reconstructed by different reconstruction algorithms from the noise-free and noisy projection (Number of projection = 36, Iteration = 1000). ((a)-(d)) Reconstructed images from noise-free projection by : (a) Landweber, (b) Cimmino, (c) TVcim, and (d) proposed TVcim-p; ((e)-(h)) reconstructed images from noisy projection : (e) Landweber, (f) Cimmino, (g) TVcim, and (h) proposed TVcim-p.

The quantitative and the qualitative evaluations of experiment data of the reconstruction algorithms are summarized in Table 1 and Table 2. We set maximum of 1000 iterations in the test.

Table 1. Evaluations results of reconstruction by different algorithms.

	Free noise projection			Noisy projection		
	Time (s)	**Error**	**MSE**	**Time (s)**	**Error**	**MSE**
Landweber	43	0.51	0.016	45	0.51	0.016
Cim	19	0.49	0.014	31	0.51	0.016
TVcim	26	0.11	0.0008	39	0.17	0.001
TVcim-p	26	0.069	0.0002	39	0.098	0.0005

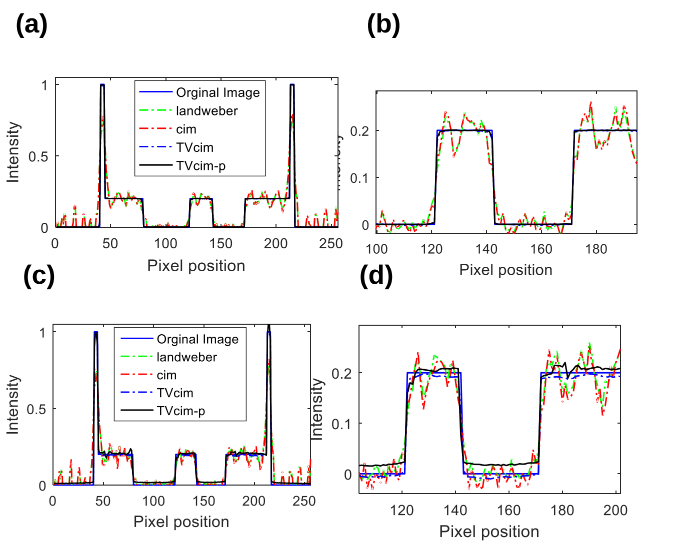
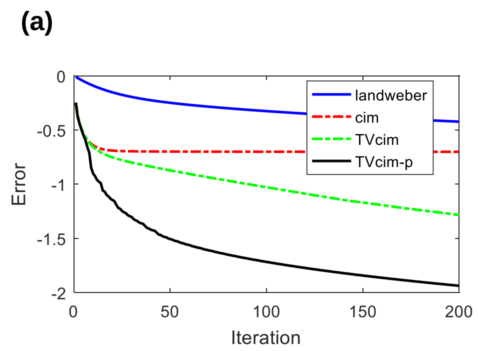


Fig. 5: (a) Line intensity profile (128th row) reconstructed by different algorithms of noise free projection, (b) zoomed image of (a), (c) Line intensity profile (128th row) reconstructed by different algorithms of noisy projection, (d) zoomed image of (c).

Table 2: Quantitative evaluation of the experimental results for the Shepp-Logan phantom with different algorithms.

		Free noise projection			
Number of projection		12	18	36	45
Landweber	PSNR	16.86	17.88	20.68	22.02
	SNR	4.69	5.71	8.17	9.17
Cim	PSNR	17.16	19.12	22.48	23.46
	SNR	4.99	6.09	8.76	9.77
TVcim	PSNR	21.81	31.41	37.76	38.76
	SNR	9.35	18.68	25.26	25.86
TVcim-p	PSNR	30.19	36.29	40.74	41.47
	SNR	16.66	23.13	28.21	54.47
		noisy projection			
Number of projection		12	18	36	45
Landweber	PSNR	16.85	17.88	17.15	17.67
	SNR	4.68	5.7	4.98	5.5
Cim	PSNR	18.84	24.7	22.24	24.54
	SNR	4.87	5.69	8.52	9.09
TVcim	PSNR	21.59	31.63	28.36	30.19
	SNR	8.66	15.3	16.09	14.96
TVcim-p	PSNR	29.7	33.68	33.91	33.53
	SNR	15.37	20.09	21.32	19.09



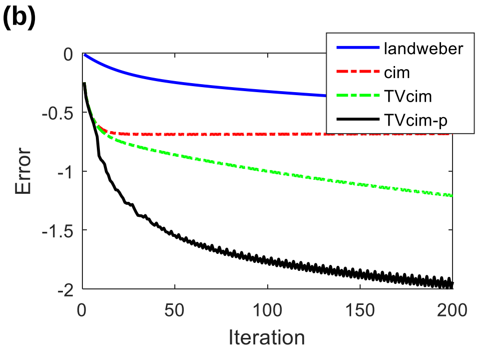


Fig. 6: Reconstruction errors as a function of iterations (iteration numbers range from 1 to 200) for different reconstruction algorithms at 12 projection angles.

5 Conclusion

In conclusion, in this paper, we have implemented a CT image reconstruction from few projections using iterative methods. The main contribution of this work is to improve the reconstruction of image using cimmino algorithm by imposing positivity constraint in the total variation (TVcim-p). We can note that the combination of the total variation regularization with the cimmino method gives more enhancement of reconstruction. Furthermore, the combination of total variation regularization and positive constraint improved significantly the reconstruction images even in the case of the limited projections with noise. After tests on numerical Shepp-Logan phantom with size 256 × 256 and different number of projections from 12 to 45 projections, the proposed method shows better performance than several commonly used methods with respect to both the quantitative and the qualitative evaluations.

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