The Analysis of Wireless Relay Communication System in the Presence of Nakagami-m Fading

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Abstract: - The wireless relay communication system with three sections operating over Nakagami-m multipath fading channel is determined in this work. The outage probability of proposed relay system is calculated for two cases. In the first case, the outage probability is defined as probability that signal envelope falls below the specified threshold at any section. In the second case, the outage probability is defined as probability that output signal envelope is lower than predetermined threshold. For the first case, the outage probability is calculated by using cumulative distribution function of minimum of three Nakagami-m random variables. For the second case, the outage probability is evaluated by using the cumulative distribution function of product of three Nakagami-m random variables. Numerical expressions for the outage probability of wireless relay communication system are presented graphically and the influence of Nakagami-m parameter from each section on the outage probability is estimated. Then, the probability density functions and cumulative distribution functions of the ratios of product of two Nakagami-m random variables and Nakagami-m random variable and the ratio of Nakagami-m random variable and product of two Nakagami-m random variables are derived.

Key-Words: - Nakagami-m short term fading; relay communication system; outage probability

1 Introduction
The outage probability, as one of the most important wireless communication systems performance, is determined in [1] for wireless relay communication system with three sections in the presence of Nakagami-m short term fading. In this paper, an extended analysis of Nakagami-m multipath fading influence on desired signal and interference is done.

Refractions, reflections, diffractions and scatterings cause multipath propagation resulting in degradation of the outage performance of wireless relay communication radio system [2], [3]. Certain numbers of channel models exist to describe the statistics of the amplitude and phase of multipath fading signals. Nakagami-m distribution has some advantages versus the other models, such as that this is a generalized distribution which can model different fading environments. It has greater flexibility and accuracy in matching some experimental data than the Rayleigh, lognormal or Rice distributions, and also, Rayleigh and one-sided Gaussian distribution are special cases of Nakagami-m model. So the Nakagami-m channel model is of more general applicability in practical fading channels [4] - [9].

Nakagami-m fading modelling in the frequency domain is investigated in [7]. For frequency-selective Nakagami-m fading channels, the magnitudes of the channel frequency responses to be Nakagami-m distributed random variables with fading and mean power parameters as explicit functions of the fading and mean power parameters of the channel impulse responses are shown. Based on this model, the bit error rate (BER) performance of an orthogonal frequency-division multiplexing system with receive diversity over correlated Nakagami-m fading channels is analytically evaluated.

BER of band-limited binary phase-shift keying in a fading and cochannel interference (CCI)
environment is derived for the case of perfect coherent detection in [8]. The assumed fading-and-interference model is general and of interest for microcellular systems. The model allows both desired signal and interfering signals to experience arbitrary amounts of fading severity.

In first part of this paper, the wireless relay system with three sections is considered. In sections, Nakagami-m channel is present. This channel can be denoted as Nakagami- Nakagami- Nakagami channel. It has three parameters which are denoted with \( m_1, m_2 \) and \( m_3 \). Also, Nakagami- Nakagami- Nakagami relay channel is general channel and several channels can be derived from this channel. For \( m_1=1 \), Nakagami- Nakagami- Nakagami channel becomes Rayleigh- Nakagami- Nakagami channel; for \( m_1=1 \) and \( m_2=1 \), Nakagami- Nakagami- Nakagami channel becomes Rayleigh- Rayleigh - Nakagami channel, and for \( m_1=1, m_2=1 \) and \( m_3=1 \), Nakagami- Nakagami- Nakagami channel becomes Rayleigh- Rayleigh – Rayleigh channel. For \( m_1=0.5 \), Nakagami- Nakagami- Nakagami channel becomes One sided Gaussian- Nakagami- Nakagami channel; for \( m_1=0.5 \) and \( m_2=0.5 \), Nakagami- Nakagami- Nakagami channel becomes One sided Gaussian-One sided Gaussian- Nakagami channel, and for \( m_1=1/2, m_2=1/2 \) and \( m_3=1/2 \), Nakagami- Nakagami- Nakagami channel becomes One sided Gaussian-One sided Gaussian- One sided Gaussian relay channel. Also, for \( m_1 \) goes to infinity, Nakagami- Nakagami- Nakagami relay channel becomes no fading- Nakagami- Nakagami channel, for \( m_1 \) goes to infinity and \( m_2 \) goes to infinity, Nakagami- Nakagami- Nakagami relay channel becomes no fading- no fading - Nakagami relay channel, and for \( m_1 \) goes to infinity, \( m_2 \) goes to infinity and \( m_3 \) goes to infinity, Nakagami- Nakagami- Nakagami relay channel becomes no fading- no fading - no fading channel.

The wireless relay system with two sections in the presence of \( \kappa-\mu \) and \( \eta-\mu \) multipath fading is processed in [10]. The wireless relay communication mobile radio system with two sections, subjected to \( \kappa-\mu \) short term fading is considered in [11]. The outage probability is derived and parameters influence is analyzed.

In this work, the outage probability of wireless communication relay radio mobile system with three sections operating over Nakagami multipath fading channel is considered. For relay system, the outage probability can be defined at two manners. In the first case, the outage probability is defined as probability that signal envelope at any section falls below the specified threshold. For this definition, the outage probability can be calculated by using cumulative distribution function of minimum of three Nakagami random variables.

At the second manner, the outage probability is defined as probability that signal envelope at output of relay mobile radio system is lower than specified threshold. Signal envelope at the output of relay system with three sections can be written as product of three Nakagami random variables. Therefore, the outage probability by the second definition can be evaluated by using cumulative distribution function of product of three Nakagami random variables.

In this paper, probability density function and cumulative distribution function of minimum of three Nakagami random variables and product of three Nakagami random variables are calculated. The joint probability density function of minimum of three Nakagami random variables can be calculated and used for calculation the level crossing rate of minimum of three Nakagami random processes. After that, the ratio of product of two Nakagami-m random variables and Nakagami-m random variable is derived and PDF and CDF of this ratio. Also, the ratio of Nakagami-m random variable and product of two Nakagami-m random variables is determined. The CDF are graphically presented and parameters influence is observed.

2 Statistics of Minimum of Three Nakagami Random Variables

Random variables \( x_1, x_2 \) and \( x_3 \) follow Nakagami-m distribution:

\[
p_{x_1}(x_1) = \frac{2}{\Gamma(m_1)} \left( \frac{m_1}{\Omega_1} \right)^{m_1} x_1^{2m_1-1} e^{-\frac{m_1}{\Omega_1} x_1^2}, \quad x_1 \geq 0,
\]

\[
p_{x_2}(x_2) = \frac{2}{\Gamma(m_2)} \left( \frac{m_2}{\Omega_2} \right)^{m_2} x_2^{2m_2-1} e^{-\frac{m_2}{\Omega_2} x_2^2}, \quad x_2 \geq 0,
\]

\[
p_{x_3}(x_3) = \frac{2}{\Gamma(m_3)} \left( \frac{m_3}{\Omega_3} \right)^{m_3} x_3^{2m_3-1} e^{-\frac{m_3}{\Omega_3} x_3^2}, \quad x_3 \geq 0,
\]

where \( \Gamma(\cdot) \) is the (complete) gamma function [12].

Cumulative distribution functions (CDF) of \( x_2, x_3 \) and \( x_3 \) are:

\[
F_{x_1}(x_1) = \frac{1}{\Gamma(m_1)} \gamma \left( m_1, \frac{m_1}{\Omega_1} x_1^2 \right), \quad x_1 \geq 0,
\]

\[
F_{x_2}(x_2) = \frac{1}{\Gamma(m_2)} \gamma \left( m_2, \frac{m_2}{\Omega_2} x_2^2 \right), \quad x_2 \geq 0,
\]
\[ F_{x_1}(x_3) = \frac{1}{\Gamma(m_1)} \gamma \left( m_1 \frac{m_3}{\Omega_3} x_3^2 \right), \quad x_3 \geq 0, \quad (6) \]

Minimum of \( x_1, x_2 \) and \( x_3 \) is:
\[ x = \min(x_1, x_2, x_3) \quad (7) \]

Probability density function (PDF) of \( x \) is:
\[ p_x(x) = p_{x_1}(x) F_{x_1}(x) F_{x_2}(x) + p_{x_2}(x) F_{x_2}(x) F_{x_1}(x) = \]
\[ = \frac{2}{\Gamma(m_1)} \gamma \left( m_1 \frac{m_3}{\Omega_3} x_1^2 \right) e^{\frac{m_x}{\Omega_1} x^2} \cdot \frac{1}{\Gamma(m_2)} \gamma \left( m_2 \frac{m_x}{\Omega_2} x^2 \right), \quad x \geq 0 \quad (8) \]

Cumulative distribution function of minimum of three Nakagami-\( m \) random variables is:
\[ F_x(x) = \int_0^x dt \; p_x(t) = \]
\[ F_x(x) = \left( 1 - \left( 1 - F_{x_1}(x) \right) \left( 1 - F_{x_2}(x) \right) \left( 1 - F_{x_3}(x) \right) \right) = \]
\[ = 1 - \left( 1 - \frac{1}{\Gamma(m_1)} \gamma \left( m_1 \frac{m_3}{\Omega_3} x^2 \right) \right) \left( 1 - \frac{1}{\Gamma(m_2)} \gamma \left( m_2 \frac{m_3}{\Omega_2} x^2 \right) \right) \]
\[ \left( 1 - \frac{1}{\Gamma(m_3)} \gamma \left( m_3 \frac{m_1}{\Omega_1} x^2 \right) \right), \quad x \geq 0 \quad (9) \]

In previous expressions, parameter \( m_1 \) is severity parameter of Nakagami-\( m \) fading in the first section, \( m_2 \) is severity parameter of Nakagami-\( m \) fading in the second section and \( m_3 \) is the severity parameter of Nakagami-\( m \) fading in the third section. The \( \Omega_1 \) is signal envelope average power in the first section, \( \Omega_2 \) is signal envelope average power in the second section and \( \Omega_3 \) is signal envelope average power in the third section.
Probability density functions of $x$ are shown in Figs. 1 and 3 versus of minimum of three Nakagami-m random variables. Severity parameters of Nakagami-m fading are $m_1 = m_2 = m_3 = 2$ in Fig. 1 and $m_1 = m_2 = m_3 = 3$ in Fig. 2. Signal envelope average powers are $\Omega_1 = \Omega_2 = \Omega_3 = 1$ in both figures.

In Figs. 2 and 4, the outage probability in terms of minimum of three Nakagami-m random variables are shown for several values of severity Nakagami parameters and several values of signal envelopes average powers in sections. The outage probability decreases when severity Nakagami parameter $m_1$ in the first section increases, severity Nakagami parameter $m_2$ in the second section increases, and severity Nakagami parameter $m_3$ in the third section increases. The influence of severity Nakagami parameter in the first section on the outage probability is the highest for higher values of severity Nakagami parameters in the second section and in the third section. The outage performance is better as signal envelope average power in the first section increases, when, also, signal envelope average power in the second section increases and when signal envelope average power in the third section increases. Signal envelope average power in the first section has higher influence on the outage probability when severity Nakagami parameters in the first, in the second and in the third section have higher values.

3 Statistics of Product of Three Nakagami Random Variables

The statistics of three random variables are very important for performance analyzing of wireless mobile communication radio systems. Sum of three random variables can be used for calculation the outage probability of wireless system with equal gain combining (EGC) receiver with three inputs operating over short term fading channel. EGC receiver is used for reduction multipath fading effects on system performance [3]. The statistics of the ratio of more random variables can be used in performance analysis of wireless communication systems operating over short term fading channel in the presence of cochannel interference [13]. In cellular radio interference limited environment, the ratio of signal envelope and interference envelope is important system performance. The statistics of product of two or more random variables have application in performance analysis of wireless relay communication systems with more sections [14]. Under determined conditions, signal envelope at output of relay system can be expressed as product of signal envelope at each section [14].

Product of three Nakagami-m random variables is:

$$x = x_1 \cdot x_2 \cdot x_3$$

(10)

Conditional probability density function of $x$ is:

$$p_x(x / x_2 x_3) = \frac{dx_1}{dx} p_{x_1}(x / x_2 x_3) =$$

$$= \frac{2}{\Gamma(m_1) \Gamma(m_2) \Gamma(m_3)} \left( \frac{m_1}{\Omega_1} \right)^m_1 \left( \frac{m_2}{\Omega_2} \right)^m_2 \left( \frac{m_3}{\Omega_3} \right)^m_3 x^{2m_1-1} \int_0^\infty dx_2 x_2^{-1-2m_1+1+2m_2-1} e^{-\frac{m_2 x_2^2}{\Omega_2}} \left( \frac{m_3 x_2^2}{\Omega_3} \right)^{m_3}$$

(11)

where:

$$\frac{dx_1}{dx} = \frac{1}{x_2 x_3}.$$  

(12)

After substituting and averaging, probability density function of $x$ becomes:

$$p_x(x) = \frac{8}{\Gamma(m_1) \Gamma(m_2) \Gamma(m_3)} \left( \frac{m_1}{\Omega_1} \right)^m_1 \left( \frac{m_2}{\Omega_2} \right)^m_2 \left( \frac{m_3}{\Omega_3} \right)^m_3 x^{2m_1-1} \int_0^\infty dx_2 x_2^{-m_1-2m_2+1+2m_3-1} e^{-\frac{m_2 x_2^2}{\Omega_2}} \left( \frac{m_3 x_2^2}{\Omega_3} \right)^{m_3}$$

(13)
where \( K_n(x) \) denotes the modified Bessel function of the second kind [15, eq. (3.471.9)]. Cumulative distribution function of \( x \) is:

\[
F_x(x) = \int_0^x dt \ p_x(t) = \\
\int_0^x dt \ \frac{8}{\Gamma(m_1) \Gamma(m_2) \Gamma(m_3)} \left( \frac{m_1}{\Omega_1} \right)^{m_1} \left( \frac{m_2}{\Omega_2} \right)^{m_2} \left( \frac{m_3}{\Omega_3} \right)^{m_3} \cdot 2^{m_3 - 1} \int_0^\infty dx_2 \ x_2^{2m_2 - 2m_1 - 1} \ e^{- \frac{m_2 x_2^2}{\Omega_2}} \cdot \int_0^\infty dx_3 \ x_3^{2m_3 - 2m_1 - 1} \ e^{- \frac{m_3 x_3^2}{\Omega_3}}.
\]

By substituting the last expression in the expression (14), we obtain for cumulative distribution function of \( x \):

\[
F_x(x) = \frac{8}{\Gamma(m_1) \Gamma(m_2) \Gamma(m_3)} \left( \frac{m_1}{\Omega_1} \right)^{m_1} \left( \frac{m_2}{\Omega_2} \right)^{m_2} \left( \frac{m_3}{\Omega_3} \right)^{m_3} \cdot \\
\frac{1}{m_1} x^{2m_1 - 1} \sum_{j_i=0}^\infty \frac{1}{(m_1 + 1)(j_1)} \left( \frac{m_1 x^2}{\Omega_1} \right)^{j_1} \cdot \int_0^\infty dx_2 \ x_2^{2m_2 - 2m_1 - 1} \ e^{- \frac{m_2 x_2^2}{\Omega_2}} \cdot \\
\left( \frac{m_1 x^2 \Omega_1^{m_1 - j_1}}{\Omega_3} \right)^{m_3 - j_1} \cdot K_{2m_3 - 2m_1 - 2j_1} \left( \frac{2 m_3 x^2}{\Omega_1 \Omega_3 \Omega_3} \right).
\]

Probability density functions of \( x \) are shown in Figs. 5. and 7 versus of product of three Nakagami-m random variables. Severity parameters of Nakagami-m fading are \( m_1 = m_2 = m_3 = 2 \) in Fig. 5. and \( m_1 = m_2 = m_3 = 3 \) in Fig. 7. Signal envelope average powers are \( \Omega_1 = \Omega_2 = \Omega_3 = 1 \) in both figures.
In this composite fading-shadowing environment, the signal envelope can be modeled by product of two random variables [17]. Also, when two fading affect together at the combiner inputs, the equivalent envelope is equal to the product of two random variables.

The ratio of product of two Nakagami-m random variables and Nakagami-m random variable is:

\[ x = \frac{x_1 x_2}{x_3}, \quad x_1 = \frac{xx_3}{x_2}, \quad (17) \]

where random variables \( x_1, x_2 \) and \( x_3 \) follow Nakagami-m distribution:

\[ p_{x_1}(x_1) = \frac{2}{\Gamma(m_1)} \left( \frac{m_1}{\Omega_1} \right)^{m_1} x_1^{m_1-1} e^{-\frac{m_1 x_1^2}{\Omega_1}}, \quad x_1 \geq 0 \]
\[ p_{x_2}(x_2) = \frac{2}{\Gamma(m_2)} \left( \frac{m_2}{\Omega_2} \right)^{m_2} x_2^{m_2-1} e^{-\frac{m_2 x_2^2}{\Omega_2}}, \quad x_2 \geq 0 \]
\[ p_{x_3}(x_3) = \frac{2}{\Gamma(m_3)} \left( \frac{m_3}{\Omega_3} \right)^{m_3} x_3^{m_3-1} e^{-\frac{m_3 x_3^2}{\Omega_3}}, \quad x_3 \geq 0. \]

Conditional probability density function of \( x \) is:

\[ p_x(x/x_2, x_3) = \frac{dx_1}{dx} p_{x_1}(\frac{xx_3}{x_2}) \quad (19) \]

where

\[ \frac{dx_1}{dx} = \frac{x_1}{x_2}. \quad (20) \]

After substituting and averaging, the expression for probability density function \( p_x(x) \) is:

\[ p_x(x) = \int_{0}^{\infty} dx_2 \int_{0}^{\infty} dx_3 p_{x_1}(\frac{xx_3}{x_2}) p_{x_2}(x_2) p_{x_3}(x_3) = \]

\[ = \frac{8}{\Gamma(m_1) \Gamma(m_2) \Gamma(m_3)} \left( \frac{m_1}{\Omega_1} \right)^{m_1} \left( \frac{m_2}{\Omega_2} \right)^{m_2} \left( \frac{m_3}{\Omega_3} \right)^{m_3} \]

\[ \int_{0}^{\infty} dx_2 x_2^{m_2-1} \int_{0}^{\infty} dx_3 x_3^{m_3-1} e^{-\frac{m_1 x_1^2}{\Omega_1} - \frac{m_2 x_2^2}{\Omega_2} - \frac{m_3 x_3^2}{\Omega_3}}. \]

The two-fold integral is:

\[ J = \int_{0}^{\infty} dx_2 x_2^{m_2-1} e^{-\frac{m_2 x_2^2}{\Omega_2}}, \quad \int_{0}^{\infty} dx_3 x_3^{m_3-1} e^{-\frac{m_3 x_3^2}{\Omega_3}} \]

\[ = \frac{\Gamma(m_2) \Gamma(m_3)}{\Omega_2 \Omega_3}. \]

4 The Ratio of Product of Two Nakagami-m Random Variables and Nakagami-m Random Variable

The distribution of the ratio of random variables is important in statistical analysis in a number of different fields of science, and also in analysis of wireless communication systems in the presence of fading [16]. The random variable in nominator represents the desired signal envelope while the random variable in denominator presents the interference signal envelope. Because desired signal in wireless communication systems can be subjected to cochannel interference (CCI) beside of fading and shadowing.
\[
\frac{1}{2} \left( \frac{m_1 \Omega_3 x^2}{m_1 \Omega_3} \right)^m \cdot \Gamma(m_1 + m_3) \cdot \frac{1}{2} \frac{m_1 \Omega_3 x^2}{m_1 \Omega_3} \frac{1}{2} \left( \frac{m_1 \Omega_3}{m_1 \Omega_3} \right)^{m_1+m_3-1}
\]
\[
\int_0^\infty dy \, y^{m_2+m_3-1} e^{\frac{m_1 \Omega_3 x^2}{m_1 \Omega_3} y} \cdot \frac{1}{(1+x)^{m_1+m_3}} = \frac{1}{2} \left( \frac{m_1 \Omega_3}{m_1 \Omega_3} \right)^{m_1+m_3} \cdot \Gamma(m_1 + m_3)
\]
\[
= \frac{1}{2} \frac{x^{-2m_2+2m_3}}{2} \left( \frac{m_1 \Omega_3}{m_1 \Omega_3} \right)^{m_2+m_3} \cdot \frac{1}{2} \left( \frac{m_1 \Omega_3}{m_1 \Omega_3} \right)^{m_2+m_3} \cdot \Gamma(m_2 + m_3)
\]
\[
U \left( m_2 + m_3, m_2 - m_1 + 1, \frac{m_1 \Omega_3 x^2}{m_1 \Omega_3} \frac{1}{2} \right)
\]

We make replacement:
\[
m_1 \Omega_3 x^2 = y, \quad x^2 = \frac{m_1 \Omega_3 x^2}{m_1 \Omega_3} y,
\]
\[
x_2 dx_2 = \frac{m_1 \Omega_3 x^2}{2 m_1 \Omega_3} dy
\]

and then, from (22) and (23), it is:
\[
J = \frac{1}{2} \left( \frac{m_1 \Omega_3}{m_1 \Omega_3} \right)^{m_1+m_3} \cdot \Gamma(m_1 + m_3)
\]

\[
\int_0^\infty dx_2 x_2^{-2m_1+2m_2+2m_3-1} e^{\frac{m_1 \Omega_3 x^2}{m_1 \Omega_3}} \cdot \frac{m_1 \Omega_3^2}{m_1 \Omega_3} dx_3 x_3^{-2m_2+2m_1-1} e^{\frac{m_2 \Omega_3^2}{m_2 \Omega_3}} = \frac{8}{\Gamma(m_1) \Gamma(m_2) \Gamma(m_3)} \left( \frac{m_1}{m_1 \Omega_3} \right)^{m_1} \left( \frac{m_2}{m_2 \Omega_3} \right)^{m_2} \left( \frac{m_1}{m_2 \Omega_3} \right)^{m_3} \cdot \frac{m_1 \Omega_3^2}{m_1 \Omega_3} \frac{m_2 \Omega_3^2}{m_2 \Omega_3} \frac{m_1 \Omega_3^2}{m_1 \Omega_3}
\]

where \( U(a, b, z) \) is Tricomi’s (confluent hypergeometric) function [18].

Cumulative distribution function of \( x \) is:
\[
F_x(x) = \int_0^x dt \, p_x(t) = \frac{4}{\Gamma(m_1) \Gamma(m_2) \Gamma(m_3)} \left( \frac{m_1}{m_1 \Omega_3} \right)^{m_1} \left( \frac{m_2}{m_2 \Omega_3} \right)^{m_2} \left( \frac{m_1}{m_2 \Omega_3} \right)^{m_3} \cdot \frac{m_1 \Omega_3^2}{m_1 \Omega_3} \frac{m_2 \Omega_3^2}{m_2 \Omega_3} \frac{m_1 \Omega_3^2}{m_1 \Omega_3}
\]

The integral \( J \) is [19]:
\[
J_1 = \int_0^\infty dx_2 x_2^{-2m_1+2m_2+2m_3-1} e^{\frac{m_1 \Omega_3 x^2}{m_1 \Omega_3}} \cdot \int_0^\infty dx_3 x_3^{-2m_2+2m_1-1} e^{\frac{m_2 \Omega_3^2}{m_2 \Omega_3}}
\]
The new substitution is:
\[
\frac{m_1^3}{\Xi^2} = t, \quad x_2^2 = \frac{m_1\Xi^2}{m_3\Omega_1} t,
\]
\[
x_2 \, dx_2 = \frac{m_1\Xi^2}{2m_3\Omega_1} \, dt
\]  \hfill (27)

After substituting, the integral \( J_1 \) becomes:
\[
J_1 = \frac{1}{2} \left( \frac{m_1^3}{m_3\Omega_1} \right)^{m_1 + m_2 + m_3} \frac{1}{(m_1\Xi^2)^{m_1 + m_2 + m_3}} \int_0^\infty dt \frac{t^{-2m_2 - m_1 - 1}}{\Omega_2^2 m_3\Omega_1} e^{-\frac{m_1\Xi^2 + m_2\Omega_1}{\Omega_2}} \frac{1}{(1 + t)^{m_1 + m_2 + m_3 + j}}.
\]  \hfill (28)

It can be solved by using the formula:
\[
\int_0^\infty \frac{dz}{z^n} e^{-\alpha z} = \Gamma(n, \alpha z)^{-1} = \Gamma(n, \alpha)^{-1} \quad \text{for} \quad n, \alpha > 0.
\]  \hfill (29)

The previous integral becomes:
\[
J_1 = \frac{1}{2} \left( \frac{m_1^3}{m_3\Omega_1} \right)^{m_1 + m_2 + m_3} \frac{1}{(m_1\Xi^2)^{m_1 + m_2 + m_3}} \cdot \Gamma(-2m_2 - m_1 + m_3)
\]
\[
\cdot \left( \frac{m_1^3}{m_3\Omega_1} \right)^{-2m_1 + m_2 + m_3} \cdot \left( \frac{m_1^3}{m_3\Omega_1} \right)^{-2m_2 + m_1 + m_3 + 1 + m_1 - m_3 - j} \cdot \left( \frac{m_1^3}{m_3\Omega_1} \right)^{-m_1 \Omega_1} \Omega_j - m_1 \Omega_j
\]  \hfill (30)

After putting obtained solution in (25), the final expression for cumulative distribution function is obtained. Then, the outage probability can be calculated by using this cumulative distribution function.

5 The Ratio of Nakagami-m Random Variable and Product of Two Nakagami-m Random Variables

The ratio of Nakagami-m random variable and product of two Nakagami-m random variables is:
\[
x = \frac{x_1}{x_2 x_3}, \quad x_1 = x x_2 x_3,
\]  \hfill (31)

where random variables \( x_1 \), \( x_2 \) and \( x_3 \) have Nakagami-m distribution given by (18).

Conditional probability density function of \( x \) is:
\[
p_x(x / x_2, x_3) = \frac{dx_1}{dx} p_{x_2}(xx_2, x_3)
\]  \hfill (32)

where
\[
\frac{dx_1}{dx} = x_2 x_3
\]  \hfill (33)

After substituting and averaging, the expression for probability density function \( p_x(x) \) becomes:
\[
p_x(x) = \int_0^\infty \int_0^\infty \int_0^\infty p_{x_2}(xx_2, x_3) p_{x_1}(x_1) p_{x_2}(x_2) p_{x_3}(x_3) dx_2 dx_3 dx_1
\]
\[
= \frac{1}{\Gamma(m_1) \Gamma(m_2) \Gamma(m_3)} \cdot \left( \frac{m_1}{\Omega_1} \right)^{m_1} \left( \frac{m_2}{\Omega_2} \right)^{m_2} \left( \frac{m_3}{\Omega_3} \right)^{m_3} \cdot \frac{1}{x^{2m_2 + 2m_1 + m_3 - 1}} \cdot \frac{1}{\Omega_1 \Omega_2 \Omega_3}
\]
\[
x^{2m_2 - 1} \int_0^\infty dx_2 x_2^{2m_1 + 2m_2 + m_3 - 1} e^{-\frac{m_1 x_2}{\Omega_1}} \cdot \int_0^\infty dx_3 x_3^{2m_2 + 2m_1 + m_3 - 1} e^{-\frac{m_2 x_3}{\Omega_2}} \cdot \int_0^\infty dx_1 x_1^{2m_1 - 1} e^{-\frac{m_3 x_1}{\Omega_3}}
\]
\[
= \frac{1}{\Gamma(m_1) \Gamma(m_2) \Gamma(m_3)} \cdot \left( \frac{m_1}{\Omega_1} \right)^{m_1} \left( \frac{m_2}{\Omega_2} \right)^{m_2} \left( \frac{m_3}{\Omega_3} \right)^{m_3} \frac{1}{x^{2m_2 + 2m_1 + m_3 - 1}} \int_0^\infty dx_2 x_2^{2m_1 + 2m_2 + m_3 - 1} e^{-\frac{m_1 x_2}{\Omega_1}} \cdot \int_0^\infty dx_3 x_3^{2m_2 + 2m_1 + m_3 - 1} e^{-\frac{m_2 x_3}{\Omega_2}} \cdot \int_0^\infty dx_1 x_1^{2m_1 - 1} e^{-\frac{m_3 x_1}{\Omega_3}}
\]  \hfill (34)

The substitution is introduced:
\[
\frac{m_1 \Xi^2}{m_3 \Omega_1} = t, \quad x_2^2 = \frac{m_1 \Xi^2}{m_3 \Omega_1} t,
\]
\[
x_2 \, dx_2 = \frac{m_3 \Omega_1}{2 m_1 \Omega_3 x^2} \, dt
\]  \hfill (35)

If we put (35) into (34), \( p_x(x) \) becomes:
\[
p_x(x) = \frac{1}{\Gamma(m_1) \Gamma(m_2) \Gamma(m_3)} \left( \frac{m_1}{\Omega_1} \right)^{m_1} \left( \frac{m_2}{\Omega_2} \right)^{m_2} \left( \frac{m_3}{\Omega_3} \right)^{m_3} \cdot \frac{1}{x^{2m_2 + 2m_1 + m_3 - 1}} \cdot \frac{1}{\Xi^2} \cdot \left( \frac{m_1 \Xi^2}{m_3 \Omega_1} \right)^{m_1 + m_3} \cdot \frac{1}{(m_3 \Omega_1)^{m_3 + m_3}}.
\]  \hfill (36)
By using the formula [19]:

\[
\int_0^\infty dz \, z^{-1} e^{-\alpha z} \frac{1}{(1+z)^n} = \Gamma(a, a+1-n, \alpha)
\]

the expression for \( p_x(x) \) from (36) is:

\[
p_x(x) = \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \left( \frac{m_1}{\Omega_1} \right)^{m_1} \left( \frac{m_2}{\Omega_2} \right)^{m_2} \left( \frac{m_3}{\Omega_3} \right)^{m_3} \cdot \frac{1}{2} \frac{m_3 \Omega_3}{m_1 \Omega_1 x^2} \int_0^\infty \Gamma(m_1+m_2)
\]

\[
= \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \left( \frac{m_1}{\Omega_1} \right)^{m_1} \left( \frac{m_2}{\Omega_2} \right)^{m_2} \left( \frac{m_3}{\Omega_3} \right)^{m_3} \cdot \frac{1}{2} \frac{m_3 \Omega_3}{m_1 \Omega_1 x^2} \int_0^\infty \Gamma(m_1+m_2)
\]

\[
U(m_1+m_2, m_1+m_3+1, m_1, m_2, m_3, \Omega_1, \Omega_2, \Omega_3, x)
\]

Cumulative distribution function of \( x \) is:

\[
F_x(x) = \int_0^x dt \, p_x(t) = \int_0^x dt \, \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \left( \frac{m_1}{\Omega_1} \right)^{m_1} \left( \frac{m_2}{\Omega_2} \right)^{m_2} \left( \frac{m_3}{\Omega_3} \right)^{m_3} \cdot \frac{1}{2} \frac{m_3 \Omega_3}{m_1 \Omega_1 x^2} \int_0^\infty \Gamma(m_1+m_2)
\]

\[
= \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \left( \frac{m_1}{\Omega_1} \right)^{m_1} \left( \frac{m_2}{\Omega_2} \right)^{m_2} \left( \frac{m_3}{\Omega_3} \right)^{m_3} \cdot \frac{1}{2} \frac{m_3 \Omega_3}{m_1 \Omega_1 x^2} \int_0^\infty \Gamma(m_1+m_2)
\]

\[
= \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \left( \frac{m_1}{\Omega_1} \right)^{m_1} \left( \frac{m_2}{\Omega_2} \right)^{m_2} \left( \frac{m_3}{\Omega_3} \right)^{m_3} \cdot \frac{1}{2} \frac{m_3 \Omega_3}{m_1 \Omega_1 x^2} \int_0^\infty \Gamma(m_1+m_2)
\]

\[
= \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \left( \frac{m_1}{\Omega_1} \right)^{m_1} \left( \frac{m_2}{\Omega_2} \right)^{m_2} \left( \frac{m_3}{\Omega_3} \right)^{m_3} \cdot \frac{1}{2} \frac{m_3 \Omega_3}{m_1 \Omega_1 x^2} \int_0^\infty \Gamma(m_1+m_2)
\]

\[
= \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \left( \frac{m_1}{\Omega_1} \right)^{m_1} \left( \frac{m_2}{\Omega_2} \right)^{m_2} \left( \frac{m_3}{\Omega_3} \right)^{m_3} \cdot \frac{1}{2} \frac{m_3 \Omega_3}{m_1 \Omega_1 x^2} \int_0^\infty \Gamma(m_1+m_2)
\]
In the next few figures, the outage probability ($P_{out}$) of the ratio of three Nakagami-$m$ random variables and Nakagami-$m$ random variable is presented. It is visible from Fig. 9 that the outage probability decreases with increasing of the signal power $\Omega_i$, $i=1,2,3$.

From Fig. 10, one can see that outage probability has smaller values for smaller values of the interference power $\Omega_4$. The system performance is better for smaller values of the outage probability.

In Fig. 11, different combinations of Nakagami-$m$ parameter and signal and interference envelope powers are presented.

From last Fig. 12, it is possible to see that $P_{out}$ increases with decreasing of Nakagami fading parameter signed by $\mu$ in these figures. This means that system performance starts to be worse. The results from this paper are helpful to wireless relay system designers to choose optimal system parameters.

6 Conclusion

In this paper, wireless mobile relay radio communication system with three sections operating over Nakagami small scale fading channel is considered. Nakagami- Nakagami- Nakagami relay channel is defined. For proposed relay system, the outage probability is determined. The outage probability can be defined in two manners. To the first case, the outage probability is
defined as probability that signal envelope at any section falls below the specified threshold. For this case, the outage probability can be calculated by using cumulative distribution function of minimum of three Nakagami random variables.

To the second manner, the outage probability is defined as probability that signal envelope at the output of wireless relay communication system with three sections is lower than the predetermined threshold. Signal envelope at the output of relay system with three sections can be written as product of signal envelopes at sections. Therefore, the outage probability for the second case can be derived from cumulative distribution function of product of three Nakagami random variables.

In this work, probability density functions and cumulative distribution functions of minimum of three Nakagami random variables and product of three Nakagami random variables are evaluated. Cumulative distribution function of minimum of three Nakagami random variables is derived in the closed form. Cumulative distribution function of product of three Nakagami random variables is obtained as expression with one integral. For the both cases, the outage probability decreases when severity parameters of Nakagami fading increase at any sections.

On the second part of this work, the distributions of the ratio of product of two Nakagami-m random variables and Nakagami-m random variable and of the ratio of Nakagami-m random variable and product of two Nakagami-m random variables are derived. These distributions of the ratios of random variables are important in statistical analysis of wireless communication systems in the presence of fading and cochannel interference (CCI). In such composite fading-shadowing environment, the signal envelope can be modelled by product of random variables. Also, when two or more fading affect together at the combiner inputs, the equivalent envelope is equal to the product of random variables.

The probability density functions and cumulative distribution functions are determined and the outage probability based on CDF. The numerical results are shown to point out the influence of different parameters.

These results are useful for designing of wireless mobile relay radio communication system with more sections in the presence of fading.

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References:


