ICI and PAPR Enhancement in MIMO-OFDM System
Using RNS Coding

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Abstract: - The Inter-Carrier-Interference (ICI) is considered a bottleneck in the utilization of Multiple-Input-Multiple-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) systems, due to the sensitivity of the OFDM towards frequency offsets which lead to loss of orthogonality, interference and system performance degradation. In this paper Residue Numbers as a coding scheme is impeded in MIMO-OFDM systems, where the ICI levels is measured and evaluated with respect to the conventional ICI mitigation techniques as pulse shaping, windowing and self-cancellation techniques implemented in MIMO-OFDM system. The Carrier-to-Interference Ratio (CIR), the system Bit-Error-Rate (BER) and the “Complementary Cumulative Distribution Function” (CCDF) for MIMO-OFDM system with Residue Number System (RNS) coding are measured and evaluated using MATLAB tool; were the results had shown the performance enhancement of the transmission model over the system without RNS implementation.

Key-Words: - BER, CIR, ICI, Mitigation techniques, MIMO-OFDM system, RNS.

1 Introduction
In MIMO systems, the signal at both sides of the communication link is combined through usage of various Space–Time Block Coding (STBC) algorithms to achieve either higher transmission data rates or enhanced system BER performance for the same data rate [1, 2]. At the same time OFDM is used as well in the communication system to take benefit from its characteristics as a multi-carrier modulation scheme to provide high transmission rates [3, 4].

For the MIMO-OFDM communication systems [5], the orthogonality seen in OFDM technique is lost within the sub-carriers due to the sensitivity of OFDM to frequency offset caused by Doppler shift between transmitter and receiver sides, resulting an ICI in transmitted symbols, that degrades the system performance [6].

Different ICI cancellation techniques are currently available like time-domain widowing, pulse shaping and frequency equalization, which reduce the ICI levels and thus improve the BER performance of MIMO-OFDM systems, still these techniques are costly and high complex either on the transmitter or receiver side.

The paper propose an efficient ICI cancellation technique based on the utilization of Residue coding scheme; were the system is analysed and compared to current mitigation techniques.

This paper starts in section 2 to provide some basic background, section 3 and 4 provide analysis of the ICI and a review for current ICI cancellation techniques respectively, section 5 describes the proposed MIMO-RNS-OFDM communication system, section 6 simulation results are provided to measure the system performance and finally in section 7 a conclusion had been provided.

2. Residue System Background
In this section a review for both Residue Number system and Redundant Residue Number system is provided.

2.1 Residue Number System Review
The RNS represent large integers by set of smaller ones, and have two unique features; a carry-free arithmetic that enable to perform parallel mathematical operations related to the individual residue symbols, and no weight-information are carried between carriers which prevent error propagation [7].
It is defined by selecting \( v \) positive pair-wise relative primes \( m_i \) (\( i = 1, 2, 3 \ldots v \)), such that any integer \( N \), describing a message, is represented by the sequence \( (r_1, r_2, \ldots, r_v) \) in the range \( 0 < N < M_1 \) in a unique matter, where:

\[
 r_i = N \pmod{m_i}
\]

Where;

\( r_i \) is the least positive remainder when \( N \) is divided by modulus \( m_i \).

\[
 M_1 = \prod m_i
\]

is the information symbols' dynamic range.

Then use the Mixed Radix Conversion (MRC) method [8], to recover symbols. Where for a given set of pair-wise relatively prime moduli \( \{m_1, m_2, \ldots, m_v\} \) and a residue state \( \{r_1, r_2, \ldots, r_v\} \) of a number \( X \), that number can be uniquely represented in mixed-radix form as seen in next:

\[
 X = [z_1, z_2, \ldots, z_n] \quad (3)
\]

And:

\[
 X = z_1 + z_2 m_1 + z_3 m_2 m_1 + \ldots + z_n m_{n-1} m_{n-2} \ldots m_1 \quad (4)
\]

Where:

\( z_i \) is represented as function of the moduli and residue representations as seen in table (1):

Table (1): Representation of \( z_i \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_1 )</td>
<td>= ( r_1 )</td>
</tr>
<tr>
<td>( z_2 )</td>
<td>= ( |m_1 - 1</td>
</tr>
<tr>
<td>( z_3 )</td>
<td>= ( |(m_2m_1 - 1</td>
</tr>
<tr>
<td>( z_n )</td>
<td>= ( |(m_{n-1} \ldots m_2m_1 - 1</td>
</tr>
</tbody>
</table>

2.2 Redundant Residue Number System

The RNS moduli utilized for error detection and correction through implementation of additional RNS moduli as redundancy symbols; being named the Redundant Residue Number System (RRNS).

In this configuration each redundant moduli selected to be greater than any of the other chosen moduli set and don’t play any role in determining the system dynamic range. So, an RRNS is obtained by appending an additional \( u - v \) number of moduli \( m_{v+1}; m_{v+2}; \ldots; m_u \), where \( m_{v+j} \geq \max\{m_1; m_2; \ldots; m_v\} \) is referred to as a redundant modulus, to the previously introduced RNS, in order to form an RRNS of \( u \) positive, pairwise relative prime moduli. [9, 10]

For the correction of the error, using the MRC method, a test on each of the information moduli with the two redundant moduli is performed and through this test we are able to identify and correct the bit which generated the error [11].

3. Inter-Carrier-Interference Analysis

In MIMO-OFDM systems, the loss of orthogonality between subcarriers, increases the ICI between subcarriers and leads to degradation for the system performance. This is due to the Doppler shift induced from the relative motion between the transmitter and receiver sensitivity and caused a frequency offset between sub-carriers, and would result in a reduced signal amplitude and ICI as presented in Fig. 1.

Fig. 1. Effect of Carrier Frequency Offset

![Figure 1](image1.png)

The frequency offset (\( \varepsilon \)) is modelled as shown in Fig. 2, where the received signal represented as:

\[
 Y(n) = x(n)e^{j2\pi \varepsilon N} + W(n) \quad (5)
\]

Fig. 2. Frequency offset model

![Figure 2](image2.png)

The effect of this frequency offset on the received symbol stream is shown in the received symbol \( Y(k) \) on the \( K \)th sub-carrier.

\[
 Y(k) = X(k) S(0) + \sum_{l=0, l \neq k}^{N-1} X(l) S(l-k) + n_k \quad (6)
\]

Where;

\( X(k), n_k \): Transmitted symbol for \( k \)th sub-carrier, and FFT of \( w(n) \), respectively.

\( N \): Total number of sub-carriers,

\( S(l-k) \): ICI components for received signal

The ICI components \( S(l-k) \), are the interfering signals on sub-carriers, where their complex coefficients are given by:

\[
 S(l-k) = \frac{\sin(\pi (1+\varepsilon-k))}{N\sin\left(\frac{\pi (1+\varepsilon-k)}{N}\right)} \exp(j\pi \left(1 - \frac{1}{N}\right) (1 + \varepsilon - k)) \quad (7)
\]
4. ICI Mitigation Techniques

The sensitivity in OFDM systems to frequency offset factors and rotation of subcarriers is a concern. Frequency-domain equalization, time-domain equalization, ICI self-cancellation, and Pulse shaping techniques are used to address this issue. These techniques are employed as well for the reduction of the Peak-Average-Power Ratio (PAPR) through the reduction of side lobes in each carrier, permitting a larger usual power to be transmitted for a constant peak power, and making improvements to the system performance. A detailed description of existing ICI mitigation techniques is provided in the coming subsections.

4.1 Self-Cancellation Technique

The input symbols are modulated to a group of subcarriers with pre-defined coefficients such that the ICI signals would cancel each other in the group. So, one data symbol is modulated into two consecutive subcarriers, such that the data symbol ‘a’ is modulated in the first sub-carrier, then ‘-a’ is modulated in to the second subcarrier. Thus, the ICI generated between the two sub-carriers would cancel each other. Through this scheme it is possible to achieve an improvement in Carrier-Interference-Ratio (CIR) of about 20dB for 0<ε<0.5, due to reduced ICI levels compared to standard OFDM [15]. Furthermore, this technique doesn’t need an estimation feedback and is simple in implementation, but on the other hand due to the redundancy introduced a higher bandwidth is needed.

4.2 Frequency domain equalization

A frequency pilot symbol is inserted between two sub-blocks, where it is able to determine the coefficients of the equalizers that are used in frequency domain [16].

![Fig.3. Pilot Sub-Carrier Arrangement](image)

Falling within this category; it is indicated techniques as Maximum Likelihood (ML) estimation and the Extended Kalman Filter (EKF) that estimate the offset and correct it at the receiver.

4.3 Windowing Technique

It is system equalization in time-domain [17], where an exponential function is multiplied to the transmitted signal before calculating its Fourier transform seen in equation (8), with the purpose to reduce the effect of discontinuities at both ends of the discrete signal.

\[ b_k = a_k (1 - \exp(j2\pi n/N)) \]  

Where:

- \( b_k \) transmitted data samples on the \( k \)th subcarrier

This mitigation technique reduces the start and ends of waveform, reducing the transients and consequently the spectral spreading. Also, it is utilized to decrease the sensitivity towards frequency errors and so reducing BER of the system. All the windows include Hanning, Nyquist, and Kaiser etc, give some reduction in the sensitivity to frequency offset.

4.4 Pulse Shaping Technique

As the peak power is associated with main lobe of the signal, while the ICI power is associated with side lobes, so the objective is to reduce the amplitude of side-lobes and increase the width of main lobe. This is done through using a new pulse shaping functions to reduce the side lops in each carrier with a target for ICI reduction [18].

This technique is very similar to the windowing technique, and even is implemented in similar ways, but their purposes are different. The pulse shaping means choosing a pulse with the desired spectral and orthogonality properties for ICI power reduction.

Several pulse shaping functions are present to perform the requirement as: Raised Cosine pulse (RC), and Square Root Raised Cosine pulse (SRRQ), and presented in equation (9) and (10) respectively:

\[ P_{RC}(f) = \text{sinc}(f) \frac{\cos(\pi (\alpha f t))}{1-(2\alpha ft)^2} \]  

Where:

- \( \alpha \) is the roll off factor ,
- \( f \) is the frequency,
- \( t \) is the time

\[ P_{SRRC}(f) = \text{sinc}(f) \left( 1 - \frac{4}{(\pi^2 f^2 t^2)} \cos(1 + \alpha) \left( \frac{at}{T} \right) + \frac{\alpha \sin(1-\alpha) \sin(\alpha t)}{1 - (4\alpha f t)^{0.5}} \right) \]  

Through this technique, side loop power is decreased leading to a decrease in the ICI between adjacent carriers and better bandwidth efficiency, which could be further enhanced through increasing the number of filter coefficients, as, indicated in previous literature [19].
5. Proposed System Model
The proposed MIMO-RNS-OFDM system is shown in Fig.4 is initialized with a binary data random source, converted to residue system then the packet is modulated, coded through the Space-Time Block Coding (STBC) encoder, passed to a Serial-To-Parallel (S/P) converter for parallel transmission, and then passed through an IFFT block then to the transmission antenna. At the receiver side the communication blocks are the reverse of the transmitter.

![Fig. 4. MIMO-OFDM System model](image)

The above system shown in Fig. 5; is evaluated by measuring the Carrier-Interference-Ratio (CIR) given in equation (11), and the Bit Error Rate (BER) of the signal shown in equation (12), respectively.

\[
CIR = \frac{|S(0)|^2}{\sum_{l=0, l \neq k}^{N-1} |S(l-k)|^2}
\]

(11)

Where;
\[S(l-k)\] Complex coefficient for ICI components in the receiving signal.

And; the probability of error for M-PSK modulated transmission is given by:

\[
P_{ERR} = \gamma \sum_{k=1}^{\min(2, \frac{M}{2})} Q\left(\sqrt{2\sigma x \sin \left(\frac{(2k-1)\pi}{M}\right)}\right)
\]

(12)

\[
\gamma = \max(\log_2 M, 2)
\]

(13)

Where;
\[M\] is the constellation size
\[\rho\] is the SNR per symbol
\[x\] is a chi-square distributed random variable

6. Simulation Results
The results obtained from the MATLAB simulations are discussed, where various analysis had been performed on MIMO-RNS-OFDM system to measure its resilience towards ICI in comparison to current MIMO-OFDM systems.

In this simulation, 1000 symbols are 512-QAM modulated and transmitted over a MIMO-OFDM communication system using RNS coding technique with redundant moduli’s (17, 13, 11, 7, 5, 3), were (11, 7, 5, 3) are the information moduli’s and the set (13, 17) are the redundant moduli’s.

6.1 BER vs. SNR for various offset values
The performance of communication system in the presence of frequency offset between the transmitter and the receiver is seen in Fig. 5.

![Fig. 5: Effect of frequency offset on Performance](image)

From the above Fig. 5, it is shown that degradation of performance increases with frequency offset. Thus, when the offset is small, the system has a lower BER (better).

6.2 ICI Measurements for MIMO-RNS
For a pre-defined SNR value (80), the transmission signal error is plotted versus the frequency offset as seen in Fig. 6 for OFDM system with and without RNS moduli’s (13, 11, 7, 5, 3) as coding scheme;

![Fig. 6: Error for MIMO-RNS-OFDM System](image)

Where; it is seen in Fig. 6 an absolute 25 dB improvement when using the RNS scheme, which is better than the achieved improvement using ICI cancelation scheme indicated in section (4.1).

In addition, it is seen that - as expected - as the frequency offset increase this would increase the error due to the increasingly loss of orthogonality between inter-carriers.

The RNS coding performance is even enhanced with low offset values due to the inherent properties of RNS that doesn’t allow the transmission of error between different moduli’s.
6.3 ICI Measurements for MIMO-RRNS
Using RNS with redundant moduli’s (17, 13, 11, 7, 5, 3), where (11, 7, 5, 3) are the information moduli’s and the set (13, 17) are the redundant moduli’s, and measuring ICI for the system comparing its value with the communication system without any redundant modus as seen in Fig. 7;

Fig 7: Error for MIMO-RRNS-OFDM Systems
Here; in Fig 7 the improvement is more than 30dB, which is better than ICI cancellation scheme and RNS coding scheme seen in sections (4.1) and (6.2) respectively.

Moreover, the system exhibit similar performance as that shown when using RNS as a coding scheme only, as seen in section (6.2).

6.4 Effect of RNS moduli selection on ICI
Increasing the order of RNS moduli set and measures the system performance to see the effect of the selection of the RNS on ICI reduction.

Fig. 8: ICI vs. RNS moduli set
From Fig 8, it is noted that each time the amplitude of the RNS set increased this would increase the ICI error, and thus the increased signal amplitude would increase directly the interference between adjacent sub-carriers.

Now; in the comings sub-sections (6.5) to (6.8) various mitigation schemes are implemented and analyzed in the MIMO-RNS-OFDM communication system to study and evaluate its performance in combination with Residue coding technique.

6.5 MIMO-RNS-OFDM with “Frequency domain equalization” Scheme
A frequency domain equalizer is used in the receiver, and the system performance is evaluated as seen in Fig. 9 over a Rician Log Normal (RLN) distribution fading channel.

Fig. 9: MIMO-RNS with/without equalization
Where; at SNR = 15, BER for the communication system with error correction is 1*10^{-3} while it reaches 3*10^{-2} for the system without error correction.

6.6 MIMO-RNS-OFDM with “Self-Cancellation” Scheme
Using data conjugate technique in self-cancellation scheme, where the system performance is evaluated as seen in Fig. 10 over a Rayleigh fading channel.

Fig. 10. MIMO-RNS with self-cancellation Scheme
Where; at SNR =20, BER for the communication system with error correction is 4*10^{-3} while it reaches 1*10^{-2} for the system without error correction.

6.7 MIMO-RNS-OFDM with “Pulse Shaping” Scheme
A raised cosine pulse shaping scheme added to MIMO-RNS-OFDM, and evaluated through the coming simulations.

6.7.1 Pulse Shape Design:
Prior to testing the communication system with pulse shaping mitigation scheme, it is essential to set-up the pulse filter to obtain the optimum performance.
Thus, the pulse roll-off parameter is adjusted and the system performance is measured using a 512QAM modulation and an RRNS moduli set of \{3, 5, 7, 11, 13\}, where information moduli are \{3, 5, 7, 11\} and redundant moduli is \{13\}, over Rician + AWGN channel, as seen in Fig 11.

From Fig. 11, the effect of the shape of the filter which is adjusted through the ‘roll-off’ parameter clearly affects the system performance through decreasing or increasing the PAPR and ICI values.

In Fig 11.a, 11.c, and 11.e, the ICI results decrease with the increase of the roll-off, while on the other hand as seen in Fig 11.b, 11.d, and 11.f the PAPR increase with the increase in the roll-off. Therefore, the selection of the optimum roll-off is a trade-off between the ICI and PAPR required.

6.7.2 CCDF Measurement:

Fig. 12: CCDF measurements

It is shown in Fig 12.a that the CCDF measured for the system with a raised cosine pulse scheme was lower than foreseen in Fig 12.b which was measured without a mitigation scheme. This is attributed to the condensation in signal spectrum resulted from RC filter that decrease symbol shape.

6.7.3 PAPR Measurement:

Perform recurrent measurement to evaluate the PAPR of the communication system with and without pulse shaping mitigation scheme as seen in Fig 13.

From Fig 13.a and 13.b, it is shown that reduction in PAPR seen over Rayleigh fading channel for wireless communication with residue system and error control when using pulse shaping mitigation scheme in comparison to that without the mitigation scheme.

6.7.4 ICI and BER Measurement:

Then measuring the ICI error and overall BER performance for the system with and without pulse shaping mitigation is measured, as seen in Fig 14 and 15.
From the above Fig. 14, it is shown that using the mitigation scheme as in Fig. 14.a the ICI reduction using RNS coding is around 40 dB while without the mitigation scheme as seen in Fig. 14.b the reduction is only 30 dB. The improved features seen in Fig. 14.a is attributed to the use of pulse shaping scheme as a mitigation technique in the communication system.

And from Fig. 15, the BER performance for the system with and without mitigation scheme is measured, it is shown that BER performance in Fig. 15.a is better than that seen in Fig. 15.b. Where, at SNR = 10 dB, the BER for the RRNS communication system with mitigation scheme seen in Fig. 15.a is $10^{-4}$ while for the same system without mitigation scheme as seen in Fig. 15.b is $10^{-3}$. This result is coherent with that obtained in Fig. 14 indicating the decrease of ICI when implementing mitigation scheme.

### 6.8 MIMO-RNS-OFDM vs. ICI Reduction Techniques

In this subsection a comparisons of various ICI cancelation schemes that are implemented within the MIMO-RNS-OFDM system are studied and analyzed as seen in Fig. 16, to determine the best choice of ICI mitigation techniques that is suitable of RNS coding scheme.

From the above Fig. 16, the equalization considers the channel information and hence gives more accurate results compared to self-cancellation scheme. Windowing/pulse shaping on the other hand would provide the best performance with respect to the other schemes.

The rationale behind this is that conventional technique for ICI Reduction like time domain equalization, self-cancelation, does not properly reduce ICI at the receiver side as through these techniques, ICI reduces only band limited channel which is not the main source of ICI. Where, the main source of ICI is due to frequency mismatch between transmitter and receiver that is corrected and reduced through pulse shaping mitigation.

### 7. Conclusion

In this paper, a review for MIMO-OFDM system performance using ICI self-cancelation, pulse shaping, windowing mitigation techniques had been provided and discussed.

An RNS coding insertion in MIMO-OFDM communication system has been proposed, and evaluated with respect to both CIR and BER performance. The usage of residue system had showed its advantage in improving the communication system features through decreasing the ICI and improving the BER performance.
The MIMO-OFDM with RNS coding further enhanced through insertion of ICI mitigation scheme in the system, where pulse shaping mitigation had proven its enhanced performance with the residue system over the equalization scheme; through the recorded improvement in the BER, PAPR and ICI parameters.

References


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