Level Crossing Rate of the Ratio of Product of Two Nakagami-*m* Random Processes and Nakagami-*m* Random Process

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Abstract: - In this work, the ratio of product of two Nakagami-*m* random processes and Nakagami-*m* random process (RP) is analyzed. Statistical characteristics of ratio of product of two RPs and RP is applied in performance analysis of relay communication systems with two sections operating in multipath fading channel in the presence of co-channel interference. Here, multipath fading is with Nakagami-*m* distribution, and co-channel interference is exposed to the influence of Nakagami-*m* small scale fading. Level crossing rate (LCR) of defined ratio is obtained using some useful mathematical manipulations and formulas. The influence of fading parameters and signal powers on the LCR is assessed based on some plotted graphs. By selection of the channel parameters, the system performance could be improved and higher reliability of wireless links achieved, what is important for command information systems.

Key-Words: - Nakagami-*m* fading, co-channel interference, random process, level crossing rate, command information systems

1 Introduction

Reflections, refractions and deviations cause multipath fading [1]. When the dominant component is not present, signal envelope can have Nakagami-*m* distribution. Rayleigh random process (RP) and one sided Gaussian random process are special cases of Nakagami-*m* random process. For m=1, Nakagami-*m* random process reduces to Rayleigh random process and for m=1/2, Nakagami-*m* random process reduces to one sided Gaussian random process [2], [3].

There are many works in technical literature considering the performance of wireless communication systems working over short term fading environment. Also, the consideration of the ratio and product of random processes is very important for wireless communication system in the presence of fading [4]-[14]. For such ratios and products, the first order system performance (the outage probability (OP), the bit error probability (BEP) and wireless communication system capacity) and second order system performance (level crossing rate (LCR) and average fade duration (AFD)) have been derived.

In [5], the expressions for the LCR and the AFD of the double Nakagami-*m* random process are presented. The same authors carried out the second order statistical characteristics (LCR and AFD) of the amplify-and-forward multihop Rayleigh fading channel in [6]. Because cascaded channel can be modelled as the product of N fading amplitudes, they performed analytical expressions for the average LCR and the AFD of the product of N Rayleigh fading envelopes (i.e. performance for so-called $N*$Rayleigh channel).

The closed-form bounds for the performance of multihop transmissions with non-regenerative relays in Nakagami-*m* fading channels are derived in [7]. The product of N statistically independent, but not necessarily identically distributed, Nakagami-*m* random variables (RVs) is analyzed in [8]. This is convenient way to model cascaded Nakagami-*m* fading channels and analyzing their performance. In that paper, the first order system performance of the $N*$Nakagami distribution are calculated in closed form using the Meijer's G function. Derivation of the probability density function (PDF) of products of Rayleigh and Nakagami-*m* RVs by using Mellin transform is shown in [9].

Further, statistical characteristics of the ratio and product of Rician random variables and its application in analysis of wireless communication systems are presented in [10]. PDF, OP and LCR are derived for the Rician fading environment. LCR of the ratio of product of two Rayleigh and one Nakagami-m RV and LCR of the ratio of Rayleigh and product of two Nakagami-*m* RVs are performed in [11], and LCR of the ratio of product of two k-μ

RVs and Nakagami-*m* RV is obtained in [12]. The LCR of the ratio of product of two RVs and one RV, with k-μ and α-k-μ distributions, are derived in [13] and [14] respectively.

In this work, we will analyze the ratio of product of two Nakagami-*m* random processes and Nakagami-*m* random process (RP) and derive formula for the level crossing rate. We will consider the case where parameters of Nakagami-*m* fading envelopes are m_1 and m_2 for distributions in nominator, and m_3 in denominator. For m_1 =1 and m_2=1, the LCR of the ratio of product of two Nakagami-*m* RPs and Nakagami-*m* RP becomes the LCR of the ratio of product of two Rayleigh RPs and Nakagami-*m* RP. For m_1 =1 or m_2 =1, the LCR of the ratio of product of two Nakagami-*m* RPs and Nakagami-*m* RP reduces to the LCR of the ratio of product of Rayleigh RP and Nakagami-*m* RP, and Nakagami-*m* RP. For m_3=1, the LCR of the ratio of product of two Nakagami-*m* RPs and Nakagami-*m* RP converts to the LCR of the ratio of product of two Nakagami-*m* RPs and Rayleigh RP. Finally, for m_1=m_2=m_3=1, the performance of the ratio of product of two Nakagami-*m* RPs and Nakagami-*m* RP turns into the performance of the ratio of product of two Rayleigh RPs and Rayleigh RP, what is shown in [4].

According to the author's knowledge, level crossing rate of the ratio of product of two Nakagami-*m* random processes and Nakagami-*m* random process is not considered in the available literature.

2 LCR of the Ratio of Product of Two Nakagami-*m* Random Processes and Nakagami-*m* Random Process

Random variables y_1, y_2 and y_3 follow Nakagami-*m* distribution [15]:

$$p_{y_1}(y_1)=\frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)^{m_1} y_1^{2m_1-1}e^{-\frac{m_1}{\Omega_1}y_1^2},\ y_1\geq 0, \quad (1)$$

$$p_{y_2}(y_2)=\frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)^{m_2} y_2^{2m_2-1}e^{-\frac{m_2}{\Omega_2}y_2^2},\ y_2\geq 0, \quad (2)$$

$$p_{y_3}(y_3)=\frac{2}{\Gamma(m_3)}\left(\frac{m_3}{\Omega_3}\right)^{m_3} y_3^{2m_3-1}e^{-\frac{m_3}{\Omega_3}y_3^2},\ y_3\geq 0, \quad (3)$$

where Ω_i , i=1,2,3, are average powers of y_i, m_i are fading severity parameters of y_i and $\Gamma(\cdot)$ denotes the Gamma function.

Random variable *y* is defined as:

$$y=\frac{y_1y_2}{y_3}, \quad (4)$$

and consequently:

$$y_1=\frac{yy_3}{y_2}. \quad (5)$$

The first derivative of *y* is:

$$\dot{y}=\frac{\dot{y}_1y_2}{y_3}+\frac{y_1\dot{y}_2}{y_3}-\frac{y_1y_2\dot{y}_3}{y_3^2}. \quad (6)$$

The first derivative of Nakagami-*m* RP is Gaussian RP. Thus, $\dot{y}_i$ are Gaussian RPs. Linear transformation of Gaussian RPs is Gaussian RP. Therefore, $\dot{y}$ is a variable with Gaussian distribution. The mean value of $\dot{y}$ is:

$$\overline{\dot{y}}=\frac{\overline{\dot{y}_1}y_2}{y_3}+\frac{y_1\overline{\dot{y}_2}}{y_3}-\frac{y_1y_2\overline{\dot{y}_3}}{y_3^2}=0, \quad (7)$$

because it applies:

$$\overline{\dot{y}_1}=\overline{\dot{y}_2}=\overline{\dot{y}_3}=0. \quad (8)$$

The variance of $\dot{y}$ is:

$$\sigma_{\dot{y}}^2=\frac{y_2^2}{y_3^2}\sigma_{\dot{y}_1}^2+\frac{y_1^2}{y_3^2}\sigma_{\dot{y}_2}^2+\frac{y_1^2y_2^2}{y_3^4}\sigma_{\dot{y}_3}^2 \quad (9)$$

where:

$$\sigma_{\dot{y}_1}^2=\pi^2 f_m^2\frac{\Omega_1}{m_1} \quad (10)$$

$$\sigma_{\dot{y}_2}^2=\pi^2 f_m^2\frac{\Omega_2}{m_2} \quad (11)$$

$$\sigma_{\dot{y}_3}^2=\pi^2 f_m^2\frac{\Omega_3}{m_3}. \quad (12)$$

Here, f_m is maximal Doppler frequency.

After substituting (10)-(12) and (5) into (9), formula for the variance $\sigma_{\dot{y}}^2$ is calculated as:

$$\sigma_{\dot{y}}^2=\pi^2 f_m^2\left(\frac{y_2^2}{y_3^2}\frac{\Omega_1}{m_1}+\frac{y_1^2}{y_3^2}\frac{\Omega_2}{m_2}+\frac{y_1^2y_2^2}{y_3^4}\frac{\Omega_3}{m}\right)=$$
$$=\pi^2 f_m^2\frac{y_2^2}{y_3^2}\frac{\Omega_1}{m_1}\left(1+\frac{y_1^2}{y_2^2}\frac{\Omega_2}{m_2}\frac{m_1}{\Omega_1}+\frac{y_1^2}{y_3^2}\frac{\Omega_3}{m_3}\frac{m_1}{\Omega_1}\right)=$$
$$=\pi^2 f_m^2\frac{y_2^2}{y_3^2}\frac{\Omega_1}{m_1}\left(1+\frac{y^2y_3^2}{y_2^4}\frac{\Omega_2}{m_2}\frac{m_1}{\Omega_1}+\frac{y^2}{y_2^2}\frac{\Omega_3}{m_3}\frac{m_1}{\Omega_1}\right) \quad (13)$$

The joint probability density function (JPDF) of y, $\dot{y}$, y_2 and y_3 is:

$$p_{y\dot{y}y_2y_3}\left(y\dot{y}y_2y_3\right)=p_{\dot{y}}\left(\dot{y}/yy_2y_3\right)p_{yy_2y_3}\left(yy_2y_3\right), \quad (14)$$

where:

$$p_{yy_2y_3}\left(yy_2y_3\right)=p_y\left(y/y_2y_3\right)p_{y_2y_3}\left(y_2y_3\right)=$$
$$=p_y\left(y/y_2y_3\right)p_{y_2}\left(y_2\right)p_{y_3}\left(y_3\right), \quad (15)$$

$$p_y\left(y/y_2y_3\right)=\left|\frac{dy_1}{dy}\right|p_{y_1}\left(\frac{yy_3}{y_2}\right), \quad (16)$$

and:

$$\frac{dy_1}{dy}=\frac{y_2}{y_3}. \quad (17)$$

Now, we obtain the JPDF of y and $\dot{y}$ as:

$$p_{y\dot{y}}\left(y\dot{y}\right)=\int_0^\infty\int_0^\infty dy_2dy_3\,p_{y\dot{y}y_2y_3}\left(y\dot{y}y_2y_3\right)=$$
$$=\int_0^\infty dy_2\int_0^\infty dy_3\,p_{\dot{y}}\left(\dot{y}/yy_2y_3\right)\cdot$$
$$\cdot\frac{y_2}{y_3}p_{y_1}\left(\frac{yy_3}{y_2}\right)p_{y_2}\left(y_2\right)p_{y_3}\left(y_3\right). \quad (18)$$

The level crossing rate is average value of the first derivative of random process and defined by [16], [17, eq. (5.80)]:

$$N_y=\int_0^\infty d\dot{y}\,\dot{y}\,p_{y\dot{y}}\left(y\dot{y}\right). \quad (19)$$

By inserting the corresponding formulas into the subintegral function of the upper integral, we have LCR of the ratio of product of two Nakagami-*m* RPs and Nakagami-*m* RP, N_y:

$$N_y=\int_0^\infty dy_2\int_0^\infty dy_3\int_0^\infty d\dot{y}\,\dot{y}\,p_{\dot{y}}\left(\dot{y}/yy_2y_3\right)\cdot$$
$$\cdot\frac{y_2}{y_3}p_{y_1}\left(\frac{yy_3}{y_2}\right)p_{y_2}\left(y_2\right)p_{y_3}\left(y_3\right)=$$
$$=\int_0^\infty dy_2\int_0^\infty dy_3\frac{y_2}{y_3}p_{y_1}\left(\frac{yy_3}{y_2}\right)p_{y_2}\left(y_2\right)p_{y_3}\left(y_3\right)\frac{1}{\sqrt{2\pi}}\sigma_{\dot{y}}=$$
$$=\int_0^\infty dy_2\int_0^\infty dy_3\cdot\frac{1}{\sqrt{2\pi}}\pi f_m\cdot$$
$$\cdot\frac{y_2}{y_3}\frac{2}{\Gamma\left(m_1\right)}\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{yy_3}{y_2}\right)^{2m_1-1}e^{-\frac{m_1}{\Omega_1}\left(\frac{yy_3}{y_2}\right)^2}\cdot$$
$$\cdot\frac{2}{\Gamma\left(m_2\right)}\left(\frac{m_2}{\Omega_2}\right)^{m_2}y_2^{2m_2-1}e^{-\frac{m_2}{\Omega_2}y_2^2}\cdot$$
$$\cdot\frac{2}{\Gamma\left(m_3\right)}\left(\frac{m_3}{\Omega_3}\right)^{m_3}y_3^{2m_3-1}e^{-\frac{m_3}{\Omega_3}y_3^2}$$
$$\cdot\frac{y_2}{y_3}\frac{\Omega_1^{1/2}}{m_1^{1/2}}\left(1+\frac{y^2y_3^2}{y_2^4}\frac{\Omega_2}{m_2}\frac{m_1}{\Omega_1}+\frac{y^2}{y_2^2}\frac{\Omega_3}{m_3}\frac{m_1}{\Omega_1}\right)^{1/2}=$$
$$=\frac{1}{\sqrt{2\pi}}\pi f_m\frac{\Omega_1^{1/2}}{m_1^{1/2}}\frac{8}{\Gamma\left(m_1\right)\Gamma\left(m_2\right)\Gamma\left(m_3\right)}\cdot$$
$$\cdot\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{m_3}{\Omega_3}\right)^{m_3}\cdot y^{2m_1-1}\cdot$$
$$\int_0^\infty dy_2\int_0^\infty dy_3\,y_2^{-2m_1+2m_2+2}y_3^{2m_1+2m_3-4}\cdot$$
$$e^{-\frac{m_1}{\Omega_1}\left(\frac{yy_3}{y_2}\right)^2-\frac{m_2}{\Omega_2}y_2^2-\frac{m_3}{\Omega_3}y_3^2}\left(1+\frac{y^2y_3^2}{y_2^4}\frac{\Omega_2}{m_2}\frac{m_1}{\Omega_1}+\frac{y^2}{y_2^2}\frac{\Omega_3}{m_3}\frac{m_1}{\Omega_1}\right)^{1/2}.$$
$$(20)$$

For solving the two-fold integral in (20), the Laplace approximation formula can be applied. The solution is defined using the [18], [19]:

$$\int_0^\infty dy_2\int_0^\infty dy_3\,g\left(y_{20},y_{30}\right)e^{\lambda f\left(y_{20},y_{30}\right)}=$$
$$=\frac{\pi}{\lambda}\frac{g\left(y_{20},y_{30}\right)}{B\left(y_{20},y_{30}\right)}e^{\lambda f\left(y_{20},y_{30}\right)}, \quad (21)$$

where y_{20} and y_{30} are solutions of the equations:

$$\frac{\partial f\left(y_{20},y_{30}\right)}{\partial y_{20}}=0,\quad\frac{\partial f\left(y_{20},y_{30}\right)}{\partial y_{30}}=0. \quad (22)$$

The matrix B is in the shape:

$$B\left(y_{20},y_{30}\right)=\begin{vmatrix}\frac{\partial^2 f\left(y_{20},y_{30}\right)}{\partial y_{20}^2} & \frac{\partial^2 f\left(y_{20},y_{30}\right)}{\partial y_{20}\partial y_{30}}\\ \frac{\partial^2 f\left(y_{20},y_{30}\right)}{\partial y_{20}\partial y_{30}} & \frac{\partial^2 f\left(y_{20},y_{30}\right)}{\partial y_{30}^2}\end{vmatrix}. \quad (23)$$

For two-fold integral from (20), it is:

$$g\left(y_2,y_3\right)=y_2^{-2m_1+2m_2+2}y_3^{2m_1+2m_3-4}\cdot$$
$$\left(1+\frac{y^2y_3^2}{y_2^4}\frac{\Omega_2}{m_2}\frac{m_1}{\Omega_1}+\frac{y^2}{y_2^2}\frac{\Omega_3}{m_3}\frac{m_1}{\Omega_1}\right)^{1/2} \quad (24)$$

$$f\left(y_2, y_3\right) = -\frac{m_1}{\Omega_1}\left(\frac{y y_3}{y_2}\right)^2 - \frac{m_2}{\Omega_2} y_2^2 - \frac{m_3}{\Omega_3} y_3^2 . \quad (25)$$

LCR of the ratio of product of two Nakagami-*m* random processes and Nakagami-*m* random process will be finalized by putting obtaining solutions of these equations into (20).

3 Graphics and Analysis of Results

In this work, the level crossing rate of the ratio of product of two Nakagami-*m* random processes and Nakagami-*m* random process is determined and presented in the next two plots.

It is possible to see from Fig. 1 that LCR increases for lower values of the resulting signal envelope and decreases for higher values of the resulting signal envelope. The influence of the signal envelope on the LCR is bigger for smaller values of this envelope.

LCR grows slower with the increase of average powers Ω_i. For smaller average powers of resulting signal envelope, LCR achieves maximum, and then start to decline.

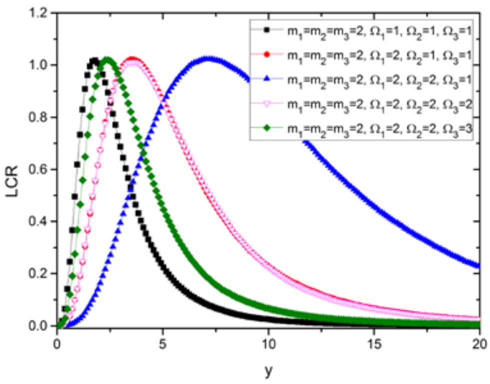

Fig.1. LCR of ratio of product of two Nakagami-*m* random processes and Nakagami-*m* random process

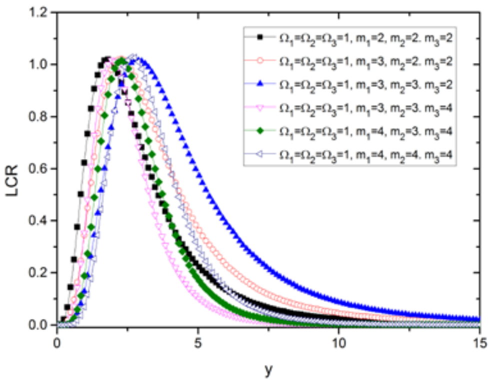

Fig.2. LCR of ratio of product of two Nakagami-*m* RPs and Nakagami-*m* RP for new set of parameters

Fig. 2 shows that LCR decreases as parameters m_1 and m_2 increase, and LCR increases when parameter m_3 decreases. The influence of parameters m_1 and m_2 on the LCR is higher for higher values of parameter m_3.

Also, the impact of parameter m_3 on the LCR is higher for lower values of parameter m_3 and higher values of the resulting signal envelope. The influence of resulting signal envelope on the LCR is bigger for lower values of Nakagami-*m* fading severity parameters m_1 and m_2, and for higher values of parameter m_3.

4 Conclusion

Here, the level crossing rate of the ratio of product of two Nakagami-*m* random processes and Nakagami-*m* random process is calculated. This result can be used for evaluation the average fade duration of wireless relay communication system with two sections operating over Nakagami-*m* short term fading channel, where co-channel interference is present at one section.

The average fade duration is defined as the ratio of outage probability and level crossing rate. Outage probability is defined as probability that signal envelope is lower than threshold [20]. Derived expression for LCR can be used for evaluation of the LCR of the ratio of product of two Rayleigh random processes and Rayleigh process. Also, from this formula, for special values of Nakagami-*m* fading severity parameter *m*, LCR of the ratio of product of Rayleigh and Nakagami-*m* random processes and Rayleigh or Nakagami-*m* process can be obtained.

The selection of the channel parameters and signal powers, done here based on the observed system performance, can seriously improved reliability of wireless links. This is also very important for command information systems.

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References:

[1] S. Panic, M. Stefanovic, J. Anastasov, P.Spalevic, *Fading and Interference Mitigation in Wireless Communications*, USA: CRC Press, 2013.

[2] W. C. Y. Lee, *Mobile Communications Engineering*, 2nd ed., McGraw-Hill, New York, 1997.

[3] G. L. Stuber, *Mobile Communication*, 2nd ed. Dordrecht: Kluwer Academic Publisher, 2003.

[4] D. Krstić, M. Stefanović, M. Peric, and S. Minic, "Analysis of ratio of one and product of two Rayleigh random variables and its application in telecommunications", *International Journal of Communications*, vol. 3, 2018, pp. 32-38.

[5] N. Zlatanov, Z. Hadzi-Velkov, G. K. Karagiannidis, Level Crossing Rate and Average Fade Duration of the Double Nakagami-*m* Random Process and Application in MIMO Keyhole Fading Channels, *IEEE Communications Letters*, Vol. 12, No. 11, November 2008, pp. 822-824.

[6] Z. Hadzi-Velkov, N. Zlatanov, K. G. Karagiannidis, On the second order statistics of the multihop Rayleigh fading channel, *IEEE Transactions on Communications*, vol. 57, No. 6, June 2009, pp. 1815–1823, DOI: 10.1109/TCOMM.2009.06.070460.

[7] G. K. Karagiannidis, T. A. Tsiftsis, R. K. Mallik, N. C. Sagias, S. A. Kotsopoulos, Closed-form bounds for multihop relayed communications in Nakagami-*m* fading, *IEEE Transactions on Communications*, vol. 54, January 2006, pp. 18–22.

[8] G. Karagiannidis, S. Nikos, and M. Takis, N*Nakagami: a novel stochastic model for cascaded fading channels, *IEEE Transactions on Communications*, vol. 55, no. 8, 2007, pp. 1453–1458.

[9] S. Ahmed, L. L. Yang, L. Hanzo, Probability distributions of products of Rayleigh and Nakagami-*m* variables using Mellin transform, *IEEE International Conference on Communications*, ICC 2011, 05 Jun - 09 Jun 2011, Kyoto, Japan

[10] D. Krstić, M. Stefanović, Z. Jovanović, R.Gerov, V. Milenković, Statistical Characteristic of Ratio and Product of Rician Random Variables and its Application in Analysis of Wireless Communication Systems", *International Journal of Mathematical and Computational Methods*, vol. 1, 2016, pp. 79-86.

[11] D. Krstic, M. Stefanovic, M. Głąbowski, M. Peric, Level Crossing Rate of Ratio of Product of Two Rayleigh and One Nakagami-m Random Variable and of Ratio of Rayleigh and Product of Two Nakagami-m Random Variables, 11th IEEE/IET International Symposium on Communication Systems, Networks and Digital Signal Processing, CSNDSP18, Budapest, Hungary, 18th to 20th July 2018, doi:10.1109/csndsp.2018.8471843

[12] D. Krstić, I. Romdhani, M. Masadeh Bani Yassein, S. Minić, G. Petković, P. Milačić, Level Crossing Rate of Ratio of Product of Two k-μ Random Variables and Nakagami-m Random Variable, 2015 IEEE International Conference on Computer and Information Technology; Ubiquitous Computing and Communications; Dependable, Autonomic and Secure Computing; Pervasive Intelligence and Computing, 26-28 Oct. 2015, Liverpool, UK, DOI: 10.1109/CIT/IUCC/DASC/PICOM.2015.244

[13] D. Krstić, M. Stefanović, N. Simić, A. Stevanović, The Level Crossing Rate of the Ratio of Product of Two k-μ Random Variables and k-μ Random Variable, 13th WSEAS International Conference on Electric Power Systems, High Voltages, Electric Machines (POWER '13), Chania, Crete Island, Greece, August 27-29, 2013, pp. 153-158.

[14] D. Krstić, M. Stefanović, V. Milenković, Dj. Bandjur, Level Crossing Rate of Ratio of Product of Two α-k-μ Random Variables and α-k-μ Random Variable, *WSEAS Transaction on Communications*, Vol. 13, 2014, Art#68, pp. 622-630.

[15] M. Nakagami, The m-distribution: A general formula of intensity distribution of rapid fading, in W. C. Hoffman (ed.), *Statistical Methods in Radio Wave Propagation*: Proceedings of a Symposium held June 18–20, 1958. Pergamon Press, New York, 1960, pp. 3–36.

[16] M. D. Yacoub, J. E. Vargas Bautista, L. Guerra de Rezende Guedes, On Higher Order Statistics of the Nakagami-m Distribution, *IEEE Transactions on Vehicular Technology*, Vol. 48, No. 3, May 1999, pp. 790-794.

[17] T. S. Rappaport, *Wireless Communications Principles and Practice*, Second Edition, Beijing Publishing House of Electronics Industry 2004, 3, pp. 1-707.

[18] M. Abramowitz, I. A. Stegun, *Handbook of mathematical functions*, National Bureau of Standards, 1964; reprinted Dover Publications, 1965.

[19] J. L. Lopez and P. J. Pagola, A simplification of the Laplace method for double integrals. Application to the second Appell function, *Electronic Transactions on Numerical Analysis*, vol. 30, pp. 224-236, 2008.

[20] Z. Cao and Y. D. Yao, Definition and drivation of level crossing rate and average fade duration in an interference-limited environment, *IEEE 54th Vehicular Technology Conference*, VTC Fall 2001, 7-11 Oct. 2001, Atlantic City, NJ, USA, DOI: 10.1109/VTC.2001.956470.