Application of Market Equilibrium Models to Optimal Resource Allocation in Telecommunication Networks

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Abstract: We consider several problems of optimal resource allocation in telecommunication networks and show that they can be formulated as market equilibrium models. This approach enables us to create simple and efficient solution methods. Next, we consider such a resource allocation problem for a provider of a wireless communication network divided into zones (clusters). The network manager aims to distribute some homogeneous resource (bandwidth) among users of several zones in order to maximize the total network profit, which takes into account payments from users and implementation costs. As a result, we obtain a convex optimization problem involving capacity and balance constraints. By using the dual Lagrangian method with respect to the capacity constraint, we reduce the initial problem to a suitable one-dimensional problem, so that calculation of its cost function value leads to independent solution of zonal problems, treated as two-side market equilibrium models with one trader. We show that solution of each zonal problem can be found exactly by a simple arrangement type algorithm even in the case where the trader price is not fixed. Besides, we suggest ways to adjust the basic problem to the case of moving nodes. Some results of computational experiments confirm the applicability of the new method.

Key–Words: Resource allocation, wireless networks, bandwidth, market equilibrium models, convex optimization, Lagrangian duality method, decomposition, non fixed prices.

1 Introduction

The current development of telecommunication systems creates a number of new challenges of efficient management mechanisms for efficient allocation of limited communication networks resources. In fact, despite the existence of powerful processing and transmission devices, increasing demand of different communication services and its variability lead to serious congestion effects and inefficient utilization of network resources (e.g., bandwidth and batteries capacity), especially in wireless telecommunication networks. This situation forces one to replace the fixed allocation rules with more flexible mechanisms, which are based on proper mathematical models; see e.g. [1]–[3]. The problem is to suggest such models and to develop suitable solution methods. Usually, the decision making processes are based on solutions of the corresponding optimization problems. At the same time, experience of dealing with these very complicated and spatially distributed systems usually shows that these problems have to utilize a proper decomposition/clustering approach, which can be based on zonal, time, frequency and other attributes of nodes/units; see e.g. [4, 5].

In this paper, we consider some problems of optimal allocation of a homogeneous resource in telecommunication networks such that the income received from users payments is maximized and the implementation costs of the network operator are minimized. We show that market equilibrium models suggested in [6, 7] can serve as a basis for these problems and enable us to create simple and efficient solution methods. We consider two-side equilibrium models with fixed prices. Being based on these properties, we consider a more general resource allocation problem for a provider of a wireless communication network divided into zones (clusters); which was formulated as a convex optimization problem in [8, 9]. Here the network manager problem consists in optimal distribution of the resource shares among zones in order to maximize the total network profit. This optimization problem involves capacity and zonal balance constraints. Unlike [8, 9], we suggest to apply the dual Lagrangian method with respect to only capacity constraint. This reduces the initial problem to a suitable one-dimensional problem, where calculation of its cost function value leads to independent solution of zonal problems, treated as above two-side market equilibrium models with one trader. We show that solution of each zonal problem can be found exactly by a simple arrangement type algorithm even in the case where the trader price is not fixed. In such a way we develop a new dual decomposition approach for solution finding, whose implementation is simpler essentially in comparison with the methods from [8, 9]. We present results of computational experiments which confirm the applicability of the new method.

2 A single commodity market equilibrium model

For the sake of clarity of exposition, we first describe a simple market equilibrium model, which was suggested in [6, 7]. The model involves a finite number of traders and buyers of a homogeneous commodity, their index sets will be denoted by I and J, respectively. For each $i \in I$, the i-th trader chooses his/her offer volume $x_i \in [0, a_i]$ and has price function g_i. Similarly, for each $j \in J$, the j-th buyer chooses his/her bid volume $y_j \in [0, b_j]$ and has price function h_j. Then we can define the feasible set of volumes

$$D = \left\{ (x,y) \;\middle|\; \begin{array}{c} \sum\limits_{i\in I} x_i = \sum\limits_{j\in J} y_j; \\ 0 \le x_i \le a_i, i \in I, \\ 0 \le y_j \le b_j, j \in J \end{array} \right\}, \qquad (1)$$

where $x = (x_i)_{i\in I}, y = (y_j)_{j\in J}$. We suppose that the prices may in principle depend on offer/bid volumes of all the participants, i.e. $g_i = g_i(x, y)$ and $h_j = h_j(x, y)$. We say that a pair $(\bar{x}, \bar{y}) \in D$ constitutes an *equilibrium point* if $(\bar{x}, \bar{y}) \in D$ and there exists a number $\bar{\lambda}$ such that

$$g_i(\bar{x},\bar{y}) \begin{cases} \ge \bar{\lambda} & \text{if } \bar{x}_i = 0, \\ = \bar{\lambda} & \text{if } \bar{x}_i \in (0, a_i), \\ \le \bar{\lambda} & \text{if } \bar{x}_i = a_i, \end{cases} \qquad (2)$$
for $i \in I$;

and

$$h_j(\bar{x},\bar{y}) \begin{cases} \le \bar{\lambda} & \text{if } \bar{y}_j = 0, \\ = \bar{\lambda} & \text{if } \bar{y}_j \in (0, b_j), \\ \ge \bar{\lambda} & \text{if } \bar{y}_j = b_j, \end{cases} \qquad (3)$$
for $j \in J$.

Observe that $\bar{\lambda}$ is a market clearing price, which equilibrates the market. In fact, the minimal offer (bid) volumes correspond to traders (buyers) whose prices are greater (less) than $\bar{\lambda}$, and the maximal offer (bid) volumes correspond to traders (buyers) whose prices are less (greater) than $\bar{\lambda}$. The prices of other participants are equal to $\bar{\lambda}$ and their volumes may be arbitrary within their capacity bounds, but should be subordinated to the balance equation.

In [6] (see also [7, 10]), the following basic relation between the equilibrium problem (1)–(3) and a variational inequality (VI, for short) was established.

Proposition 1 (a) If $(\bar{x}, \bar{y}, \bar{\lambda})$ satisfies (2)–(3) and $(\bar{x}, \bar{y}) \in D$, then $(\bar{x}, \bar{y})$ solves VI: Find $(\bar{x}, \bar{y}) \in D$ such that

$$\begin{array}{l} \sum\limits_{i\in I} g_i(\bar{x},\bar{y})(x_i - \bar{x}_i) - \\ \sum\limits_{j\in J} h_j(\bar{x},\bar{y})(y_j - \bar{y}_j) \ge 0 \end{array} \quad \forall (x,y) \in D. \qquad (4)$$

(b) If a pair $(\bar{x}, \bar{y}) \in D$ solves VI (4), then there exists $\bar{\lambda}$ such that $(\bar{x}, \bar{y}, \bar{\lambda})$ satisfies (2)–(3).

Therefore, we can apply various results from the theory of VIs or more general equilibrium problems (see, e.g., [7]) for its investigation and solution.

However, we feel that the model is essentially incomplete without the indication of an implementation mechanism for attaining the equilibrium point defined above, which is clearly attributed to a suitable information exchange scheme.

In [6, 7], the auction market mechanism was described, where all the traders and buyers submit their offers and bids (prices and capacities) to an auction manager within a fixed time period (session). After closing the session, the manager determines the cutting price and reports it to the participants, which also yields all the actual commodity volumes.

In the potential case where prices are partial derivatives of some differentiable function f, i.e.

$$\begin{aligned} g_i(x,y) &= \frac{\partial f(x,y)}{\partial x_i}, \ i \in I; \quad \text{and} \\ h_j(x,y) &= -\frac{\partial f(x,y)}{\partial y_j}, \ j \in J; \end{aligned}$$

VI (4) is rewritten as follows:

$$\langle \nabla f(\bar{x},\bar{y}), (x,y) - (\bar{x},\bar{y}) \rangle \ge 0 \quad \forall (x,y) \in D;$$

and it yields the optimality condition for the optimization problem:

$$\min_{(x,y)\in D} \to f(x,y). \qquad (5)$$

This is the case if the price functions are separable, i.e. $g_i(x, y) = g_i(x_i)$ for each $i \in I$ and $h_j(x, y) = h_j(y_j)$ for each $j \in J$. Then, $g_i(x_i) = \mu_i'(x_i)$ and

$h_j(y_j) = \eta_j(y_j)$ where μ_i and η_j are treated as utility functions of the participants and

$$f(x,y) = \sum_{i\in I} \mu_i(x_i) - \sum_{j\in J} \eta_j(y_j) \tag{6}$$

gives the total profit of the system; see, e.g., [11]. Note that problems (4) and (5) are equivalent if the function f is convex. Problem (5)–(6) can be solved within the centralized planning scheme, where some upper level unit is able to receive the necessary information for this optimal allocation of network resources.

Moreover, it was shown in [11] that the same equilibrium solution of (4) can be attained by a completely decentralized mechanism of bilateral transactions.

The preference of all these mechanisms related to equilibrium problem (1)–(3) is that they admit implementation with minimal information requirements, which is very significant for telecommunication network applications. In fact, we can treat (1)–(3) as a telecommunication network resource allocation problem including several providers (traders) and many users (buyers) with their maximal capacity values a_i and b_j. The price functions of providers must cover their implementation costs, whereas users indicate their willingness to pay for the service in these functions. In such a way, (1)–(3) yields a solution of this problem, which can be found within different mechanisms.

We however have to describe rather rapid and simple iterative methods which can be applied for calculation of this solution. We give several illustrative examples in the next section.

3 Simple models and methods

First we take the previous two-side model (1)–(3) in the case where all the prices are fixed, i.e., $g_i(x,y) = \alpha_i$ for each $i \in I$ and $h_j(x,y) = \beta_j$ for each $j \in J$. Then (5)–(6) becomes a linear programming (LP) problem since

$$f(x,y) = \sum_{i\in I} \alpha_i x_i - \sum_{j\in J} \beta_j y_j.$$

It follows that one can find very easily an exact solution of this problem in a finite number of iterations by a simple ordering algorithm. One should rearrange the sellers indices such that $i < j$ implies $\alpha_i \le \alpha_j$ and rearrange the buyers indices such that $i < j$ implies $\beta_i \ge \beta_j$. Then one finds any intersection point for staircase supply (ascending) and demand (descending) lines, which gives the desired clearing price; see Figure 1.

Figure 1: The ordering method in the case of fixed prices.

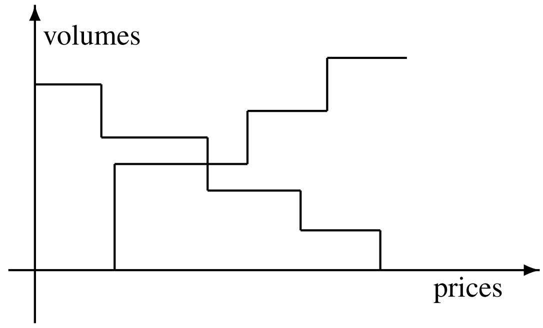
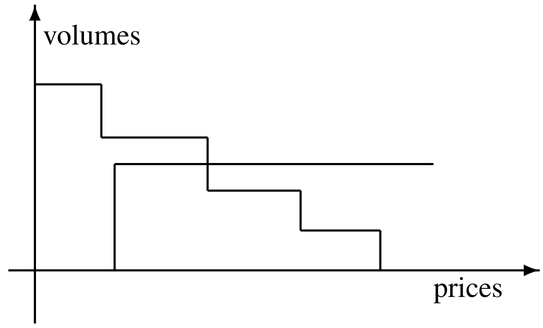


Figure 2: The ordering method for the case of one trader.



The problem becomes simpler in case of only one trader. As above, we suppose that all the prices are fixed, i.e., $g_1(x,y) = \alpha$ and $h_j(x,y) = \beta_j$ for each $j \in J$. Then both (4) and (5)–(6) coincide with the LP problem

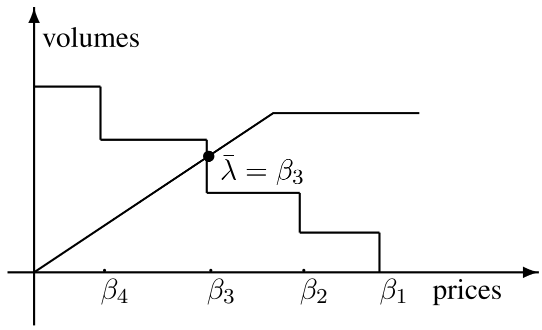
$$\min_{(x,y)\in D} \to \alpha x - \sum_{j\in J} \beta_j y_j. \tag{7}$$

where

$$D = \left\{ (x,y) \;\middle|\; \begin{array}{c} x = \sum_{j\in J} y_j; \\ 0 \le x_i \le a, \\ 0 \le y_j \le b_j, j \in J \end{array} \right\}, \tag{8}$$

If α is the network expense for a resource unit, then (7)–(8) is the network manager problem of the provider profit maximization. That is, the solution may be found within the centralized planning scheme. Again, it can be calculated very easily by a simple ordering algorithm. Here, we should re-arrange only buyers' prices to be non-increasing and then find easily an intersection point of the staircase-wise common demand and supply lines; see Figure 2.

Figure 3: The case of one trader with linear price.



If the network expense of some quantity x is determined by a non-linear function $f(x)$, the network manager problem becomes

$$\min_{(x,y)\in D} \to f(x) - \sum_{j\in J} \beta_j y_j. \tag{9}$$

where D is defined in (8). If the function f is convex and smooth, then (9)–(8) becomes equivalent to VI: Find $(x, y) \in D$ such that

$$f'(\bar{x})(x-\bar{x}) - \sum_{j\in J} \beta_j (y_j - \bar{y}_j) \geq 0 \quad \forall (x,y) \in D. \tag{10}$$

cf. (4). Again, we can find its solution by using the above ordering of buyers' prices. Similar models with a solution algorithm were considered in [12]. The illustration for the case of the linear marginal cost function $f'(x)$ is given in Figure 3. Therefore, all these problems admit very efficient and simple solution methods.

4 Multi-zonal network problem

Let us consider a more general model where a telecommunication network with nodes attributed to users (consumers) is divided into several zones (clusters). The problem of a manager of the network is to find the optimal allocation of a limited homogeneous network resource among the zones. That is, the optimal shares should maximize the value of the total profit containing the total income received from consumers' fees and negative resource implementation costs.

Let us use the following notation:

- n is the number of zones;
- J_k is the index set of users (currently) located in zone k ($k = 1, \dots, n$);
- B is the total resource supply (the total bandwidth) for the system (network);
- x_k is an unknown quantity of the resource allotted to zone k with the upper bound a_k and $f_k(x_k)$ is the cost of implementation of this quantity of the resource for zone k ($k = 1, \dots, n$);
- y_j is the resource amount received by user j with the upper bound b_j and $\varphi_j(y_j)$ is the charge value paid by user j for the resource value y_j.

The problem of the network manager can be written as follows:

$$\max \to \mu(x,y) = \sum_{k=1}^{n} \left[\sum_{j\in J_k} \varphi_j(y_j) - f_k(x_k) \right], \tag{11}$$

subject to

$$\sum_{j\in J_k} y_j = x_k, \ k = 1, \dots, n; \tag{12}$$

$$0 \leq y_j \leq b_j, \ j \in J_k, \ k = 1, \dots, n; \tag{13}$$

$$\sum_{k=1}^{n} x_k \leq B; \tag{14}$$

$$0 \leq x_k \leq a_k, \ k = 1, \dots, n. \tag{15}$$

That is, (12) provides the balance for demand and supply in each zone, (13) and (15) are capacity constraints for users and network supply values in each zone, respectively, and (14) gives the upper bound for the total resource supply. The goal of the network manager is to maximize the total network profit subject to all these constraints.

In what follows we assume that there exists at least one feasible point satisfying conditions (12)–(15), each function $f_k(x_k)$ is convex and differentiable, and all the functions $\varphi_j(y_j)$ are affine, i.e.

$$\varphi_j(y_j) = \beta_j y_j + \gamma_j, \ \beta_j > 0, \ j \in J_k, \ k = 1, \dots, n. \tag{16}$$

This means that the prices (marginal utilities) β_j of the users are fixed, but the manager can vary the prices depending on volumes, so that each zonal price is a non-increasing function.

5 Dual solution method

Under the basic assumptions of the previous section, (11)–(15) is a differentiable convex optimization problem, which has a solution since its feasible set is bounded. Hence it can be found by a great number of iterative methods; see e.g. [13, 14]. However, the problem of selection of an efficient decomposition method here is not trivial task since problem (11)–(15) has $n+1$ functional constraints (12) and (14) and many box type ones. For instance, we can utilize the standard duality approach and define the Lagrangian function with respect to all the functional constraints:

$$\begin{aligned}\Lambda(x,y,u,v) &= \mu(x,y) - u\left(\sum_{k=1}^{n} x_k - B\right)\\ &\quad - \sum_{k=1}^{n} v_k\left(\sum_{j\in J_k} y_j - x_k\right)\end{aligned}$$

Then we can write the dual problem:

$$\min_{u\geq 0, v\in R^n} \to \theta(u,v), \tag{17}$$

where

$$\theta(u,v) = \sup_{(x,y)\,\in\, H} \Lambda(x,y,u,v)$$

and

$$H = \left\{ (x,y) \;\middle|\; \begin{array}{c} 0 \leq y_j \leq b_j,\ j\in J_k, \\ 0\leq x_k \leq a_k,\ k=1,\dots,n \end{array} \right\}.$$

By duality (see e.g. [13, 14]), problems (11)–(15) and (17) have the same optimal value. Problem (17) has simple constraints, calculation of the value of the cost function $\theta(u,v)$ is rather simple since it reduces to several independent one-dimensional optimization problems, moreover, this function is convex. However, $\theta(u,v)$ is non-smooth in general, hence we replace here problem (11)–(15) with a non-smooth convex optimization problem (17) in $n+1$ dual variables, whose solution may cause certain difficulties. For this reason, we intend to apply a special dual method, which takes into account peculiarities of this problem and does not require hard implementation procedures.

Let us now define the other Lagrange function of problem (11)–(15) as follows:

$$L(x,y,\lambda) = \mu(x,y) - \lambda\left(\sum_{k=1}^{n} x_k - B\right),$$

i.e. we insert only the term corresponding to the the upper bound constraint for the total resource supply (14) with the Lagrangian multiplier λ. At the same time, we keep the zonal balance constraints (12) as well as the box capacity constraints (13) and (15).

Hence, we can write the one-dimensional dual problem:

$$\min_{\lambda\geq 0} \to \psi(\lambda), \tag{18}$$

where

$$\begin{aligned}\psi(\lambda) &= \sup_{(x,y)\,\in\, W} L(x,y,\lambda)\\ &= \sup_{(x,y)\,\in\, W} \sum_{k=1}^{n}\left[\sum_{j\in J_k}\varphi_j(y_j) - f_k(x_k) - \lambda x_k\right]\\ &\quad +\lambda B;\end{aligned}$$

and

$$W = \left\{ (x,y) \;\middle|\; \begin{array}{c} \sum_{j\in J_k} y_j = x_k, \\ 0 \leq y_j \leq b_j,\ j\in J_k, \\ 0\leq x_k \leq a_k,\ k=1,\dots,n \end{array} \right\}.$$

By duality (see e.g. [13, 14]), problems (11)–(15) and (18) also have the same optimal value. However, solution of (18) can be found by one of well-known one-dimensional optimization algorithms based on calculation of values of $\psi(\lambda)$. We now discuss this problem in more detail. The main element in calculation of $\psi(\lambda)$ is a solution of the problem:

$$\max \to \sum_{k=1}^{n}\left[\sum_{j\in J_k}\varphi_j(y_j) - f_k(x_k) - \lambda x_k\right] \tag{19}$$

subject to

$$\begin{aligned}&\sum_{j\in J_k} y_j = x_k,\ 0\leq y_j\leq b_j,\ j\in J_k,\\ &\qquad 0\leq x_k\leq a_k,\ k=1,\dots,n.\end{aligned}$$

However, this problem decomposes into n independent zonal convex programming problems

$$\max \to \left[\sum_{j\in J_k}\varphi_j(y_j) - f_k(x_k) - \lambda x_k\right] \tag{20}$$

subject to

$$\begin{aligned}&\sum_{j\in J_k} y_j = x_k,\ 0\leq y_j\leq b_j,\ j\in J_k,\\ &0\leq x_k\leq a_k,\end{aligned}$$

for $k=1,\dots,n$; cf. (9). Hence, we have to suggest a simple and efficient algorithm for the basic problem (20).

Set $y(k) = (y_j)_{j\in J_k}$,

$$W_k = \left\{ (x_k, y(k)) \;\middle|\; \begin{array}{l} \sum_{j\in J_k} y_j = x_k, \\ 0 \leq y_j \leq b_j,\ j\in J_k, \\ 0\leq x_k \leq a_k \end{array} \right\},$$

then

$$W = \prod_{k=1}^{n} W_k.$$

The necessary and sufficient optimality condition for problem (20) in view of (16) is written in the form of VI: find $(\bar{x}_k, \bar{y}(k)) \in W_k$ such that

$$\begin{aligned}&(f'_k(\bar{x}_k)+\lambda)(x_k-\bar{x}_k) - \sum_{j\in J_k}\beta_j(y_j-\bar{y}_j)\geq 0\\ &\forall (x_k, y(k))\in W_k.\end{aligned} \tag{21}$$

This is nothing but the two-sided market equilibrium problem with one trader and several buyers; see (4) and (10). Due to Proposition 1, it is equivalent to the problem of finding a feasible vector $(\bar{x}_k, \bar{y}(k)) \in W_k$ and a cutting price $\bar{p}_k$ such that

$$f'_k(\bar{x}_k) + \lambda \begin{cases} \geq \bar{p}_k & \text{if} \quad \bar{x}_k = 0, \\ = \bar{p}_k & \text{if} \quad \bar{x}_k \in (0, a_k), \\ \leq \bar{p}_k & \text{if} \quad \bar{x}_k = a_k, \end{cases} \tag{22}$$

and

$$\beta_j \begin{cases} \leq \bar{p}_k & \text{if} \quad \bar{y}_j = 0, \\ = \bar{p}_k & \text{if} \quad \bar{y}_j \in (0, b_j), \\ \geq \bar{p}_k & \text{if} \quad \bar{y}_j = b_j, \end{cases} \quad j \in J_k; \tag{23}$$

cf. (1)–(3). As indicated in Section 2, since buyers' prices are fixed, we can re-arrange them to be non-increasing and then find easily an intersection point of the staircase-wise inverse common demand and offer price $f'_k(x_k) + \lambda$ lines; see also Figure 3. Therefore, an exact solution of problem (21), (22)–(23), or (20) (hence (19)) can be found explicitly by simple ordering type algorithms, although (20) contain a non-linear function in general. In other words, calculation of values of $\psi(\lambda)$ can be accomplished by several independent simple ordering type algorithms. Notice that the re-arrangement of bid prices β_j in each zone should be made only one time that reduces the computational expenses essentially in comparison with the general duality approach. So, having the optimal value λ^* of problem (18), we can find a solution of problem (11)–(15) by solving problem (19) with $\lambda = \lambda^*$, i.e. it is accomplished within the main calculation process for (18).

6 Adjustment for the case of moving nodes

In the above model it was assumed that users locations were fixed. We now intend to suggest some adjustments of the above model to networks with more complex and non-stationary behavior of users (nodes), which is typical for various modern wireless telecommunication systems; see e.g. [15, 2].

We consider the above problem of the network manager for some time slot. In this case we need some additional information about the behavior of users (nodes). It was suggested by I. Konnov (see e.g. [16]) to treat each moving node in a wireless network as a separate Markovian chain.

In order to create such a model, we determine a suitable grid $\mathcal{G}$ covering the domain of the network so that $\mathcal{G}_k$ denotes the index set of all the cells belonging to zone k. Next, we consider the discrete time model and suppose that, given a user (node) j, we can determine the starting probability vector $\pi^{j,(0)}$, whose components $\pi^{j,(0)}_\sigma$ give its probabilities to be in cell $\sigma \in \mathcal{G}$ by time slot (stage) 1, and the probability $\tilde{\pi}^j_{\sigma\tau}$ (for the simplicity of exposition, it is supposed to be independent of time) of the one stage transition $\sigma \to \tau$ for each pair $\sigma, \tau \in \mathcal{G}$. Knowing the starting and transition probability vectors for each node j, we can calculate its probability $\pi^{j,(t-1)}_\sigma$ to be in cell $\sigma \in \mathcal{G}$ by a selected slot t via the standard Markovian chain technique (see e.g. [17]). Afterwards we calculate the value

$$\tilde{p}^{j,(t-1)}_k = \sum_{\sigma \in \mathcal{G}_k} \pi^{j,(t-1)}_\sigma$$

for each zone k and assign user j to zone l where the probability $\tilde{p}^{j,(t-1)}_l$ is maximal, i.e. then $j \in J_l$.

Therefore, we can solve the same problem (11)–(15) with this assignment and obtain the desired resource allocation for time slot t in the case of moving nodes.

Similarly, if

$$\lim_{m \to \infty} (\Pi^j)^m = \bar{\Pi}^j \tag{24}$$

for each probability matrix $\tilde{\Pi}^j = (\tilde{\pi}^j_{\sigma\tau})_{\{\sigma,\tau \in \mathcal{G}\}}$, behavior of each user is stable and we can evaluate the optimal resource allocation for a long-time stationary period by calculation of the limit probabilities

$$\bar{\pi}^j_\tau = \sum_{\sigma \in \mathcal{G}} \pi^{j,(0)}_\sigma \bar{\pi}^j_{\sigma\tau} \text{ for } \tau \in \mathcal{G}$$

and set

$$\bar{p}^j_k = \sum_{\sigma \in \mathcal{G}_k} \bar{\pi}^j_\sigma$$

for each j. Then we can assign user j to zone l where the probability $\bar{p}^j_l$ is maximal, i.e. then $j \in J_l$ and solve problem (11)–(15) with this assignment and obtain the long-time resource allocation strategy.

However, this approach can not be used if the limit in (24) does not exist. Then we can apply the statistical approach and calculate the probabilities online as it was suggested in [18]. After t time slots we can determine the value

$$p^{j,t}_k = s_{j,k}(t)/t,$$

for each user j and for each zone k, where $s_{j,k}(t)$ denotes the number of time slots when user j was in zone k. It is treated as some approximation of the probability of user j to be in zone k. We set

$$\bar{p}^j_k = p^{j,t}_k$$

if

$$\left\{ \sum_{j \in J} \sum_{k=1}^{n} (p_k^{j,t} - p_k^{j,t-1})^2 \right\}^{1/2} \leq \delta$$

for $\delta > 0$ small enough, where J denotes the index set of all the users. Then we utilize the values $\bar{p}_k^j$ as above in order to assign each user j to some zone l. Solution of problem (11)–(15) with this assignment gives the long-time resource allocation strategy.

7 Numerical experiments

In order to evaluate efficiency of the new method we made several series of computational experiments. Since the case of moving nodes yields the same mathematical model (11)–(15) we restricted ourselves with the fixed case.

We utilized the golden section method for solving the single-dimensional optimization problem (18). The programs were coded in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.

The initial intervals for choosing the dual variable λ were taken as [0,1000]. Values of a_k were chosen by trigonometric functions in $[1, 11]$, values of b_j were chosen by trigonometric functions in $[1, 2]$. The functions $f_k(x_k)$ were chosen to be convex quadratic, all the coefficients of $f_k(x_k)$ and $\varphi_j(y_j)$ were chosen with the help of trigonometric functions. The number of zones was varied from 5 to 105, the number of users was varied from 210 to 1010. Users were distributed in zones either uniformly or according to the normal distribution. The processor time and number of iterations, which gave an approximate solution of problem (18) within the same accuracy, were not significantly different for these two cases of distributions. We made calculations with 1000 test examples for each set of parameters, their average values are indicated in each row of the tables.

Further we report the results of tests, which include the time and number of iterations needed to find a solution of problem (18) within some accuracies. Let ε and δ denote the desired accuracy of finding an approximate solution of problem (18). Let M denote the total number of users, N_ε the number of upper iterations in λ, T_ε the total processor time in seconds. The results of computations are given in Tables 1–3. In Table 1, we vary the accuracy ε, in Tables 2 and 3 we vary the total number of users and the number of zones, respectively. From the results we can conclude that the performance of the new method is satisfactory for applications.

Table 1: Results of testing with $M = 510$, $n = 70$, $\delta = 10^{-2}$

ε	N_ε	T_ε
10^{-1}	20	0.0003
10^{-2}	24	0.0004
10^{-3}	29	0.0008
10^{-4}	34	0.0007

Table 2: Results of testing with $n = 70$, $\varepsilon = 10^{-2}$, $\delta = 10^{-2}$.

M	N_ε	T_ε
210	24	0.0001
310	24	0.0003
410	24	0.0004
510	24	0.0004
610	24	0.0008
710	24	0.0012
810	24	0.0011
910	24	0.0014
1010	24	0.0018

Table 3: Results of testing with $M = 510$, $\varepsilon = 10^{-2}$, $\delta = 10^{-2}$.

n	N_ε	T_ε
5	24	0.0001
15	24	0.0002
25	24	0.0001
35	24	0.0001
45	24	0.0002
55	24	0.0002
65	24	0.0002
75	24	0.0002
85	24	0.0003
95	24	0.0004
105	24	0.0002

8 Conclusions

In this work, we showed that market equilibrium models suggested in [6, 7] can serve as a basis for resource allocation problems in telecommunication networks and enable us to apply simple and efficient solution methods. We also considered a problem of managing limited resources in a multi-zonal wireless telecommunication network and gave its constrained convex optimization problem formulation. We proposed a new dual decomposition method, which reduces the initial problem to a sequence of simple zonal convex optimization problems. Each of these problem corresponds to a two-side market equilibrium model and can be solved by efficient ordering type algorithms despite the nonlinear cost network functions. The results of the numerical experiments confirmed the rapid convergence of these methods. We also suggested ways to adjust the problem to the case of moving nodes.

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Acknowledgements: The research was supported by the RFBR grant, project No. 16-01-00109a and by grant No. 297689 from Academy of Finland.