Weight Optimization for Adaptive Antenna Arrays Using LMS and SMI Algorithms

Mohammed Al-Sadoon[1], Raed A. Abd-Alhameed[1], I.T.E.Elfergani [2] ,J. M Noras[1] , Jonathan Rodriguez[2] and S. M. R. Jones[1]
[1]School of Electrical Engineering and Computer Science, University of Bradford, Bradford, BD7 1DP, UK
[2]Instituto de Telecomunicações – Aveiro, Portugal

m.a.g.al-sadoon@bradford.ac.uk, r.a.a.abd@bradford.ac.uk, j.m.noras@bradford.ac.uk
i.t.e.elfergani@av.it.pt, jonathan@av.it.pt

Abstract: - Wireless communication systems often suffer problems from multipath and interfering signals. This paper compares two types of adaptive beamforming algorithms for optimising and setting the weights of smart antenna systems, namely the Least Mean Square algorithm (LMS) and Sample Matrix Inversion (SIM). The direction of the main beam of the antenna system is adjusted to enhance the desired signal, while interfering signals are mitigated by having nulls pointed in their directions. The mathematical formulations and necessary conditions for the above algorithms are derived and verified by numerical examples, with overall behaviour obtained by MATLAB simulation. Performance evaluations are made for different cases of arrival angles, and in conclusion, the advantages and disadvantages of each algorithm are presented.

Key-Words: - Adaptive beam forming, Antenna array, Least Mean Square algorithm (LMS), Sample Matrix Inversion (SIM), Smart Antennas, Wireless communications.

1 Introduction

A smart antenna system is an array with a digital signal processing ability to receive and transmit in an adaptive manner [1-2]. Although it may seem that such smart systems are novel, the fundamentals are not new [3]; in the 1970s and 1980s two special issues of the *IEEE Transactions on Antennas and Propagation* were devoted to smart antenna arrays and associated signal processing algorithms [4-5]. The use of smart antennas in telecommunication systems initially attracted attention in military applications, finding usage for several years in electronic warfare (EWF) in countermeasures to electronic jamming. In military radar systems, similar approaches were already used through the conventional Bartlett beamformer [3]. However, today's advanced powerful low-cost digital signal processors, Application Specific Integrated Circuits (ASICs), and general purpose processors (DSP), in addition to innovative adaptive signal processing methods, now make commercial smart antenna systems increasingly viable.

Smart antenna patterns are controlled via algorithms based upon certain criteria: maximizing the signal-to interference ratio plus noise (SINR), minimizing the variance (MV), minimizing the mean square error (MSE), steering toward a useful source, nulling interfering signals, or tracking moving users [1, 2, 6, 7, 8]. Among proven benefits of smart antenna use in cellular system base stations are increased spectrum efficiency and channel capacity arising from extended range coverage and reduction of co-channel interference [1, 9, 10]. Furthermore, reduction of multipath fading can be achieved. Perhaps two of the most important advantages are the ability to minimize co-channel interference, by isolating signals originating from different directions, and the ability to avert radiation of signals in directions where another mobile user or base station is known already. This isolation of signals with different angles of arrival or spatial signatures is the essential property exploited by antenna arrays [11, 12].

This paper is organized as follows: the principles of adaptive beamforming are given in section 2, while the necessary equations and conditions used to realize smart antenna systems are given in section 3. The basic theory of both LMS and SMI algorithms is presented in sections 4 and 5 respectively. Section 6 discusses the simulation results and the performance of the proposed system with each algorithm. Finally, section 7 gives a concluding summary and comparison of performance.

2 Adaptive Beam Forming

Adaptive Beam Forming is a mechanism which uses an array of antennas to accomplish maximum transmission and reception in a particular direction on the basis of directional estimations, while signals, even of the same frequency, from other directions are rejected [13]. This can be accomplished by varying the weights of each antenna element within the array. Although signals from different transmitters may use the same frequency, they will arrive at different angles. This spatial separation is utilized to isolate the desired signal from undesired signals. Thomas Biedka in his work (2001) provided a framework for the analysis and development of blind adaptive beamforming algorithms: by using adaptive antenna arrays, the control system has complete flexibility and determines how the gains of the arrays are adjusted [18]. In this method, gains are adjusted such that the control system can maximize the gain from a desired source while attenuating the signal from an interfering user, as shown in Figure 1 [6]. In 2007, Shubair et al. explained how to apply the least mean square (LMS) and multiple signal classification (MUSIC) methods, giving a practical design of a smart antenna system employing direction-of-arrival estimation and adaptive beamforming technique [16]. In their work, Rani et al. (2009) examined the use of beam forming for adaptive antennas, with adaptive methods to determine the weights in WCMDA mobile communication [14]. Bahri and Bendimerad (2009) suggested a downlink multi-input multi-output multiple carrier CDMA system which integrated the LMS method for adaptive beamforming purposes [15]. In [17], Susmita Das (2009) describes and compares various reference-signal based methods in addition to blind adaptive methods.

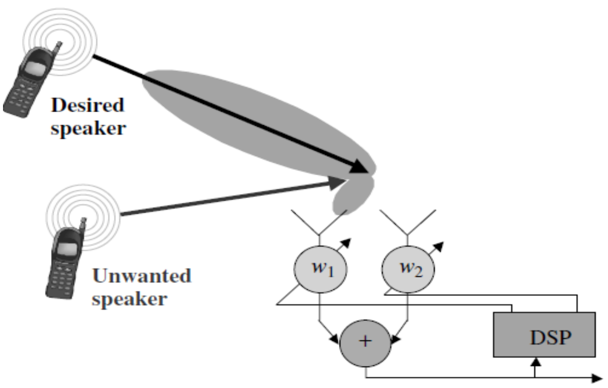

Fig1: Adaptive antenna array.

3 Mathematical Model of Adaptive Antenna Array

In general, adaptive beam forming is an effective technique because it makes use of a digital algorithm which dynamically optimizes the array pattern according to the changing electromagnetic environment [6]. Therefore, adaptive arrays maximize the signal-to-interference-plus-noise ratio (SINR) and not just the signal-to noise ratio (SNR). This dynamic adaptation of the antenna array response directs focused beams to specific users and constitutes a new mechanism for multiuser access to the base station [1]. A generic adaptive beam former is shown in Fig.2, where the weight vector w is calculated using the signal vector $x(k)$ received by multiple antennas

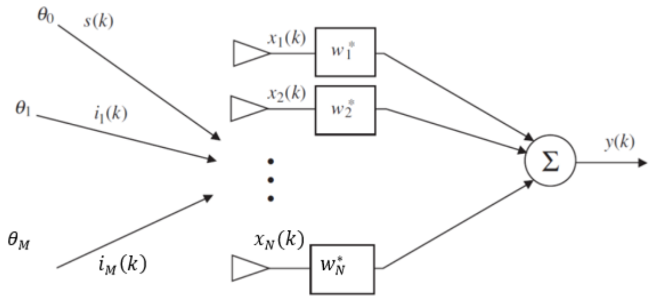

Fig.2: Traditional beam former array.

Let one desired signal be arriving from angle θ_0 with M interferers at angles θ_1, ... , θ_M as shown in Fig.2: the signals and interferers are received by an array of elements with N potential weights. Each received signal also includes additive Gaussian noise. Time is represented by the k time samples.
The weighted array output can be given [8]:

$$y(k) = \bar{w}^H . \bar{x}(k) \quad (1)$$

with $\bar{w}$, the array weights, given by:

$$\bar{w} = [w_1 \, w_2 \, \dots\dots \, w_N]^T \quad (2)$$

$$\bar{x}(k) = \bar{a}_0 s(k) + [\,\bar{a}_1 \, \bar{a}_2 \dots \bar{a}_M].\begin{bmatrix} i_1(k) \\ i_2(k) \\ . \\ . \\ i_M(k) \end{bmatrix} + \bar{n}(k) \quad (3)$$

$$\bar{x}(k) = \bar{x}_s(k) + \bar{x}_i(k) + \bar{n}(k) \quad (4)$$

where
$\bar{x}_s(k)$ = desired signal vector
$\bar{x}_i(k)$ = interfering signals vector, and
$\bar{n}(k)$ = zero mean Gaussian noise for each channel.

$\bar{a}_i$ is the N-element array steering vector for each θ_i, direction of arrival.

We can rewrite Equation (1) in the expanded notation of Equation (4) as:

$y(k) = \bar{w}^{\,H} \cdot [\bar{x}_s(k) + \bar{x}_i(k) + \bar{n}(k)]$, or

$$y(k) = \bar{w}^{H} \cdot [\bar{x}_s(k) + \bar{u}(k)] \quad (5)$$

where $\bar{u}(k) = \bar{x}_i(k) + \bar{n}(k)$ is the undesired signal.

The weighted array output power for the desired signal is given by [2]

$$\sigma_s^2 = E[|\,\bar{w}^H.\bar{x}_s\,|^2] = \bar{w}^H\,.\bar{R}_{ss}\,.\bar{w} \quad (6)$$

where $\bar{R}_{ss}$ is the signal correlation matrix, given by

$$\bar{R}_{ss} = E[\,\bar{x}_s\,\bar{x}_s^H\,] \quad (7)$$

The weighted array output power for undesired signals is

$$\sigma_u^2 = E[|\,\bar{w}^H.\bar{u}|^2] = \bar{w}^H.\ \bar{R}_{uu}\,.\bar{w} \quad (8)$$

$$\bar{R}_{uu} = \bar{R}_{ii} + \bar{R}_{nn} \quad (9)$$

where $\bar{R}_{ii}$ = correlation matrix for interferers, and $\bar{R}_{nn}$ = correlation matrix for noise.

Taking the first element as a reference, the steering vector a(θ) of signal N is given by following equation:

$$\mathrm{a}(\theta) = \begin{bmatrix} 1 \\ \exp(j\beta d\sin\theta) \\ . \\ . \\ . \\ \exp(j\beta d(N-1)\sin\theta) \end{bmatrix} \quad (10)$$

where β denotes the propagation constant, and d denotes the distance between every two adjacent elements.

Finally, the array factor can be expressed as:

$$AF = w^H\,a(\theta) \quad (11)$$

4 Least Mean Square Algorithm

One of the simplest algorithms for adaptive processing is based on Least Mean Square (LMS) error. The LMS algorithm belongs to the trained algorithm category in which a reference signal is used to update the weights at each iteration [19], so that we search for the optimal weight, which would make the array output as close as possible to the reference signal. This is the weight that minimizes the mean square error (MSE) as shown in Fig. 3. The algorithm contains three steps in each recursion:

- compute the processed signal with the current weights,
- generate the error between the processed signal and the desired signal, and
- adjust the weights using the new error information by the gradient method [6].
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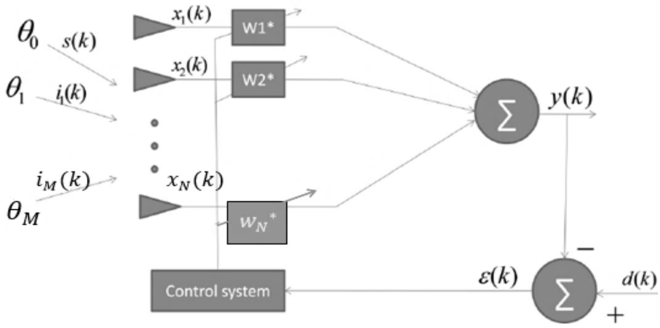

Fig.3: Block diagram of MSE adaptive system.

The error can be defined as the desired signal minus the weighted output of the array:

$$\varepsilon(k) = d(k) - \bar{w}^H\,\bar{x}(k) \quad (12)$$

The squared error is given by

$$|\varepsilon(k)|^2 = |d(k) - \bar{w}^H\,\bar{x}(k)|^2 \quad (13)$$

The cost function is defined as

$$J(\,\bar{w}\,) = D - 2\,\bar{w}^H\,\bar{r} + \bar{w}^H\,\bar{R}_{\,xx}\,\bar{w} \quad (14)$$

$$\bar{r} = E[d^*.(\bar{x}_s\,(k) + \bar{x}_i\,(k) + \bar{n})] \quad (15)$$

$$\bar{R}_{xx} = E[\bar{x}\bar{x}^H] = \bar{R}_{ss} + \bar{R}_{uu} \quad (16)$$

where $D = |d(k)|^2$ and $\bar{r} = E[d^*.\bar{x}(k)]$

The quantity in equation (14) is usually called the cost function and it is convenient to find its minimum value in order to find the optimum weights of the system. This achieved by taking the gradient of Equation (14) with respect to the weight vector $\nabla_{\bar{w}}(.)$
and equating it to zero [6]:

$$\nabla_{\bar{w}}(E[|\varepsilon|^2]) = 2\,\bar{R}_{xx}\bar{w} - 2\,\bar{r}$$

The optimum weights providing the minimum MSE can be found by simplifying the above equation as:

$$\bar{w}_{opt} = \bar{R}_{xx}^{-1}\,\bar{r} \qquad (17)$$

By using the gradient of the cost function, the LMS solution is found as [3, 6]:

$$\bar{w}(k+1) = \bar{w}(k) + \mu\,\varepsilon^*(k)\,\bar{x}(k) \qquad (18)$$

where μ is the step size parameter controlling the rate of adaptation, and

$$\varepsilon(k) = d(k) - \bar{w}^H\,x(k) \qquad (19)$$

The convergence speed of the LMS method in Equation (18) is essentially proportional to the μ value. If the step-size value is very small, the convergence rate will become slow, the over-damped case. If the convergence is too slow for the varying angles of arrival, it is possible that the adaptive array would not catch the desired signal quickly enough to track the varying signal. On the other hand, if the step-size is very large, the LMS algorithm will overshoot the optimum weights, exhibiting under-damping: the attempted convergence is too fast, and this will make the weights oscillate about the optimum, so the desired signal will not be precisely tracked. Therefore, it is necessary to choose a step-size in a range that ensures convergence [8]:

$$0 \le \mu \le \frac{1}{2\,\mathrm{trace}[\,\hat{\mathrm{R}}_{\mathrm{xx}}]} \qquad (20)$$

where “trace” means sum the diagonal elements of $\hat{\mathrm{R}}_{\mathrm{xx}}$, the estimated correlation matrix.

5 Sample Matrix Inversion [SMI] Algorithm

An alternative to the relatively slowly converging LMS scheme is the Sample Matrix Inversion (SMI) algorithm, also known as direct matrix inversion [20]. The sample matrix is a time-averaged estimate of the array correlation matrix using K samples. With random ergodic noise in the correlation, the time-averaged estimate will equal the actual correlation matrix. As we use a K–long block of data, this method is called a block-adaptive approach [2, 8, 21], which adapts the weights block by block. K samples of signal vector X define the $\mathrm{N} \times \mathrm{K}$ matrix :

$$\bar{X}_K(k) = \begin{bmatrix} x_1(1+kK) & x_1(2+kK) & \dots & x_1(K+kK) \\ x_2(1+kK) & x_2(2+kK) & \dots & x_2(K+kK) \\ & & \vdots & \\ x_N(1+kK) & x_N(2+kK) & \dots & x_N(K+kK) \end{bmatrix} \qquad (21)$$

where k is the block index and K is the block length. Then the estimate of the array correlation matrix is

$$\hat{R}_{xx}(k) = \frac{1}{K}\bar{X}_K(k)\,\bar{X}_K^H(k) \qquad (22)$$

and the estimate of the correlation vector is

$$\hat{r}(k) = \frac{1}{K}(d^*(k)\;\;\bar{X}_K^H(k)\;) \qquad (23)$$

In addition, the desired signal vector can be defined by

$$d(k) = [d(1+kK)d(2+kK).\,d(K+kK)] \qquad (24)$$

The SMI weights can be calculated for the *k*th block of length *K* as:

$\bar{w}_{SMI}(k) = \quad \bar{R}_{xx}^{-1}(k)\,\bar{r}(k)$, and

$$\bar{w}_{SMI}(k) = [\bar{X}_K(k)\;\;\bar{X}_K^H(k)]^{-1}d^*(k)\;\;\bar{X}_K(k) \qquad (25)$$

6 Simulation Results and Discussion

6.1 The LMS Algorithm

A uniform linear array system with N = 10 elements of spacing $\mathrm{d} = 0.5\lambda$ is adopted here. The desired signal arrives at the antenna at θ_0 and there is one interference signal at θ_1 with additive white noise. The angular range of interest is $[-90^0, 90^0]$.

Case (1): Desired signal (D) at $\theta_0 = 0^0$ and interference signal (I) at $\theta_1 = 30^0$.

Case (2): Desired signal (D) at $\theta_0 = 40^0$ and interference signal (I) at $\theta_1 = 60^0$.

Case (3): Desired signal (D) at $\theta_0 = -30^0$ and interference signal (I) at $\theta_1 = -80^0$.

To find the instantaneous weights vector of the LMS algorithm:

(1) Assume that the initial array weights are all zero.

(2) Find steering vectors for desired user (a_0) and interferer (a_1) using Equation (10); the steering vectors for the three cases are shown below respectively.

$$a_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad a_1 = \begin{bmatrix} 1.0000 + 0.0000i \\ 0.0000 + 1.0000i \\ -1.0000 + 0.0000i \\ -0.0000 - 1.0000i \\ 1.0000 - 0.0000i \\ 0.0000 + 1.0000i \\ -1.0000 + 0.0000i \\ -0.0000 - 1.0000i \\ 1.0000 - 0.0000i \\ 0.0000 + 1.0000i \end{bmatrix}$$

$$a_0 = \begin{bmatrix} 1.0000 + 0.0000i \\ -0.4337 + 0.9011i \\ -0.6238 - 0.7816i \\ 0.9748 - 0.2232i \\ -0.2217 + 0.9751i \\ -0.7825 - 0.6226i \\ 0.9004 - 0.4351i \\ 0.0015 + 1.0000i \\ -0.9017 - 0.4323i \\ 0.7806 - 0.6250i \end{bmatrix} \quad a_1 = \begin{bmatrix} 1.0000 + 0.0000i \\ -0.9127 + 0.4086i \\ 0.6661 - 0.7458i \\ -0.3033 + 0.9529i \\ -0.1125 - 0.9936i \\ 0.5087 + 0.8609i \\ -0.8161 - 0.5780i \\ 0.9810 + 0.1941i \\ -0.9747 + 0.2236i \\ 0.7982 - 0.6024i \end{bmatrix}$$

$$a_0 = \begin{bmatrix} 1.0000 + 0.0000i \\ 0.0000 - 1.0000i \\ -1.0000 - 0.0000i \\ -0.0000 + 1.0000i \\ 1.0000 + 0.0000i \\ 0.0000 - 1.0000i \\ -1.0000 - 0.0000i \\ -0.0000 + 1.0000i \\ 1.0000 + 0.0000i \\ 0.0000 - 1.0000i \end{bmatrix} \quad a_1 = \begin{bmatrix} 1.0000 + 0.0000i \\ -0.9989 - 0.0477i \\ 0.9954 + 0.0953i \\ -0.9898 - 0.1427i \\ 0.9818 + 0.1898i \\ -0.9717 - 0.2364i \\ 0.9593 + 0.2825i \\ -0.9447 - 0.3279i \\ 0.9280 + 0.3726i \\ -0.9092 - 0.4165i \end{bmatrix}$$

(3) Find the total array factor as: $X = a_0 + a_1$

(4) Find the total received signal correlation matrix as: $R_{xx} = X \times X^H$

(5) Find the suitable value of the convergence parameter (μ) using Equation (20) for the three cases:

$\mu_1 = 0.0114$, $\mu_2 = 0.0113$, $\mu_3 = 0.0112$

(6) Find the instantaneous value of the total received signal vector $x(k)$ by using Equation (3).

(7) Find the instantaneous value of the array output $y(k)$ using Equation (1).

(8) Find the instantaneous value of the error signal $\varepsilon(k)$ between the reference signal and the array output using Equation (19).

(9) Calculate the weights vector of the next iteration using Equation (18).

(10) Repeat steps (6) to (9) until iteration 100.

The resulting weights vector for the three cases are:

Case (1): w_{LMS}	Case (2): w_{LMS}	Case (3): w_{LMS}
$\begin{bmatrix} 0.0915 + 0.0101i \\ 0.0916 - 0.0102i \\ 0.1118 - 0.0101i \\ 0.1118 + 0.0102i \\ 0.0915 + 0.0101i \\ 0.0916 - 0.0102i \\ 0.1118 - 0.0101i \\ 0.1118 + 0.0102i \\ 0.0915 + 0.0101i \\ 0.0916 - 0.0102i \end{bmatrix}$	$\begin{bmatrix} 0.0904 + 0.0002i \\ -0.0343 + 0.0865i \\ -0.0698 - 0.0709i \\ 0.1014 - 0.0325i \\ -0.0210 + 0.1087i \\ -0.0844 - 0.0717i \\ 0.0995 - 0.0380i \\ -0.0101 + 0.0990i \\ -0.0808 - 0.0461i \\ 0.0705 - 0.0566i \end{bmatrix}$	$\begin{bmatrix} 0.0897 - 0.0077i \\ 0.0116 - 0.0935i \\ -0.1130 - 0.0088i \\ 0.0108 + 0.1111i \\ 0.0914 - 0.0099i \\ 0.0099 - 0.0914i \\ -0.1111 - 0.0108i \\ 0.0089 + 0.1130i \\ 0.0935 - 0.0117i \\ 0.0077 - 0.0897i \end{bmatrix}$

(11) Finally, the array factor can be found using Equation (11).

The variation of the magnitude of each weight vs. iteration number of the LMS algorithm for case 2 is shown in Fig.4. Fig.5 shows the resulting mean square error which converges to near zero after 53 iterations. Fig.6 shows how the array output acquires and tracks the desired signal after some 65 iterations.

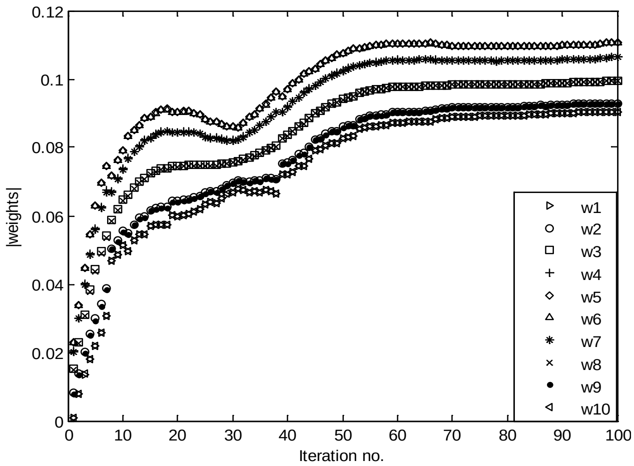

Fig.4: The variations of the magnitude of each weight of the LMS algorithm for case 2.

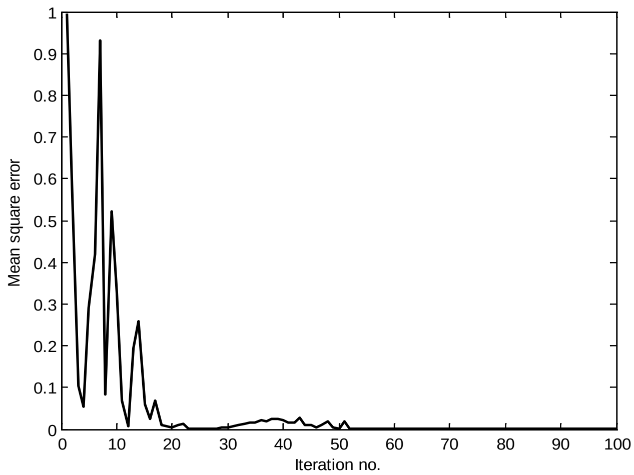

Fig.5: The MSE of the LMS algorithm.

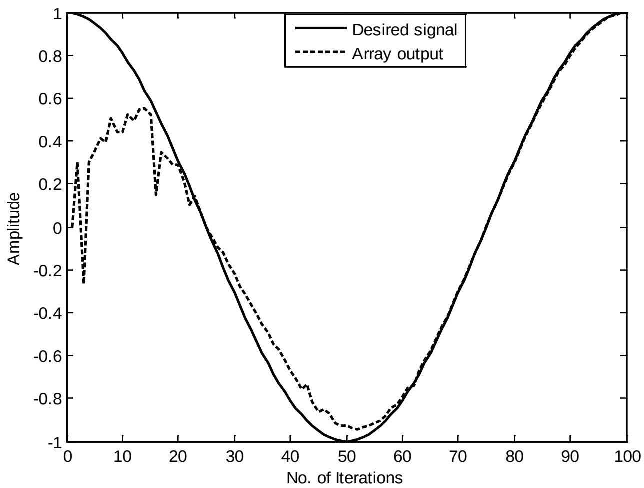

Fig.6: s(k) and y(k) of the LMS algorithm.

Finally, the resultant array factor for all three cases is shown in Fig.7, which has a peak at the desired direction and a null at the interfering direction, independent of the statistical characteristics of the arriving signals. Slow convergence limits the usefulness of this algorithm in dynamic environments where the signal must be captured quickly, also a limitation when channel conditions are rapidly changing. As shown in Fig.6, the LMS algorithm did not converge until after 65 iterations, after more than half of the duration of the signal of interest.

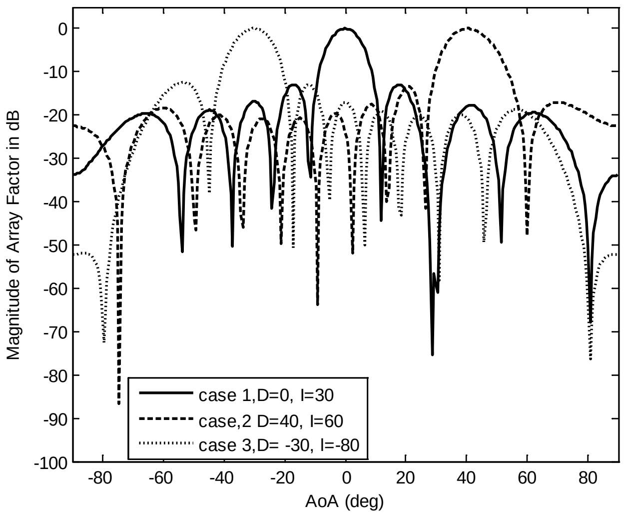

Fig.7: The normalized magnitude of the array factor of the smart antenna system using the LMS algorithm.

6.2 The SMI Algorithm

A uniform linear array system with N=10 elements array and spacing $d = 0.5\lambda$ is adopted here. The desired signal reaches the antenna at $\theta_0 = 0^0$ and one interference signal arrives at $\theta_1 = 40^0$, with additive white noise. Assume the reference signal is exactly the same as the desired signal, namely $d(k) = s(k)$. The angular range of interest is $[-90^0, 90^0]$.

Case (1): Desired signal (D) at $\theta_0 = 20^0$ and interference signal (I) at $\theta_1 = 40^0$.

Case (2): Desired signal (D) at $\theta_0 = 50^0$ and interference signal (I) at $\theta_1 = 80^0$.

Case (3): Desired signal (D) at $\theta_0 = -30^0$ and interference signal (I) at $\theta_1 = -60^0$.

To calculate the weights vector:

(1) Find steering vectors for the desired user (a_0) and interferer (a_1) using Equation (10) as:

$$a_0 = \begin{bmatrix} 1.0000 + 0.0000i \\ 0.4762 + 0.8793i \\ -0.5465 + 0.8375i \\ -0.9967 - 0.0818i \\ -0.4027 - 0.9153i \\ 0.6132 - 0.7900i \\ 0.9866 + 0.1630i \\ 0.3265 + 0.9452i \\ -0.6757 + 0.7372i \\ -0.9700 - 0.2432i \end{bmatrix} \quad a_1 = \begin{bmatrix} 1.0000 + 0.0000i \\ -0.4337 + 0.9011i \\ -0.6238 - 0.7816i \\ 0.9748 - 0.2232i \\ -0.2217 + 0.9751i \\ -0.7825 - 0.6226i \\ 0.9004 - 0.4351i \\ 0.0015 + 1.0000i \\ -0.9017 - 0.4323i \\ 0.7806 - 0.6250i \end{bmatrix}$$

$$a_0 = \begin{bmatrix} 1.0000 + 0.0000i \\ 0.4762 + 0.8793i \\ -0.5465 + 0.8375i \\ -0.9967 - 0.0818i \\ -0.4027 - 0.9153i \\ 0.6132 - 0.7900i \\ 0.9866 + 0.1630i \\ 0.3265 + 0.9452i \\ -0.6757 + 0.7372i \\ -0.9700 - 0.2432i \end{bmatrix} \quad a_1 = \begin{bmatrix} 1.0000 + 0.0000i \\ -0.4337 + 0.9011i \\ -0.6238 - 0.7816i \\ 0.9748 - 0.2232i \\ -0.2217 + 0.9751i \\ -0.7825 - 0.6226i \\ 0.9004 - 0.4351i \\ 0.0015 + 1.0000i \\ -0.9017 - 0.4323i \\ 0.7806 - 0.6250i \end{bmatrix}$$

$$a_0 = \begin{bmatrix} 1.0000 + 0.0000i \\ 0.0000 - 1.0000i \\ -1.0000 - 0.0000i \\ -0.0000 + 1.0000i \\ 1.0000 + 0.0000i \\ 0.0000 - 1.0000i \\ -1.0000 - 0.0000i \\ -0.0000 + 1.0000i \\ 1.0000 + 0.0000i \\ 0.0000 - 1.0000i \end{bmatrix} \quad a_1 = \begin{bmatrix} 1.0000 + 0.0000i \\ -0.9127 - 0.4086i \\ 0.6661 + 0.7458i \\ -0.3033 - 0.9529i \\ -0.1125 + 0.9936i \\ 0.5087 - 0.8609i \\ -0.8161 + 0.5780i \\ 0.9810 - 0.1941i \\ -0.9747 - 0.2236i \\ 0.7982 + 0.6024i \end{bmatrix}$$

(2) Find total received signal $X(k)$ and noise using Equation (21).

(3) Find the estimated signal correlation matrix$(\hat{R}_{xx})$ using Equation (22).

(4) Find the estimated signal correlation vector $(\hat{r})$ using Equation (23).

(5) Calculate the inverse of the signal correlation matrix.

(6) The instantaneous weights vector can be found using Equation (25).

(7) Repeat steps (2) to (6) until iteration 100. These yields:

Case (1): w_{SMI}	Case (2): w_{SMI}	Case (3): w_{SMI}
$\begin{bmatrix} 0.0867 + 0.0224i \\ 0.0386 + 0.0677i \\ -0.0312 + 0.0899i \\ -0.1196 + 0.0140i \\ -0.0570 - 0.1210i \\ 0.0959 + 0.0460i \\ 0.0122 + 0.0815i \\ -0.0466 + 0.0666i \\ -0.1044 + 0.0007i \\ -0.1044 + 0.0007i \end{bmatrix}$	$\begin{bmatrix} 0.1047 - 0.0004i \\ -0.0790 + 0.0674i \\ 0.0152 - 0.1000i \\ 0.0539 + 0.0814i \\ -0.0925 - 0.0214i \\ 0.0808 - 0.0489i \\ -0.0249 + 0.0932i \\ -0.0464 - 0.0883i \\ 0.0962 + 0.0368i \\ -0.0987 + 0.0349i \end{bmatrix}$	$\begin{bmatrix} 0.1025 - 0.0087i \\ -0.0053 - 0.0938i \\ -0.0933 - 0.0041i \\ -0.0084 + 0.1017i \\ 0.1090 + 0.0025i \\ -0.0066 - 0.1066i \\ -0.0973 + 0.0081i \\ 0.0003 + 0.0922i \\ 0.0972 + 0.0076i \\ 0.0063 - 0.1065i \end{bmatrix}$

(8) Finally, the array factor can be found using Equation (11).

The variation of the magnitude of each weight against iteration number is shown in Fig.8. Fig.9 shows the resulting mean square error which converges to near zero after 10 iterations, although Fig.10 shows how the array output converges quickly after ten iterations, but never acquires the desired signal. Finally, the resultant array factor is shown in Fig.11, with a peak at the desired direction and a null at the interfering direction. Although SMI is faster than the LMS algorithm, it has several drawbacks. The correlation matrix may be ill-conditioned resulting in errors or singularities when inverted. Table 1 compares the two algorithms.

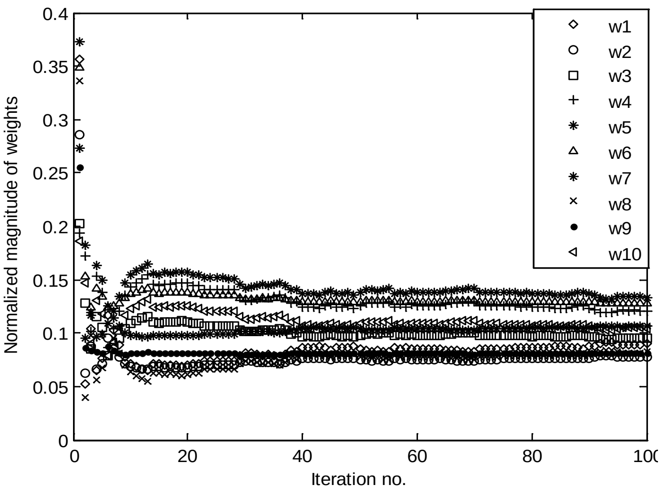

Fig.8: The magnitude variation of each weight of the SMI algorithm for case 1.

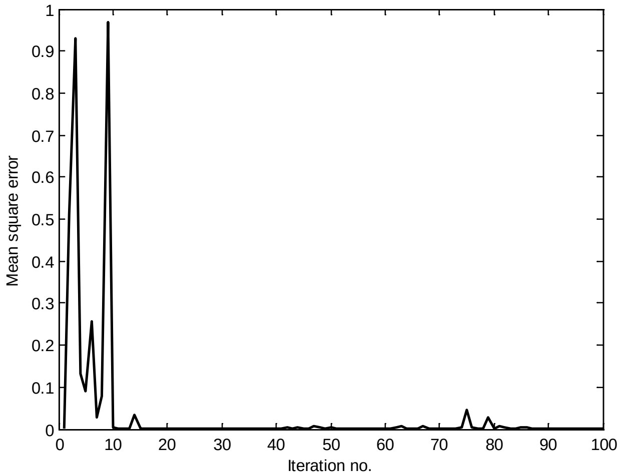

Fig.9: The MSE of the SMI algorithm.

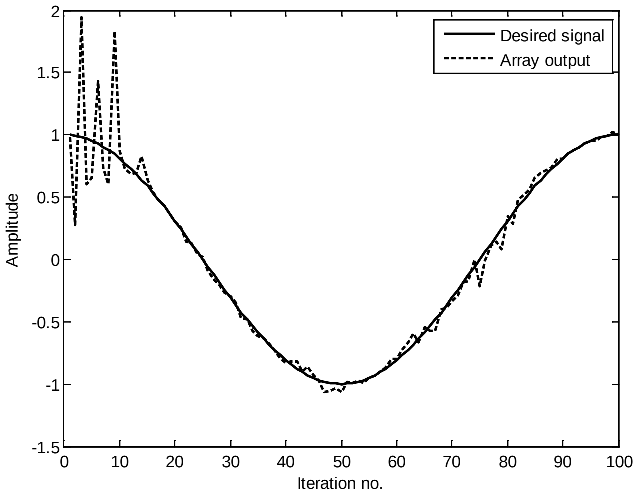

Fig.10: s(k) and y(k) of the SMI algorithm.

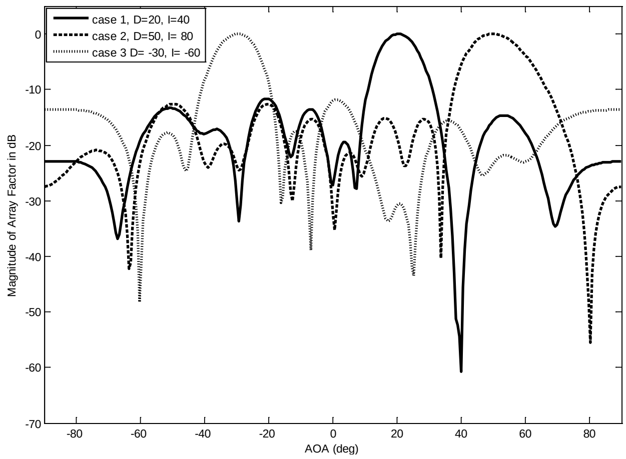

Fig.11: The normalized magnitude of the array factor of smart antenna system using SMI.

Table 1: Comparison between the LMS and SMI Algorithms.

Criterion	LMS Algorithm	SMI Algorithm

Working principle	Minimizing the mean square error between the received signal and a reference signal.	Estimation of the array weights by using the correlation matrix
Convergence	Slow convergence.	Faster than the LMS algorithm.
Complexity and accuracy	Lower complexity than SMI: does not require direct matrix inversion or memory: offers better stability.	Several drawbacks as it depends on direct matrix inversion: may result in errors or singularities.
Direction of desired signal	It depends on a reference signal which is similar to or highly correlated with the desired signal	It depends on the correlation matrix to estimate the desired signal direction.

7 Conclusion

The proposed weight optimization techniques for smart antenna wireless communication systems, to direct the main beam in the direction of the desired signal and place nulls in the direction of interfering signals using LMS and SMI adaptive beamforming algorithms have been presented. It was concluded that the LMS algorithm was a less complex algorithm and does not need direct matrix inversion or memory, but it is slow in convergence time. The SMI algorithm was faster than the LMS algorithm, but has several drawbacks: its computational complexity and potential to create singularities can cause problems.

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References:

[1] A. E. Zooghby, Smart antenna engineering, Artech House, Inc. 2005.

[2] F. Gross, Smart Antennas for Wireless Communication, McGraw-Hill, 2005.

[3] H. Krim and M. Viberg, “Two Decades of Array Signal Processing Research: The Parametric Approach”, IEEE Signal Process. Mag., pp. 67–94, July 1996.

[4] Special Issue on Adaptive Antennas, IEEE Transactions on Antennas and Propagation, Vol. Ap-24, No.2, 1976.

[5] Special Issue on Adaptive Processing Antenna Systems, IEEE Transactions on Antennas and Propagation, Vol. Ap-34, No.3, 1986.

[6] C. A. Balanis, Antenna Theory Analysis and Design, Third Edition, John Wiley & Sons, Inc., 2005.

[7] Godara, L., Smart Antennas, CRC Press, Boca Raton, FL, 2004.

[8] F. Gross, Smart Antennas with MATLAB, Second, Edition McGraw-Hill, 2015.

[9] R. H. ROY, “An overview of Smart Antenna Technology: The next Wave in Wireless Communications”, in Proc. 1998 IEEE Aerospace Conference, Vol. 3, May 1998, pp. 339-345.

[10] Joseph C. Liberti, Theodore S. Rappaport. “Smart Antennas for Wireless Communications: IS-95 and Third Generation CDMA Applications”, Prentice Hall PTR, 12 April, 1999.

[11] Carl B. Dietrich, Jr., Warren L. Stutzman, Byung-Ki Kim, and Kai Dietze, “Smart Antennas in Wireless Communications: Base-Station Diversity and Handset Beam forming”, lEEE Antennas and Propagation Magazine, Vol. 42, No. 5, October 2000.

[12] Michael Chryssomallis, “Smart Antennas” lEEE Antennas and Propagation Magazine, Vol. 42, No. 3**,** June 2000.

[13] G. V. Tsoulos, “Smart Antennas for Mobile Communication Systems: Benefits and Challenges”, Electronic and Communication Engineering Journal, pp. 1-2, April 1999.

[14] Santhi Rani, Subbaiah P. V., Chennakesava R.K, Sudha Rani S. “LMS and RLS Algorithms for Smart Antennas in a W-CDMA mobile Communication Environment”, ARPN Journal of Engineering and Applied Sciences, Vol. 4, No 6, August 2009.

[15] Sidi Bahiri, Fethi Bendimerad. “Performance of Adaptive Beamforming Algorithm for LMS-MCCDMA MIMO Smart Antennas”. The international Arab Journal of Information Technology, Vol. 6, No. 3, July 2009.

[16] Raed M. Shubair, Mahmoud A. AL-Qutayri, Jassim M. Samhan. “A Setup for the Evaluation of MUSIC and LMS Algorithms for a Smart

Antenna System", Journal of Communications, Vol. 2, No. 4, June 2007.

[17] Susmita Das. "Smart Antenna Design for Wireless Communication using Adaptive Beamforming approach", Department of Electrical Engineering, National Institute of Technology Rourkela-769008, Orissa, India, May, 2009.

[18] Thomas E. Biedka, "Analysis and Development of Blind Adaptive Beamforming Algorithms", Department of Electrical Engineering Virgina Polytechnic Institute. October, 2001.

[19] Rameshwar Kawitar, D. G. Wakde, "Advances in Smart Antenna System", Journal of Scientific & Industrial Research, Vol. 64, September 2005, pp. 660-665.

[20] G.C Nwalozie, V.N Okorogu, S.S "Maduadichie, A. Adenola, "A Simple Comparative Evaluation of Adaptive Beam forming Algorithms" International Journal of Engineering and Innovative Technology (IJEIT), Vol. 2, Issue 7, January 2013.

[21] D. G. Dimitris, K.Vinay, and M. Stephen, "Statistical and adaptive signal processing", McGraw Hill Publication, 2005.