Performance of SC Receiver over Weibull Multipath Fading Channel

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Abstract: - The wireless communication system with dual selection combining (SC) diversity receiver operating over Weibull short term fading channel in the presence of cochannel interference affected to Weibull multipath fading, is considered in this paper. Weibull fading can be used to describe signal envelope variation in nonlinear fading environment. Cumulative distribution function and the level crossing rate of the ratio of two Weibull random variables are evaluated and these expressions are used for calculation the level crossing rate of dual SC receiver output signal envelope. These results can be availed for evaluation of the average fade duration of proposed communication system. The series in derived expressions rapidly converge for every values of fading parameters. The influence of Weibull fading parameter on level crossing rate is analyzed and studied.

Key-Words: - Level crossing rate, outage probability, ratio of two random variables, SC receiver, Weibull fading

1 Introduction

Short term fading and cochannel interference degrade outage probability, symbol error probability, channel capacity and average fade duration of wireless communication radio systems [1]. Small scale fading causes signal envelope variation resulting in system performance degradation. There are more distributions which can be used to model signal envelope variation in fading channels. Weibull distribution can describe signal envelope in nonlinear and nonline-of-sight short term fading channels [2]. This distribution has parameter α which is in relation to nonlinearity of fading environment. Fading is more severe for lower values of parameter α. For parameter $\alpha=2$, Weibull multipath fading channel becomes Rayleigh multipath fading channel, and when parameter α goes to infinity, Weibull multipath fading channel becomes no fading channel.

Cochannel interference is from channels operating at the same carrier frequency. In interference limited environment, cochannel interference power is significantly higher than Gaussian noise power, so Gaussian noise effects on the outage probability and symbol error probability can be ignored. It is important to determine the i-th moment of cochannel interference. In interference limited environment, the ratio of signal envelope and cochannel interference envelope is important performance measure which can be used for calculation the outage probability and symbol error probability. The SC receiver selects the branch with the highest desired signal envelope and interference envelope to provide service to user resulting in reduction of multipath fading effects.

There are many papers in open technical literature treating impact of short term fading and cochannel interference effects on performance of the wireless communication systems with SC receiver. In [3], average level crossing rate and average fade duration of wireless system with SC receiver operating over Rayleigh, Rician and Nakagami-*m* multipath fading channels. The wireless communication systems with SC receiver operating over Nakagami-*m* multipath fading channel in the presence of Nakagami-*m* cochannel interference is considered in [4]. The probability density function, cumulative distribution function and moments of SC receiver output signal envelope are calculated, and also outage probability and average symbol error probability of proposed system are evaluated. Performance analysis of such system with selection combining over correlated Weibull fading channels in the presence of cochannel interference is given in [5].

The novel general, simple and closed-form expressions for the probability density function (PDF) and cumulative distribution function (CDF) of the ratio of the products of two independent α-μ variates, where all variates have identical values of α parameter are presented in [6].

In [7], the authors present novel closed-form expressions for the PDF and CDF of the ratio of random variable and product of two random variables for the cases where random variables are Rayleigh, Weibull, Nakagami-*m* and α-μ distributed.

In this paper, wireless mobile radio communication system with SC receiver used to reduce short term fading effects on system performance will be considered. Desired signal and cochannel interference experience Weibull multipath fading resulting in desired signal envelope variation and cochannel interference envelope variation. The probability density function, cumulative distribution function and level crossing rate of the ratio of two Weibull random processes are calculated. These formula are used for calculation the level crossing rate of SC receiver output signal envelope, Also, the average fade duration of considered system can be calculated as the ratio of outage probability and level crossing rate. The obtained infinite series in derived expressions rapidly converge since 5-10 terms should be summed.

2 Level Crossing Rate of Ratio of Two Weibull Random Variables

The ratio of two Weibull random variables is:

$$\eta = \frac{x}{y}, \quad x = \eta \cdot y \tag{1}$$

The probability density function of η is:

$$p_\eta(\eta) = \int_0^\infty dy\; y\, p_x(\eta y) \cdot p_y(y) \tag{2}$$

where x and y follow Weibull distribution [1]:

$$p_x(x) = \frac{\alpha}{\Omega} x^{\alpha-1} e^{-\frac{1}{\Omega}x^\alpha}, \; x \ge 0 \tag{3}$$

$$p_y(y) = \frac{\alpha}{S} y^{\alpha-1} e^{-\frac{1}{S}y^\alpha}, \; y \ge 0 \tag{4}$$

After substituting, the expression for η becomes:

$$p_\eta(\eta) = \frac{\alpha^2}{\Omega S}\eta^{\alpha-1}\int_0^\infty dy\; y^{2\alpha-1} e^{-y^\alpha\left(\frac{\eta^\alpha}{\Omega}+\frac{1}{S}\right)} =$$

$$= \frac{\alpha^2}{\Omega S}\eta^{\alpha-1}\cdot\frac{1}{\alpha}\frac{\Omega^\alpha S^\alpha}{\left(\Omega+Sy^\alpha\right)^\alpha}\Gamma(\alpha) =$$

$$= \alpha\Omega^{\alpha-1}S^{\alpha-1}\Gamma(\alpha)\cdot\frac{\eta^{\alpha-1}}{\left(\Omega+Sy^\alpha\right)^\alpha} \tag{5}$$

The cumulative distribution function of η is:

$$F_\eta(\eta) = \int_0^\eta dt\; p_\eta(t) =$$

$$= \alpha\Omega^{\alpha-1}S^{\alpha-1}\Gamma(\alpha)\cdot\frac{\eta^{\alpha-1}}{\left(\Omega+Sy^\alpha\right)^\alpha} \tag{6}$$

After substituting:

$$\Omega + St^\alpha = z, \quad t^{\alpha-1}dt = \frac{1}{S\alpha}dz,$$

expression (6) becomes:

$$F_\eta(\eta) = \frac{\alpha^2\Omega^{\alpha-1}S^{\alpha-1}\Gamma(\alpha)}{S\alpha}\int_\Omega^{\Omega+Sy^\alpha}\frac{dz}{z^\alpha} =$$

$$= \alpha\Omega^{\alpha-1}S^{\alpha-2}\Gamma(\alpha)\frac{1}{\alpha-1}\left(\frac{1}{\Omega^{\alpha-1}} - \frac{1}{\left(\Omega+Sy^\alpha\right)^{\alpha-1}}\right) =$$

$$= \frac{\alpha S^{\alpha-2}\Gamma(\alpha)}{\alpha-1}\left(1 - \frac{\Omega^{\alpha-1}}{\left(\Omega+Sy^\alpha\right)^{\alpha-1}}\right) \tag{7}$$

The first derivative of ratio of two Weibull random variables is:

$$\dot{\eta} = \frac{\dot{x}_1}{y_1} - \frac{x_1\dot{y}_1}{y_1^2} \tag{8}$$

where x_1 and y_1 follow Rayleigh distribution:

$$p_{x_1}(x_1) = \frac{2x_1}{\Omega_1}e^{-\frac{x_1^2}{\Omega_1}}, \; x_1 \ge 0 \tag{9}$$

$$p_{y_1}(y_1) = \frac{2y_1}{S_1}e^{-\frac{y_1^2}{S_1}}, \; y_1 \ge 0 \tag{10}$$

The first derivative of Rayleigh random variables are Gaussian random variables with zero mean. The first derivative of linear transformation of the first derivative of Rayleigh random variables Gaussian random variables has Gaussian distribution. Therefore, the random variable $\dot{\eta}$ follows conditional Gaussian distribution with $\overline{\dot{\eta}}=0$ and variance:

$$\sigma_{\dot{\eta}}^2=\frac{4}{\alpha^2\eta^{\alpha-2}}\left(\frac{1}{y_1^2}\sigma_{\dot{x}_1}^2+\frac{x_1^2}{y_1^4}\sigma_{\dot{y}_1}^2\right) \tag{11}$$

where

$$\sigma_{\dot{x}_1}^2=\pi^2 f_m^2\Omega_1, \tag{12}$$

$$\sigma_{\dot{y}_1}^2=\pi^2 f_m^2 S_1. \tag{13}$$

After substituting, the expression for variance becomes:

$$\sigma_{\dot{\eta}}^2=\frac{4}{\alpha^2\eta^{\alpha-2}}\frac{\pi^2 f_m^2}{y_1^2}\left(\Omega_1+\eta^\alpha S_1\right) \tag{14}$$

The joint probability density function of η, $\dot{\eta}$ and y_1 is:

$$p_{\eta\dot{\eta}y_1}\left(\eta\dot{\eta}y_1\right)=p_{\dot{\eta}}\left(\dot{\eta}/\eta y_1\right)p_{\eta y_1}\left(\eta y_1\right)=$$

$$=p_{\dot{\eta}}\left(\dot{\eta}/\eta y_1\right)p_{y_1}\left(y_1\right)p_{\eta}\left(\eta/y_1\right) \tag{15}$$

where

$$p_{\eta}\left(\eta/y_1\right)=\left|\frac{dx_1}{d\eta}\right|p_{x_1}\left(y_1\eta^{\alpha/2}\right)$$

$$\frac{dx_1}{d\eta}=\frac{\alpha}{2}y\eta^{\frac{\alpha}{2}-1}$$

After substituting, the expression for η becomes:

$$p_{\eta\dot{\eta}y_1}\left(\eta\dot{\eta}y_1\right)=\frac{\alpha}{2}\eta^{\frac{\alpha}{2}-1}y_1 p_{\dot{\eta}}\left(\dot{\eta}/\eta y_1\right)p_{x_1}\left(y\eta^{\frac{\alpha}{2}}\right)p_{y_1}\left(y_1\right) \tag{16}$$

The joint probability density function of η and $\dot{\eta}$ is:

$$p_{\eta\dot{\eta}}\left(\eta\dot{\eta}\right)=\int_0^\infty dy_1\, p_{\eta\dot{\eta}y_1}\left(\eta\dot{\eta}y_1\right)=$$

$$=\frac{\alpha}{2}\eta^{\frac{\alpha}{2}-1}\int_0^\infty dy_1\, y_1 p_{\dot{\eta}}\left(\dot{\eta}/\eta y_1\right)p_{x_1}\left(y_1\eta^{\frac{\alpha}{2}}\right)p_{y_1}\left(y_1\right) \tag{17}$$

The level crossing rate of the ratio of two Weibull random variables is [2]:

$$N_\eta=\int_0^\infty d\dot{\eta}\,\dot{\eta}\;p_{\eta\dot{\eta}}\left(\eta\dot{\eta}\right)=$$

$$=\frac{\alpha}{2}\eta^{\frac{\alpha}{2}-1}\int_0^\infty dy_1\, y_1\, p_{x_1}\left(y_1\eta^{\frac{\alpha}{2}}\right)p_{y_1}\left(y_1\right)\int_0^\infty d\dot{\eta}\,\dot{\eta}\;p_{\dot{\eta}}\left(\dot{\eta}/\eta y_1\right)=$$

$$=\frac{\alpha}{2}\eta^{\frac{\alpha}{2}-1}\int_0^\infty dy_1\, y_1\, p_{x_1}\left(y_1\eta^{\frac{\alpha}{2}}\right)p_{y_1}\left(y_1\right)\frac{\sigma_{\dot{\eta}}}{\sqrt{2\pi}}=$$

$$=\frac{\alpha}{2}\eta^{\frac{\alpha}{2}-1}\frac{1}{\sqrt{2\pi}}\frac{2\pi f_m}{\alpha\eta^{\alpha/2}}\sqrt{\Omega_1+\eta^\alpha S_1}\,\cdot$$

$$\cdot\frac{4}{\Omega_1 S_1}\eta^{\frac{\alpha}{2}}\int_0^\infty dy_1\, y_1^2\, e^{y_1^2\left(\frac{\eta^\alpha}{\Omega_1}+\frac{1}{S_1}\right)}=$$

$$=\frac{\pi f_m}{\sqrt{2\pi}}\sqrt{\Omega_1+\eta^\alpha S_1}\,\cdot$$

$$\cdot\frac{4}{\Omega_1 S_1}\eta^{\frac{\alpha}{2}}\frac{1}{2}\frac{\left(\Omega_1 S_1\right)^{3/2}}{\left(\Omega_1+\eta^\alpha S_1\right)^{3/2}}\Gamma\left(3/2\right)=$$

$$=\frac{2\pi f_m}{\sqrt{2\pi}}\left(\Omega_1 S_1\right)^{1/2}\Gamma\left(3/2\right)\frac{\eta^{\alpha/2}}{\Omega_1+\eta^\alpha S_1} \tag{18}$$

3 Statistics of SC Receiver Output Signal Envelope

Dual SC receiver output signal is:

$$z=\max\left(z_1,z_2\right) \tag{19}$$

where z_1 and z_2 are denoted signal to interference ratios (SIRs) at inputs of SC receiver and x is output SC receiver SIR. Probability density function of z is:

$$p_z\left(z\right)=p_{z_1}\left(z\right)F_{z_2}\left(z\right)+p_{z_2}\left(z\right)F_{z_1}\left(z\right) \tag{20}$$

For considered case it is valid:

$$p_z\left(z\right)=2p_{z_1}\left(z\right)F_{z_2}\left(z\right)=$$

$$=2\alpha\Omega^{\alpha-1}S^{\alpha-1}\Gamma\left(\alpha\right)\cdot\frac{z^{\alpha-1}}{\left(\Omega+Sz^\alpha\right)^{\alpha-1}}\cdot$$

$$\cdot\frac{\alpha S^{\alpha-2}\Gamma\left(\alpha\right)}{\alpha-1}\left(1-\frac{\Omega^{\alpha-1}}{\left(\Omega+Sz^\alpha\right)^{\alpha-1}}\right) \tag{21}$$

The cumulative distribution function of z is:

$$F_z\left(z\right)=F_{z_1}\left(z\right)\cdot F_{z_2}\left(z\right)=$$

$$=\frac{\alpha^2 S^{2\alpha-4}\Gamma\left(\alpha\right)^2}{\left(\alpha-1\right)^2}\left(1-\frac{\Omega^{\alpha-1}}{\left(\Omega+Sz^\alpha\right)^{\alpha-1}}\right)^2 \tag{22}$$

The joint probability density function of z and $\dot z$ is:

$$p_{z\dot z}(z\dot z) = p_{z_1\dot z_1}(z\dot z_1)\cdot F_{z_2}(z) + p_{z_2\dot z_2}(z\dot z_2)F_{z_1}(z) =$$

$$= 2p_{z_1\dot z_1}(z\dot z_1)\cdot F_{z_2}(z) \qquad (23)$$

The level crossing rate of random process *z(t)* is:

$$N_z = \int_0^\infty d\dot z\, \dot z\, p_{z\dot z}(z\dot z) =$$

$$= 2F_{z_2}(z)\int_0^\infty d\dot z_1\, \dot z_1 p_{z_1\dot z_1}(z\dot z_1) =$$

$$= 2F_{z_2}(z)N_{z_1} =$$

$$= 2\frac{\alpha S^{\alpha-2}\Gamma(\alpha)}{\alpha-1}\left(1-\frac{\Omega^{\alpha-1}}{\left(\Omega+Sz^\alpha\right)^{\alpha-1}}\right)\cdot$$

$$= \frac{2\pi f_m}{\sqrt{2\pi}}(\Omega S)^{1/2}\Gamma(3/2)\frac{z^{\alpha/2}}{\Omega+z^\alpha S}. \qquad (24)$$

Previous expression can be used for calculation the average fade duration of wireless communication system with SC receiver operating over Weibull multipath fading channels in the presence of cochannel interference subjected to Weibull small scale fading.

4 Numerical Results

In Fig. 1, the histogram of the ratio of two Weibull random variables is shown. The abscissa of the histogram presents the amplitude value of Weibull random process; the ordinate is the number of samples in the interval of abscissa.

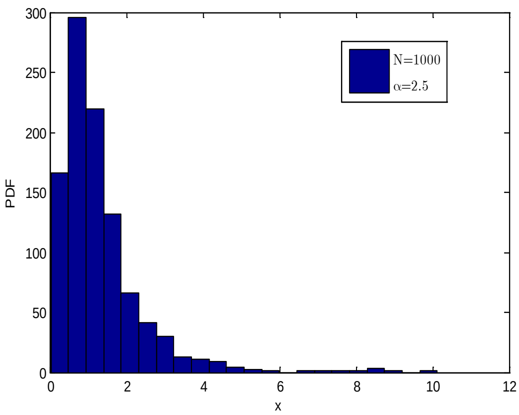

Fig.1. Histogram of the ratio of two Weibull random variables

In Fig. 2, the cumulative distribution function of the ratio of two Weibull random variables is given. This is actually the outage probability of proposed system. One can see from this figure that the outage probability is bigger for smaller values of Weibull fading parameter α.

The level crossing rate of the ratio of two Weibull random variables is presented in Fig. 3. The level crossing rate of SC receiver output SIR decreases when Weibull fading parameter α increases. For lower values of the level crossing rate system performance are better.

In Fig. 4, the histogram of the maximum of two Weibull random variables is shown. The abscissa of the histogram presents the amplitude value of Weibull random process.

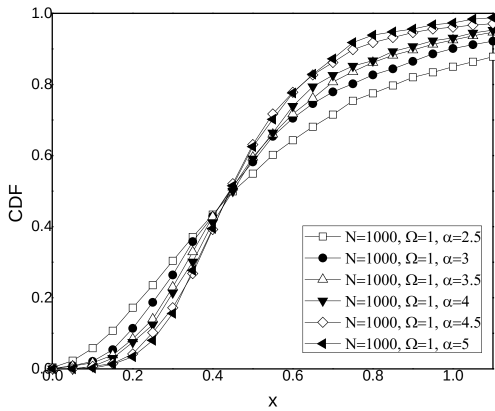

Fig.2. The outage probability of the ratio of two Weibull random variables

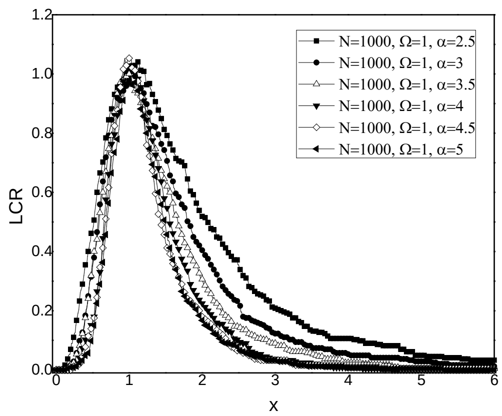

Fig.3. The level crossing rate of the ratio of two Weibull random variables

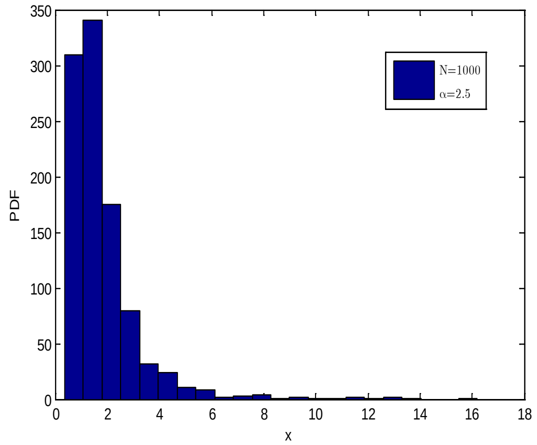

Fig.4. Histogram of the maximum of two Weibull random variables

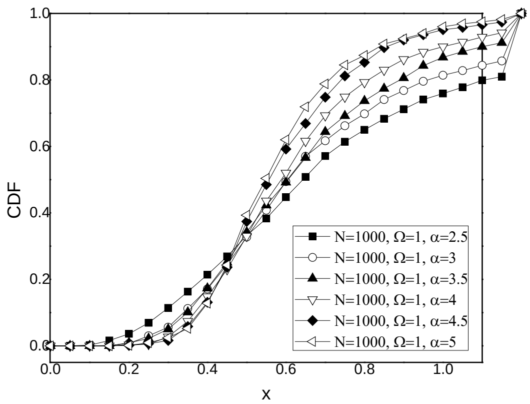

Fig.5. The outage probability of the maximum of two Weibull random variables

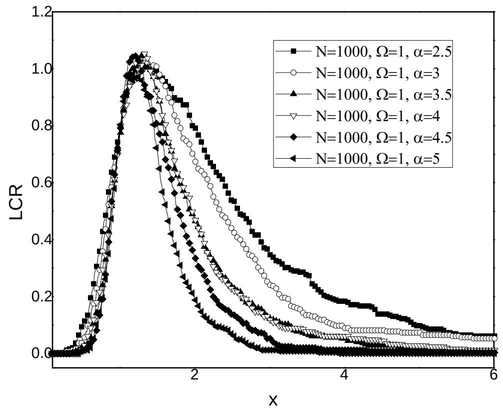

Fig.6. The level crossing rate of the maximum of two Weibull random variables

In Fig. 5, the cumulative distribution function of the ratio of two Weibull random variables is given. It is visible from this figure that the outage probability is bigger for smaller values of Weibull fading parameter α.

The level crossing rate of the ratio of two Weibull random variables is presented in Fig.6. The level crossing rate of SC receiver output SIR is bigger when Weibull fading parameter α is smaller. For lower values of the level crossing rate the system performance are better. This fact gives opportunity to the designers of wireless systems to choose the best options for system parameters.

5 Conclusion

In this paper, wireless radio communication system operating over Weibull multipath fading channel in the presence of Weibull cochannel interference is considered. Weibull distribution describes signal envelope in nonlinear short term fading channel when signal propagates under one cluster. In interference limited environment, the ratio of signal envelope and interference envelope is important performance measure of wireless communication system. The outage probability, channel capacity and bit error probability can be calculated by using this ratio. In this paper, the ratio of two Weibull random variables is analyzed and probability density function, cumulative distribution function and level crossing rate are efficiently evaluated.

By setting α=2 in derived expressions, the expressions for probability density function, cumulative distribution function and level crossing rate of the ratio of two Rayleigh random processes can be obtained. Also, by application statistics of the ratio of two Weibull random processes, the level crossing rate of SC receiver output SIR is calculated. This formula can be used for calculation the average fade durationof wirwless system with SC receiver in the presence of Weibull short term fading and Weibull cochannel interference. The level crossing rate of SC receiver output SIR decreases when Weibull fading parameter of desired signal increases resulting in system performance improvement. The series in derived expressions are rapidly convergent and 10 terms should be summed to achieve accuracy at fourth significant digit for all values of fading parameters.

The influence of desired signal average power, interference average power and Weibull fading parameter on the level crossing rate is presented in some figures and analysis is given.

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