Achievable Unified Performance Analysis of Orthogonal Space-Time Block Codes with Antenna Selection over Correlated Rayleigh Fading Channels

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Abstract: - In this paper we evaluated the performance of orthogonal space-time block codes with transmit antenna selection (TAS/OSTBCs) for frequency-flat correlated Rayleigh fading channels. By effective use of the probability density function (pdf), the bit error probability (BEP) for binary phase shift keying (BPSK) and M-ary Quadrature amplitude modulation (M-QAM) is derived first. Further we evaluated the ergodic capacity and outage probability over correlated Rayleigh fading channels. Numerical results illustrate the impact of fading correlation on the BEP, channel capacity and outage probability.

Key-Words: - Antenna selection, bit error probability (BEP), ergodic capacity, multiple-input multiple-output (MIMO), outage probability, orthogonal space-time block codes (OSTBCs).

1 Introduction

MULTIPLE-INPUT MULTIPLE OUTPUT (MIMO) systems can offer robust paradigm to enhance the throughput and performance of wireless communication system [1, 2]. In this view, OSTBCs is favorable technique introduced in [3,4] that realize full diversity order using a simple linear computation at the output. Nevertheless, a considerable drawback in the deployment of multiple antennas and STBCs is the high cost of multiple radio frequency (RF) chains at both terminals of a wireless channel.

The practical approach to solve the implementation complexity of MIMO system, antenna selection technique is a practical solution while preserving the benefits of all antennas, was presented in [5]-[9]. The fundamental objective behind antenna selection is to select only the optimal subset of antennas either at one or both link ends in MIMO systems and this selection norm is based on to enhance the channel capacity and minimize the error probability. The difficulty of antenna subset selection with Alamouti scheme was presented in [5]. The MIMO system with antenna selection was outlined in [6], where either one or both link ends use the signal from the optimal set of antennas. In [7], closed-form bit error rate (BER) of the TAS/MRC scheme was evaluated. The TAS was applied in [8] and the BER has been investigated for M-ary signal by using antenna selection impact. The closed-form BER expression of TAS/ OSTBC was analyzed in [9]. In this paper, we derive analytical exact BEP for BPSK and M-QAM modulation, ergodic capacity and outage probability of the TAS/OSTBC systems over correlated Rayleigh fading channels.

The remainder of this paper is organized as follows: First, in Section 2, the system and channel model characterization is presented. In Section 3, we derive the performance analysis in terms of BEP, ergodic capacity and outage probability by the effective use of pdf over correlated Rayleigh fading channels. Finally, Section 4 provides numerical results, followed by concluding remarks in Section 5.

2 System and Channel Model

Considering a wireless point to point communication system in Rayleigh frequency flat fading channel environment with N_t transmit and N_r receive antennas. We assume that the channel state information (CSI) is known at the receiver such that it selects the optimal M_t out of N_t transmit antennas, further this information is fed back to the transmitter via a error-free feedback channel. Assume the

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matrix S transmit the R symbols $s_1, s_2, \ldots., s_R$ in T time slots, the symbol rate of the OSTBC scheme is defined as $R_c = R/T$. Consider $\boldsymbol{H} = [h_{ij}] \in C^{N_r \times N_t}$ be the complex channel matrix and follows the Rayleigh fading model with channel coefficient are modeled as $h_{ij} \sim \mathcal{CN}(0,1)$. Let after the selection of antenna, $N_r \times M_t$ sub-matrix is $\widetilde{\boldsymbol{H}}$. The received signal at a time nT can be expressed as

$$\boldsymbol{Y_{nT}} = \sqrt{\frac{E_s}{N}} \widetilde{\boldsymbol{H}} \boldsymbol{X} + \boldsymbol{V_{nT}} \tag{1}$$

where matrix $\boldsymbol{Y_{nT}} \in C^{N_r \times T}$ is received signal matrix, $\boldsymbol{X} \in C^{M_t \times T}$ is the complex transmitted signal matrix and $\boldsymbol{V_{nT}} \in C^{N_r \times T}$ is a receiver noise matrix and are modeled as $\boldsymbol{V_{nT}} \sim \mathcal{CN}(0, N_0)$. The subset of $N = \binom{N_t}{M_t}$ alternate transmit antennas is selected in order to maximize the total received signal power for data transmission. Thus $\widetilde{\boldsymbol{H}}_s (1 \le s \le N)$ be the sub matrix of N channel with respect to N subset of antenna. By using OSTBC, the maximum likelihood decoder decomposes the MIMO system to R independent scalar additive white Gaussian noise (AWGN) channels [4].

$$z_r = \sqrt{\frac{E_s}{N}} \left(\frac{1}{R_c} \left\| \widetilde{\boldsymbol{H}}_s \right\|_F^2 \right) x_r + v_r, r = 1, \ldots., R \tag{2}$$

where $v_r \in \mathcal{CN}\left(0, \frac{1}{R_c} \left\| \widetilde{\boldsymbol{H}}_s \right\|_F^2 N_0\right)$. Therefore, the instantaneous SNR at the receiver defined as

$$\gamma_s = \frac{E_s}{N_0} \frac{1}{N R_c} \left\| \widetilde{\boldsymbol{H}}_s \right\|_F^2 = a \bar{\gamma} \left\| \widetilde{\boldsymbol{H}}_s \right\|_F^2 \tag{3}$$

where $\bar{\gamma} = E_s / N_0$ is the SNR per channel and $a = 1/N R_c$. Let $\gamma_{(k)} = a\,\bar{\gamma} \|\boldsymbol{h}_k\|^2, 1 \le k \le N_t$ and are arranged as $\gamma_{(N_t)} \ge \cdots . \ge \gamma_{(2)} \ge \gamma_{(1)}$. In TAS, we choose the optimal set of antennas with the maximum $\gamma_{(N_t)}$ for transmission. Thus, practical SNR at the receiver (3) can be written as

$$\gamma = \sum_{k=1}^{N} \gamma_{(k)} \tag{4}$$

where $\gamma_{(k)} = a\,\bar{\gamma} \|\boldsymbol{h}_{ik}\|^2$, which represents order statistics sum and statistical distribution of γ_k are accessible in an independent channel fading condition. Let correlated MIMO channel $\boldsymbol{H}$ can be characterize by the following covariance matrix with size $t \times t$ as [10]

$$\boldsymbol{\mathcal{R}}_{\text{cc}} = E\{\boldsymbol{h}\boldsymbol{h}^H\} \tag{5}$$

where $t = N_r \times N_t, \& \ \boldsymbol{h} = vec(\boldsymbol{H})$.

The moment generating function (MGF) of correlated Rayleigh fading, have been realized in [11]

$$\phi(t_1, \ldots, t_N) = E\left(e^{j t_1 \gamma_1, \ldots, j t_N \gamma_N}\right)$$
$$= det(\boldsymbol{I} - \boldsymbol{S}\boldsymbol{\mathcal{R}}_{\text{cc}})^{-1} \tag{6}$$

where $\boldsymbol{S} = diag(jt_1, \ldots, jt_N)$ is the diagonal matrix. For all branches, the cumulative density function (cdf) and pdf of largest SNR are given in [11]

$$F_\gamma(\gamma) = \frac{1}{(2\pi)^N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \phi(t_1, \ldots, t_N)$$
$$\times \prod_{k=1}^{N} \left(\frac{1 - e^{-j t_k \gamma}}{j t_k} \right) dt_1, \ldots, dt_N \tag{7}$$

and

$$f_\gamma(\gamma) = \frac{1}{(2\pi)^N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \phi(t_1, \ldots, t_N) \prod_{k=1}^{N} (j t_k)^{-1}$$
$$\times \sum_{n=1}^{N} (-1)^{n+1} \sum_{b_1 + \cdots .. + b_N = n} \frac{j T_N}{exp(j \gamma T_N)} dt_1, \ldots, dt_N \tag{8}$$

where $T_N = b_1 t_1 + \cdots + b_N t_N$ and binary variables $b_1, \ldots, b_N$ get the equivalents of 0 or 1.

3 Performance Analysis of TAS /OSTBC in Correlated Rayleigh Fading Channels

In this section, we derive closed-form BEP of TAS/OSTBC with binary signal and M-QAM signal constellations and the channel capacity and outage probability in the presence of correlated Rayleigh fading channels.

3.1 Probability of Error for Binary Signals:

The BEP of TAS/OSTBCs over Rayleigh fading is evaluated by expectation the error probability of M-ary signal $\{P_M(\gamma)\}$ over the pdf γ.

$$P_M^{OSTBC} = \int_0^{\infty} P_M(\gamma) f_\gamma(\gamma) d\gamma \tag{9}$$

Inserting (8) into (9), we get the expression of error probability as

$$P_M^{OSTBC} = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \phi(t)\, w(t) dt_1, \dots, dt_N \quad (10)$$

with

$$w(t) = \frac{1}{(2\pi)^N} \int_0^{\infty} P_M(\gamma) \prod_{k=1}^{N} (jt_k)^{-1}$$

$$\times \sum_{n=1}^{N} (-1)^{n+1} \sum_{b_1+\cdots+b_N=n} \frac{jT_N}{exp(j\gamma T_N)} d\gamma \quad (11)$$

In an AWGN channel, the error probability for binary signal $P_M(\gamma)$ is given by [12]

$$P_M(\gamma) = \beta Q\left(\sqrt{2\alpha\gamma}\right) \quad (12)$$

where $Q(x) = \frac{1}{\pi}\int_0^{\pi/2} exp\left(-\frac{x^2}{2sin^2\theta}\right) d\theta$ [13] and $\alpha = 1$ & $\beta = 1$ for BPSK and $\alpha = 1/2$ & $\beta = 1$ for BFSK. Inserting (12) into (11) and using the fact that $\sum_{n=1}^{N}(-1)^{n+1}\binom{N}{n} = 1$, we get

$$w(t) = \frac{\beta}{2(2\pi)^N} \prod_{k=1}^{N} (jt_k)^{-1}$$

$$\times \left(1 + \sum_{n=1}^{N} (-1)^n \sum_{b_1+\cdots+b_N=n} \sqrt{\frac{\alpha}{\alpha + jT_N}}\right) \quad (13)$$

Thus, the exact BEP for BPSK system is

$$P_M^{OSTBC} = \frac{1}{2(2\pi)^N} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} det(\boldsymbol{I} - \boldsymbol{S}\boldsymbol{\mathcal{R}}_{\text{cc}})^{-1} \prod_{k=1}^{N} (jt_k)^{-1} \left(1 + \sum_{n=1}^{N} (-1)^n \sum_{b_1+\cdots+b_N=n} \sqrt{\frac{1}{1 + jT_N}}\right) dt_1, \dots, dt_N \quad (14)$$

3.2 Probability of Error for M-QAM:

The error probability of rectangular M-QAM constellation is given in [12]

$$P_{M-QAM}^{OSTBC} = \left[1 - \left(1 - P_{\sqrt{M}-PAM}^{OSTBC}\right)^2\right] \quad (15)$$

The $P_{\sqrt{M}\_PAM}^{STBC}$ can be obtained as

$$P_{\sqrt{M}\_PAM}^{STBC} = \int_0^{\infty} P_{\sqrt{M}}(\gamma)\, f_\gamma(\gamma) d\gamma \quad (16)$$

where $P_{\sqrt{M}}(\gamma) = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\gamma}\right)$. Inserting $P_{\sqrt{M}}(\gamma)$, into (11) and using the fact that $\sum_{n=1}^{N}(-1)^{n+1}\binom{N}{n} = 1$, we get

$$w(t) = \frac{\left(1 - \frac{1}{\sqrt{M}}\right)}{(2\pi)^N} \prod_{k=1}^{N} (jt_k)^{-1} \left(1 + \sum_{n=1}^{N} (-1)^n \sum_{b_1+\cdots+b_N=n} \sqrt{\frac{3}{3 + (M-1)jT_N}}\right) \quad (17)$$

Thus, by inserting the value of (17) into (10), the exact BEP of $P_{\sqrt{M}-PAM}^{OSTBC}$ can be evaluated as

$$P_{\sqrt{M}-PAM}^{OSTBC} = \frac{\left(1 - \frac{1}{\sqrt{M}}\right)}{(2\pi)^N} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} det(\boldsymbol{I} - \boldsymbol{S}\boldsymbol{\mathcal{R}}_{\text{cc}})^{-1} \prod_{k=1}^{N} (jt_k)^{-1} \left(1 + \sum_{n=1}^{N} (-1)^n \sum_{b_1+\cdots+b_N=n} \sqrt{\frac{3}{3 + (M-1)jT_N}}\right) dt_1, \dots, dt_N \quad (18)$$

Finally, by inserting (18) into (15), we obtained the exact BEP of M-QAM.

3.3 Channel Capacity:

The channel capacity is given by [14]

$$C(\gamma) = R_c' \int_0^{\infty} ln(1+\gamma)\, f_\gamma(\gamma)\, d\gamma \quad (19)$$

where $R_c' = R_c/ln2$ and the weighting function is

$$w(t) = \prod_{k=1}^{N} (jt_k)^{-1} \sum_{n=1}^{N} (-1)^{n+1} \sum_{b_1+\cdots+b_N=n} jT_N\, \phi(t_1, \dots, t_N) \int_0^{\infty} C(\gamma)\, e^{-j\gamma T_N} d\gamma \quad (20)$$

The channel capacity is

$$C(\gamma) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \phi(t)\, w(t) dt_1, \dots, dt_N \qquad (21)$$

3.4 Outage Probability:

The outage probability is given as [13]

$$P_{out}(\gamma_{th}) = Pr\{R_c log_2(1+\gamma) \leq \gamma_{th}\}$$

$$= Pr\left\{\gamma \leq 2^{\frac{\gamma_{th}}{R_c}} - 1\right\} = F_\gamma\left(2^{\frac{\gamma_{th}}{R_c}} - 1\right) \quad (22)$$

$$P_{out}(\gamma_{th}) = \frac{1}{2(2\pi)^N} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} det(\boldsymbol{I} - \boldsymbol{S}\boldsymbol{\mathcal{R}}_{\rm cc})^{-1}$$

$$\times \prod_{k=1}^{N} \frac{1 - e^{-jt_k\left(2^{\frac{\gamma_{th}}{R_c}} - 1\right)}}{jt_k} dt_1, \dots, dt_N \qquad (23)$$

4 Numerical Results

In this section the numerical results of TAS/OSTBC system over Rayleigh fading channels is illustrated and validated.

Fig.1 presents the BEP for BPSK in TAS/OSTBC over uncorrelated Rayleigh fading channels. First TAS system chooses $M_t = 2$ out of $N_t = 3$ transmit antennas and $N_r = 2$ receive antenna. Second TAS system selects $M_t = 2$ out of $N_t = 3$ transmit antennas and $N_r = 1$ receive antenna. By using truncated Riemann sum of points, integrals in (10) with 0.2 equi-spaced are estimated between -10 and 10. Expanding the summation limits will not promote the accuracy as the integrant is extremely potent within the range $[-8, 8]$.

It is significant to know the dependence of the bit error performance on the spatial correlation which depends on the antenna structure and the operation element. By using the practical channel model transmit correlation matrix $\boldsymbol{R}_t$ and the receive correlation matrix $\boldsymbol{R}_r$ are generated, is described in [10, 15]. The model considers that there are uniform linear arrays at both link ends, and that the angular spectrum at both transmitter & receiver follows a Gaussian distribution. Assume the antenna separation between contiguous antennas as d_r at the receiver and d_t at the transmitter. d_r and d_t are measured in wavelengths units $\lambda = c/f_c$,where f_c is the center frequency of the narrowband signal. Furthermore, we assign $\overline{\theta_r}, \overline{\theta_t}, \sigma_r, \sigma_t$ as the mean angle of arrival (AOA), mean angle of departure (AOD), receive angle spread and transmit angle spread, respectively. The true random AOA (θ_r) and AOD (θ_r) can be written as $\theta_r = \overline{\theta_r} + \widehat{\theta_r}$ and $\theta_t = \overline{\theta}_t + \widehat{\theta}_t$with $\widehat{\theta_r} \sim \mathcal{N}(0, \sigma_r^2)$ and $\widehat{\theta_t} \sim \mathcal{N}(0, \sigma_t^2)$. Thus, $\boldsymbol{R}_r$ and $\boldsymbol{R}_t$ can be given as

$$\boldsymbol{R}_r(m,n) = exp\{-j2\pi(m-n)d_r cos\overline{\theta_r}\}$$

$$exp\{-(\pi(m-n)d_r sin\theta_r \sigma_r)^2\} \qquad (24)$$

$$\boldsymbol{R}_t(m,n) = exp\{-j2\pi(m-n)d_t cos\overline{\theta}_t\}$$

$$exp\{-(\pi(m-n)d_t sin\theta_t \sigma_t)^2\} \qquad (25)$$

The correlation matrix $\boldsymbol{\mathcal{R}}_{\rm cc} = \boldsymbol{R}_t \otimes \boldsymbol{R}_r$ where $\otimes$ is Kronecker product. In correlated Rayleigh fading channels, the impacts of the parameters on the BEP can be obtained by inserting the correlation matrix $\boldsymbol{\mathcal{R}}_{\rm cc}$ and the diagonal matrix $\mathbf{S}$ into the MGF (6) and the average BEP (10).

Fig. 2 depicts the effect of transmit antenna spacing with $d_t = 1$, receive antenna spacing $d_r = 1/3\lambda$, $\overline{\theta_t} = \overline{\theta_r} = \pi/2$, $\sigma_r = \pi/6$. A TAS/OSTBC system is selecting $M_t = 2$ out of $N_t = 3$ transmit antennas with $(a) N_r = 1$ & $(b)\, N_r = 2$ receive antennas. The error performance enhances by increasing the antenna configuration and decreases with spatial correlation. Nevertheless, as the separation between antenna d_t is raised apart from λ, the BEP starts approaching its maximal attainable performance.

Fig.3 illustrates the capacity of TAS/OSTBC system over correlated Rayleigh fading channel with $N = 6$, correlation matrix $\boldsymbol{\mathcal{R}}_{\rm cc} = \begin{bmatrix} 1 & 0.4 \\ 0.4 & 1 \end{bmatrix}$. The capacity performance decreases with the spatial correlation, however increases with increasing the diversity order.

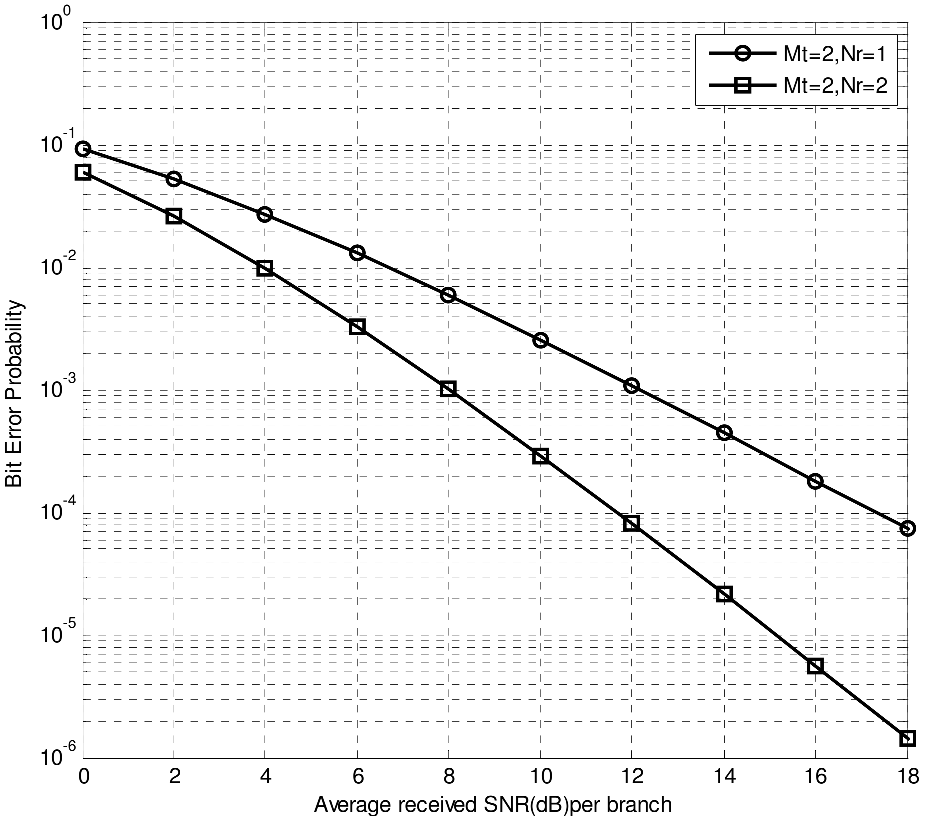

Fig. 1. BEP for BPSK in TAS/OSTBC against $\bar{\gamma}$ over uncorrelated Rayleigh fading channels

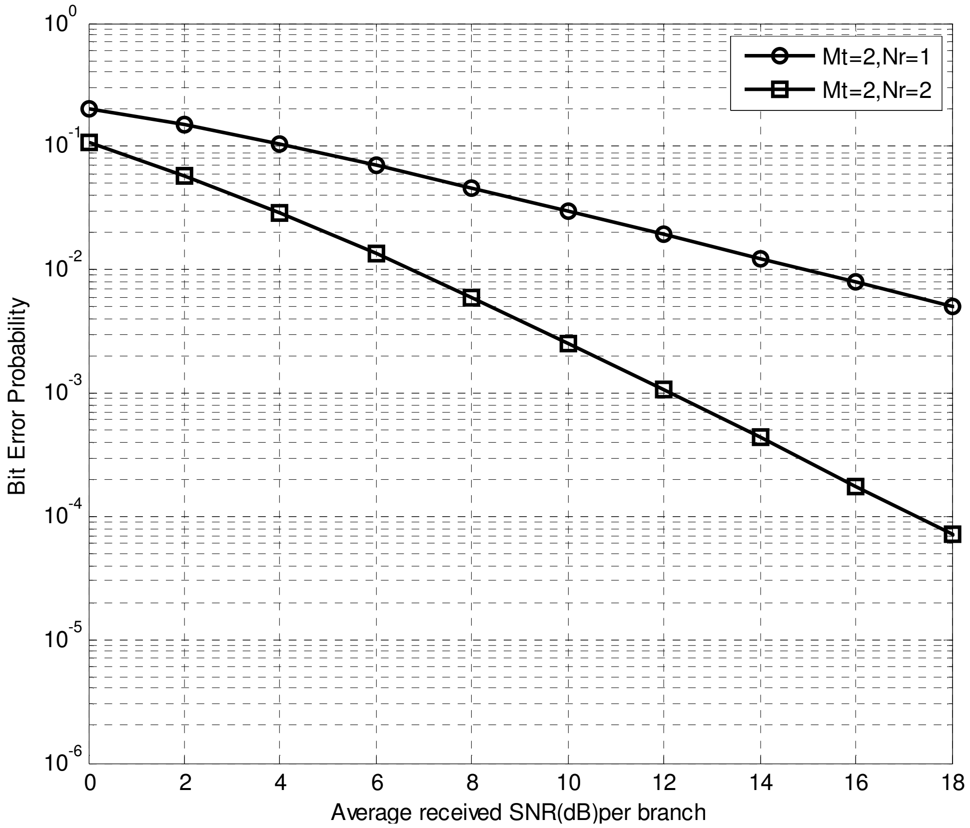

Fig. 2. BEP for BPSK in TAS/OSTBC against $\bar{\gamma}$ over correlated Rayleigh fading channels

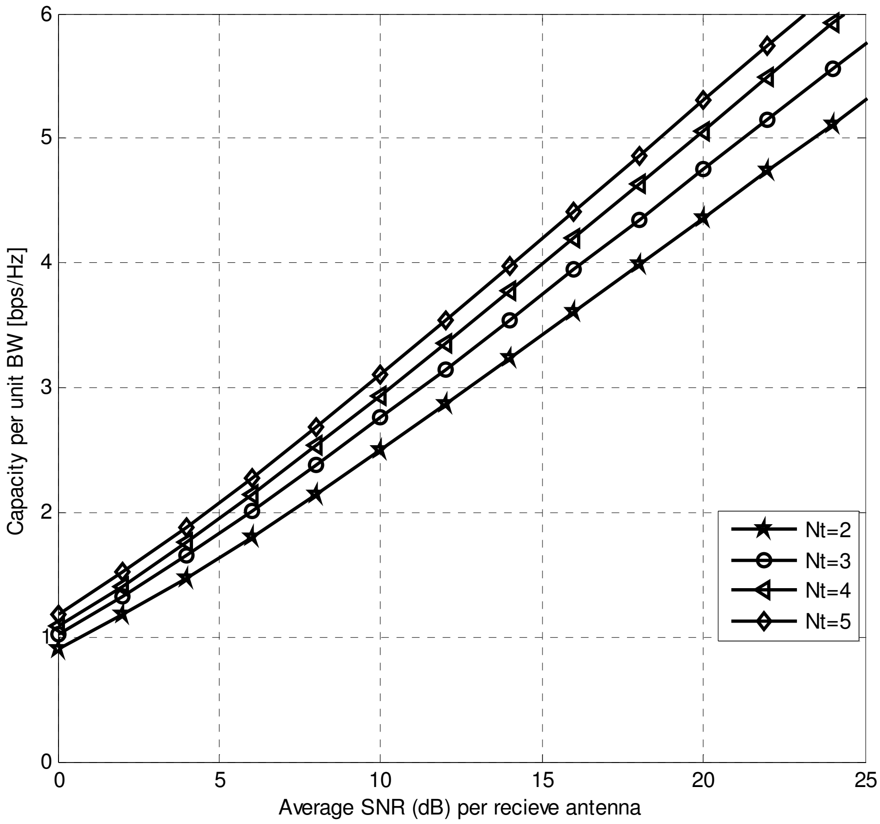

Fig. 3. Ergodic capacity of TAS/OSTBC against $\bar{\gamma}$ over correlated Rayleigh fading channels with $N_r = 1$ & $2 \leq N_t \leq 5$

5 Conclusion

In this paper, we evaluate the performance of the TAS/OSTBC systems over correlated Rayleigh fading channels. The main issue is the correlation between the different antenna subsets occurs from antenna selection in conjunction with the spatial channel correlation. The problem can be reduced by demonstrating the pdf of the largest output SNR as a function of the MGF of all feasible output SNRs. Finally, we derived average BEP, outage probability and ergodic capacity over correlated Rayleigh fading channels. It can be noticed that channel correlation deteriorates the channel capacity and reliability performance.

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