# **Iterative Doppler Shift Estimation in Radio propagation**

AMIRHOSSEIN FEREIDOUNTABAR, GIAN CARLO CARDARILLI

Department of Electronic Engineering University of Rome (Tor Vergata) ITALY Fereidountabar@ing.uniroma2.it,g.cardarilli@uniroma2.it

*Abstract:*-This paper analyzes the bias of a Doppler shift estimator based on adaptive autocorrelation function (ACF) computations and proposes a process for reduction the bias of estimation at high speed movements. Furthermore, a signal-to-noise ratio (SNR) estimator is proposed according to the varying bias in the iterations, where the estimates of the Doppler shift in the first and the final stages of iteration are merged to construct a bias ratio related to the SNR. Simulations show the proposed algorithm achieves accurate estimation within a wide range of velocities and SNRs.

*Key-Words:* -Iterative Estimation; Signal to Noise Ratio; Doppler Shift; High Vehicle Speed; Radio Propagations; High Speed Approximation

## **1** Introduction

In radio communication systems, multipath fading in channels causes power fluctuations in random amounts and therefore decreases the performance of the receiver [1]. Understanding of the fading process has been important research field in radio communications. comprehensive Among considerations on channel fading, accurate estimations of channel parameters, such as signalto-noise ratio (SNR) and Doppler shift  $(f_d)$ , play important roles in communication systems. For instance, in radio systems, accurate SNR and Doppler shift estimations are critical for dynamic channel parameters estimation [3]. More than the SNR and Doppler shift, there are many channel parameters to be considered, such as the time delay [4]. However, our focus is only on the SNR and Doppler shift estimation in this paper.

For estimation of the Doppler shift, lot of techniques have been employed, such as the level crossing rate [3], the channel autocorrelation function (ACF) [5] and the phase difference [6]. Unfortunately, the performance of those techniques will decreased by additive white Gaussian noise (AWGN) significantly.

In [7] a method, can reduce the AWGN influence is proposed. But, when the AWGN is small (i.e. the high SNR region), method of [7] suffers from large estimation bias under the case of high speeds (speeds that usually use by planes), since its series-approximation error is large in this situation. For solving this problem, a linear curve fitting is proposed to refine the estimate of the Doppler shift, which is based on the fact that the process of [7] method shows almost the same estimation curve for different SNRs. Monte Carlo simulations confirm the accuracy of the proposed method. Conventionally, study of the receivers is under this condition that SNR is known. Therefore, only a few methods for the SNR estimation, such as the moment-based SNR estimators in [8, 9, 10] and the correlationfunction-based SNR estimator are exist [11]. Also both kinds of methods suffer from large estimation biases at high SNR regions in high speeds. Furthermore, as pointed out in [5-11], the SNR and Doppler shift must be estimated by two independent estimators, which obviously increases the implementation-cost. Therefore, it is beneficial to combine the two estimators together, which simplifies the implementation. In [7], authors found the estimation bias of different iterations were related to the SNR therefore, it is possible to estimate the SNR according to these varying bias. In fact, when the iterative process ends, the final estimate of Doppler shift is close to

the actual Doppler shift. Then, the Doppler shift estimation ratio of noisy scenarios to noise-free scenarios, i.e., the ratio between the estimate of the Doppler shift in the first round of iteration and that in the final round of iteration, can be approximately obtained. Moreover, the analytical expression of this ratio can be given by a polynomial form with only one variable [7], i.e., the SNR. Consequently, the SNR estimator can be derived by solving the polynomial equation. Simulations are done to test the proposed algorithm, and accurate estimations have been observed in a wide range of velocities and SNRs. The rest of this paper is organized as follows: Section 2 presents the system model under consideration together with the conventional ACF-based estimator of Doppler shift. The estimation bias, the linear fitting as well as the SNR estimator are presented in Section 3. Finally, numerical results are presented and analyzed in Section 4, and conclusions are summarized in Section 5.

# 2 Signal Model

It is assumed that a bandwidth-limited pilot signal is transmitted over Rayleigh fading channels, and the resolvable multi-path fading channels are wide-sense stationary and mutually uncorrelated scattering (WSSUS) processes. After synchronously matching the pilot signal, it is easy to derive the expression of channel estimates according to [12] and the references therein:

$$\hat{c}(n) = c(n) + z(n) \tag{1}$$

where c(n), c(n) and z(n) represent the actual channels, the channel estimates and AWGN (with variance  $\sigma_z^2$ ) at time index *n*, respectively. Generally, the actual channels (c(n)) are modeled as zero-mean complex Gaussian random processes with variance  $\sigma^2$ .

#### 2.1 ACF-based Doppler shift estimator

According to [7], the ACF of Rayleigh channel has the following form:

$$R(k) = \sigma^2 J_0(2\pi f_d k T_s), k \neq 0$$
<sup>(2)</sup>

$$R(0) = \sigma^2 + \sigma_z^2 \tag{3}$$

where  $T_s$  denotes the pilot symbol interval and  $J_0(0)$  is the first kind Bessel function of zero orders. By neglecting the AWGN and using series expansion, there exists:

$$J_0(2\pi f_d kT_s) \approx [1 - (\pi f_d kT_s)^2]$$
 (4)

After collecting *K* samples, the original ACFbased Doppler shift estimator can be presented as follows:

$$\lambda_{M} = \frac{\sqrt{1 - \frac{\gamma}{\gamma + 1}J_{0}(2\pi M f_{d}T_{s})}}{\pi M f_{d}T_{s}}$$
(5)

where  $\hat{f}_{d,M}$  denotes the estimate of Doppler shift using  $\hat{R}(M)$ , and  $\hat{R}(M)$  denotes the estimate of channel ACF at correlation of lag *M*.

#### 2.2 Adaptive ACF method

The estimator of equation (4) is biased due to the influence of AWGN (finite SNR) and the approximation error of R(k). In order to quantify the bias, in [7] defined a ratio between the estimate of the Doppler shift under the noisy scenario and that under the noisy-free scenario is defined as follows:

$$\lambda_1^{SNR=\infty} = \frac{\sqrt{1 - J_0(2\pi M f_d T_s)}}{\pi f_d T_s} \tag{6}$$

where  $\gamma_s = \sigma^2 / \sigma_z^2$  denotes the symbol SNR.  $\hat{f}_d(noisy)$  and  $\hat{f}_d(noisy - free)$  denote the estimates of the Doppler shift with and without influence from AWGN, respectively. In equation (6), the numerator  $\hat{f}_d(noisy)$  and the denominator  $\hat{f}_d(noisy - free)$  are obtained by calculating A R(M) / R(0) through equation (2), where the former requires  $\sigma_z^2 > 0$  and the latter satisfies  $\sigma_z^2 = 0$ . Specifically, if  $\eta(M) = 1$ , which corresponds to the situation of no estimation error, then we have:

$$\eta(M) = \sqrt{\frac{1}{\gamma_s + 1}} \times \sqrt{\gamma_s + \frac{1}{(\pi f_d M T_s)^2}} = 1 \quad (7)$$

By solving the above equation (7), we have:

$$M = \frac{1}{\pi f_d T_s} \tag{8}$$

Then, an adaptive ACF method can be realized by an iterative process illustrated as follows where  $\hat{f}_d(i)$  and M(i) denote the estimate of Doppler shift and the computed correlation lag at the *i*-th iteration, respectively. Due to its iterative characteristic, this method can be called as either the adaptive ACF method or the iterative ACF method.

# **3** The SNR Estimator and the Refined Doppler Shift Estimator

Compared with the original ACF method, the adaptive ACF method in [7] already showed a great performance improvement. However, when the moving-speed increases, the adaptive ACF method produces large estimation biases due to the approximation error, which was caused by the series expansions.

Figure 1 shows the simulation results, which verifies the increases of the estimation bias with the increases of moving-speed.



Fig.1Performance of the iterative ACF method in comparison with the conventional ones

Figure 1 illustrates the accuracy of the iterative ACF estimator for the Doppler shift in comparison with the non-iterative ones. The results verify the improvements in estimation accuracy achieved by the adaptive (iterative) process. Nevertheless, the improvement decreases as the moving-speed increases, which is caused by the series approximation error when the movingspeed is high. However, it is apparent that the adaptive process yields almost the same estimation curve for different SNRs, which implies that a single fitting process can be applied to refine the Doppler shift estimations at different SNRs. In addition, although the estimation curve resulting from the iterative process seems to be nonlinear to some extent, the linear fitting technique can still be used to produce a simple estimator structure and maintain small fitting errors.

#### 3.1 The refined Doppler shift estimator

Next, denote  $f_d^k$  and  $\hat{f}_d^k$  as the *k*-th Doppler shift (unknown) and its estimation in simulations. Here, k = 1; 2;....;Q, and Q represents the total number of Doppler shifts, which is ten in Fig.1. Given an estimation curve of a certain SNR, the following linear expression can be derived approximately:

$$\frac{\hat{f}_{d}^{Q} - \hat{f}_{d}^{1}}{f_{d}^{Q} - f_{d}^{1}} = \frac{\hat{f}_{d}^{k} - \hat{f}_{d}^{1}}{f_{d}^{k} - f_{d}^{1}}$$
(9)

where  $f_d^k$  is provided by the adaptive ACF

method. In equation (9),  $f_d^Q$ ,  $f_d^1$ ,  $f_d^Q$  and  $f_d^1$ are determined in advance based on the simulation results, and thus are known. Note that in fact, equation (8) does not require any SNR information because the proposed adaptive process yields similar estimations at different SNRs. According to equation (8), the Doppler shift estimation can be refined as follows In general, only one single fitting formula is required for all SNRs, since different SNRs lead to almost the same estimation curve as shown in Fig.1. In this study, the fitting formula is set up according to the estimation curve at SNR 5 dB. Moreover, in real-world applications,  $f_d^1$  and  $f_d^Q$  will be chosen as the minimum possible value and the maximum possible value of the Doppler shift, and then their estimates will be obtained according to the method described in Section 2.2.

Finally equation (10) will be used to refine the estimation at other actual Doppler shifts. Note that in [7] the stability and convergence of the adaptive ACF method is proved. The proposed linear fitting technique in our study will not modify these good properties.

#### 3.2 The SNR estimator

As mentioned before in [7], it has been proven that the adaptive ACF method converges after Niterations, resulting in the final estimation  $\hat{f}_d(N) \approx f_d$  at appropriate speeds. Moreover, by equation (10), this relation can be extended to the scenarios of high moving-speed. Specifically, we have:

$$\lambda_1 = \frac{\sqrt{1 - \frac{\gamma}{\gamma + 1} J_0(2\pi f_d T_s)}}{\pi f_d T_s}$$
(10)

Through solving the equation (10), the SNR estimator can be given by

$$\gamma_{s} = \frac{\frac{1}{\pi^{2} f_{d}^{2} T_{s}^{2}} - 1}{\eta(1)^{2} - 1} - 1$$
(11)

$$\lambda_{optM} = \sqrt{1 - \frac{\gamma}{\gamma + 1} J_o(2)}$$
(12)

### **4** Numerical Results and Analysis

Each user transmits quadrature phase shift keying (QPSK) modulated symbols  $b_k(m)$  in blocks of length M. Each symbol is spread by a random spreading sequence  $s_k$  of length N and each chip is transmitted over an individual subcarrier. The number of subcarriers is equal to the length of the spreading sequence. The elements of the spreading sequence are randomly chosen from the

QPSK constellation set  $(\{\pm 1 \pm j\})/\sqrt{2N}$  satisfying:

$$\sum_{n=0}^{N-1} \left| s_k(n) \right|^2 = 1$$
 (13)

Due to the limited space, only the proposed iterative (adaptive) ACF method and a few typical methods developed in [3, 5, 6, 7] are measured in terms of their MSE and accuracy. The detailed simulation parameters are shown in Table 1. The total simulation duration lasts for one thousand slots. The ITU-R M.1225 channel model with six independent paths is utilized at the carrier frequency of 2.11 GHz. In addition, each slot consists of 1056 bits with a bit rate of 3.84 Mbits/s. In each slot, five pilot symbols, each of a length of thirty two bits, are time-multiplexed with four data blocks.



Fig. 2 The refined Doppler shift estimation with a linear curve fitting

Figure 2 illustrates the accuracy of the proposed Doppler shift estimator. The results show that the proposed method yields excellent performance and the bias is very small even when SNR is 0 dB. Comparing Figure 2 with Figure 1, we explicitly see that the proposed linear fitting processing produces trivial deviations between the ideal curve and the estimated curve, which means that it is insensitive to the approximating error in equation (3). Therefore, the proposed method significantly improves the adaptive ACF method [7] in a wide range of mobile speeds, which will be further confirmed by the following MSE comparisons. In addition, the maximum Doppler shift in our simulation is 586 Hz, which corresponds to a speed of 300km/h. Also direction of the transmitter is towards to the receiver in our simulations.

Figure 3 compares the MSE of the proposed method with some typical estimators. Specifically, all results are averaged over the measure of moving-speed. In comparison, the iterative ACF method maintains its MSE performance irrespective of SNR value, and obtains at least one order of magnitude gain when SNR ranges from 0 dB to 20 dB. Moreover, the proposed method further improves the MSE performance and obtains at least one order of magnitude gain compared with the iterative ACF method. Therefore, the proposed method is more suitable for practical mobile systems than the other four methods. In addition, for the sake of clear presentation, we only show the comparison between our proposed method and the other four methods. In fact, our extensive simulations show that our iterative ACF method also outperforms many other estimators, such as logarithmic envelope methods in [2, 13], where their MSEs at SNR 15 dB are about 0.68 and 0.04, respectively.



Fig.3 The mean square error (MSE) performance comparison of Doppler shift estimators



Fig.4 The accuracy and the normalized MSE performance for the proposed SNR estimator in double coordinates

Figure 4 demonstrates the SNR estimation of the proposed estimator. Double coordinates are used to illustrate both the accuracy and MSE from Fig.4, we find that the SNR estimates are very accurate within a wide range of velocities and SNRs. Specifically, the MSE curve shows a U-shape with its bottom at SNR 5 dB. This special shape can be explained for the following two reasons. Firstly, the linear fitting of equation (10) is derived at SNR 5 dB but is applied to all SNRs. Secondly, when the SNR is high, the AWGN influence becomes trivial, but the approximation error of R(k) is still significant.



Fig.5 The ratio of estimation at infinite SNR (M=1)

In Fig.5 when the SNR is infinite we can clearly find that the ratio fluctuates trivially it means less than 1.8%.Hence we can conclude that the effect of the approximation error is negligible for the speed range of 10kmh to 300kmh which is also the usually concerned plane speed range.



Fig.6 The ratio of estimation in different SNR(M=1)

Fig.6 concerns the case of infinite SNRs which we see that the ratio gaps are observable for different

SNRs at low speeds. Hence we can conclude that the effect of the additive noise is the dominant factor for the estimation error of Doppler shift. This effect can be effectively reduced by the adaptive process as shown in our paper. With the help of the adaptive process and the linear fitting the influence of the approximation error is removed effectively and therefore accurate Doppler shift estimates are obtained regardless of the actual approximation error. The order of approximation we have choosen has some errors in Doppler shift estimation but in the higher speedswill be more eerors so the following derivations will reveal that the influence of these errors can be reduced significantly for an appropriate choice of M.then theproposed method has a good performance in wide speed range.

# **5** Conclusion

A joint estimator of the Doppler shift and SNR has been proposed. The estimator is derived according to the varying bias between iterative processes of the adaptive ACF method. The simulation results show that our proposed method achieves accurate estimation for the SNR and Doppler shift in a wide range of velocities and SNRs.

#### References:

- T. Zemen, J. Wehinger, and R. Muller, "Iterative detection and channel estimation for MC-CDMA," in IEEE International Conference on Communications, ICC, Anchorage (Alaska), USA, May 2003.
- [2] A. Sampath, J. Holtzman, 'Estimation of maximum Doppler frequency for handoff decisions', Proc. of IEEE VTC'93, pp.859-862, 1993.
- [3] Z. Ma, Y. Yan, C. Zhao,X. You, 'An improved channel Estimation Algorithm Based on Estimating Level Crossing Rate for CDMA Receiver', Chinese Journal of Electronics, vol.12, no.2, pp.235-238, 2003.
- [4] J. Wen, P. Li, Y. Wen, 'A new method for unbiased time-delay estimation in noisy environments', International Journal of Adaptive Control and Signal Processing, vol.21, no.7, pp.623-634, 2007.
- [5] C. Xiao, K. Mann, J. Olivier, 'Mobile speed estimation for TDMA based hierarchical

cellular systems', IEEE Trans.Vech. Tech., vol.50, no.4, pp.981-991, 2001.

- [6] J. Hua, B. Sheng, X. You, 'The phase probability distribution of general clarke model and its application in Doppler shift estimation', IEEE AWP. Letters, vol. 4, no. 4, pp. 373-377, 2005.
- [7] J. Hua, L. Meng, G. Li, D. Wang, X. You,
  'An Accurate Scheme for Channel Parameter Estimation in Mobile Propagations', IEICE Trans. Electro., vol.E92C, no.1, pp.116-120, 2009.
- [8] N.C. Beaulieu, A.S. Toms, D. R. Pauluzzi, 'Comparison of four SNR estimators for QPSK modulations', IEEE Commun. Letters, vol.4, pp.43-45, 2000.
- [9] Y. Chen, N.C. Beaulieu, 'Estimation of Ricean K parameter and local average SNR from noisy correlated channel samples', IEEE Trans. Wirel. Communication, vol.6, no.2, pp.640-648, 2007.
- [10] Z.J. Xu, K. Wang, J.Y. Hua, G.M. Zhong, 'Blind SNR estimation with empirical characteristic function derivative in highorder modulation system', Proc. IEEE ICCCAS'08, pp.242-245, 2008.
- [11] E. Karastergios, M. Sumanasena, B. Evans, 'Simple SNR estimator for mobile fading channels', IEEE Electronics Letters. vol.39, no.2, pp.244-245, 2003.
- [12] X. Gao, B. Jiang, X. You, Z. Pan, Y. Xue, E. Schulz, 'Efficient channel estimation for MIMO single-carrier block transmission with dual cyclic timeslot structure', IEEE Transactions on Communications, vol.55, no.11, pp.2210-2223, 2007.
- [13] J. Hua, X. You, 'Doppler shift estimation methods in mobile communication systems', J. Southeast Univ., vol.20, no.4, pp.405-412, 2004.