Virtual network resource allocation based on Stackelberg game research

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Abstract:-In view of the network resources allocation problem in network virtualization environment, combined with the Stackelberg game model, this paper proposes a resources allocation scheme that can satisfy the maximum utility of the substrate network and virtual network at the same time. Firstly, it designed a utility function of virtual network based on revenue and cost, and proved in the price of the substrate network is determined, the utility function meet the conditions of concave function, which guarantee the Nash equilibrium point of non-cooperative game between the virtual network. In order to obtain the optimal bandwidth strategy in virtual network and the optimal pricing strategy in substrate network, the paper proposed a distributed iterative algorithm. Finally verified the effectiveness of the algorithm by numerical simulation experiments, and obtained the optimal strategy of players and the sub-game perfect Nash equilibrium.

Keywords:-network virtualization; Stackelberg game; Nash equilibrium; resource allocation

1 Introduction

The Internet has been a great success in the past few decades and has provided a whole new way to access and exchange information. Its success has stimulated enormous growth and wide deployment of network technology and applications. However, the growth and deployment itself is now creating obstacles to future innovations. Specifically, due to the multi-provider nature of the Internet, adopting a new network architecture require not only changes in individual routers and hosts, but also joint agreements among ISPs. The size and scale of today's Internet make the introduction and deployment of new network technology difficult[1-2].

Network virtualization provides a promising way for addressing the ossification of the Internet[1]. Network virtualization means under the premise of retaining the existing Internet architecture, by constructing a virtual network (VN) on the existing network to meet the diverse application requirements[3]. Network virtualization is the virtualization of network equipment, that is, to enhance traditional routers, switches and other equipment, which can support a large number of extensible applications, and the same network equipment can run multiple virtual network devices, such as firewall, VoIP, mobile business. Because the network virtualization shielded a lot of information of the substrate infrastructure resources, which is more convenient to use.

At present, network virtualization technology has been recognized as an effective means to solve the Internet ossification. In a virtualized network infrastructure, diverse virtual networks share a common physical substrate consisting of both links and flexible network platforms capable of hosting multiple virtual routers[4]. The core idea of the network virtualization is using virtual technology to divide the existing network service providers into two separate roles: infrastructure provider (InP) and service provider (SP)[5]. InPs deploy and manage the resources of substrate network (SN), operate and maintain substrate infrastructure, lease network resources to the virtual network, and get charge according to the number of resources. SPs lease resources from one or more InPs to establish a virtual network (VN) and make a profit through the sale of network services to users. There are competition relationships between the InPs. InPs set an appropriate price to attract more virtual network SPs to buy resources to recover the cost. At the same time, there are also competition relationships between SPs, and its goal is to determine the amount of resources rented from the substrate network to maximize their benefits. Competitors' strategies must be taken into account when virtual networks determine the amount of resources leased. The interaction between the underlying network and virtual network is a Stackelberg game problem.

In network virtualization environment, virtual network resources selection on the substrate network is the focus of present study.

A distributed virtual network embedding algorithm achieves embedding through communicating and exchanging messages between agent based physical nodes. Although centralized algorithms could suffer from a single point of failure, the performance and scalability of the proposed distributed algorithm compare unfavorably with those of the centralized algorithms[20].

To maximize the aggregate performance across virtual networks, He et al. Propose an architectural framework called DaVinci to dynamically adapt virtual networks for a customized network substrate, where each physical link periodically reassigns bandwidth among its virtual links. While on a smaller timescale, a distributed protocol is run in each virtual network to maximize the virtual network's own performance objective independently[21].

Three branches of Game Theory are introduced , leader-follower, cooperative, and two-person non-zero sum games, to the study of the Internet pricing issue[22]. In addition, both non-cooperative ans cooperative game are applied to the Internet pricing framework, especially the resource allocation problem. Recently, many resources have used game theoretical methods to analyze the resource allocation problem in computer networks, especially wireless network.

The static resource allocation algorithm[6-8], which is relatively simple and deployment cost is small, but it need to limit special circumstances, such as not considering the dynamic change of the users' requirements, ignoring the limited capacity of the physical nodes and the current situation of the physical nodes and links. Therefore there is no superiority in terms of guarantee the resources equilibrium between virtual networks with this algorithm. A virtual network mapping algorithm is divided into two stages: using the greedy algorithm to map the virtual nodes to physical nodes, and then using the K-shortest path and multi-commodity flow algorithm to map the virtual links to physical paths[9]. The mixed integer programming method put forward to Deterministic VN Embedding (D-VINE) and Randomized VN Embedding (R-VINE)[10]. The virtual network resources allocation algorithm above is to maximize the revenue of substrate network as the premise, does not take the requirement of virtual network into account. A model of resource allocation in virtual network based on non-cooperative game analyzed the existence of Nash equilibrium, and proposed an iterative algorithm to demonstrate the convergence and the effectiveness of the scheme with the experiments[11].

According to the problem of pricing and allocation of resources in network virtualization environment, combing with Stackelberg game model, this paper proposes a scheme that satisfy the maximal revenue of both substrate network and virtual network. In network virtualization environment, using Stackelberg game mechanism, the substrate network serve as a leader of this game while the virtual network is the follower. According to the market information, the substrate network take the lead in making pricing strategy, then the virtual network determine their resources requirement after obtaining the decision of the substrate network, and we maximize the revenue of both VN and SN through dynamic interaction.

2 Related Knowledge

2.1 Basic concept of game theory

Game theory is the mathematical analysis of any situation involving a conflict of interest, with the intent of indicating the optimal choices that, under given conditions, will lead to a desired outcome[12]. It attempts to determine mathematically and logically the actions that players should take to secure the best outcomes for themselves in a wide array of games.Game theory studies the equilibrium problem of the impact of the players in their decision making process and final decision. That is, a player will consider a strategy selection of other players in the choice of strategy. Meanwhile, the strategy this player chose is likely to affect the strategy choice of other players. Generally, game theory include three elements: players, strategy spaces and payoff function.

Players: in game theory, they are the subjects of decision-making that choose their own actions reasonably in order to obtain maximum revenue. In general, we use $i=\{1,2,...,n\}$ to represent the players.

Strategy space: in game theory, the action plan that players can choose are called strategy. We denote s_i as the specific strategy of player i, and denote $S_i=\{s_i\}$ as all of the optional strategy set of player i (also known as the strategy space of player i). If each of the players choose a strategy, the n dimensional vector $s=(s_1,s_2,\ldots\ldots,s_n)$ is called a strategy combination, where s_i is a strategy player i chooses.

Utility function: in game theory, revenue is the expected utility that players get in a specific strategy combination. A basic feature of the game theory is the player's revenue depends not only on its own strategic choice, but also depends on the strategic choice for all players. Or that, revenue is a function of all players in their chosen strategy formed a strategy combination. The revenue of player i is usually denoted by u_i. If one strategy combination is （$s_1,\ldots\ldots,s_n$）, the revenue of each player can be expressed as $u_i=u_i$（$s_1,\ldots\ldots,s_n$）, i=1,2,……,n.

2.2 Nash equilibrium

The most basic equilibrium in game theory is Nash equilibrium[12]. In game theory, Nash equilibrium is a solution concept of a game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players and no player has anything to gain by changing only his own strategy unilaterally. If each player has chosen a strategy and no player can benefit by changing his or her strategy while the other players keep their's unchanged, then the current set of strategy choices and the corresponding payoff constitute a Nash equilibrium.

In the strategy game G= ｛ $S_1,\ldots\ldots,S_n$; $u_1,\ldots\ldots,u_n$｝ with n players, for each player i （i=1, 2,……, n）, s_i^* is a optimal strategy for the strategy combination which is chose by all players except for player i, namely

u_i（$s_1^*,\ldots\ldots,s_{i-1}^*, s_i^*, s_{i+1}^*,\ldots\ldots,s_n^*$）>

ui（$s_1^*,\ldots\ldots,s_{i-1}^*, s_i , s_{i+1}^*,\ldots\ldots,s_n^*$）,

For all the s_i in S_i are established. That is, s_i^* is the solution of the following optimization problem:

$$\max_{s_i\in S_i} u_i\left(s_1^*,\cdots,s_{i-1}^*,s_i,s_{i+1}^*,\cdots,s_n^*\right), \text{ i=1,2,...,n.}$$

Thereupon, strategy combination s*= （ $s_1^*,\ldots\ldots,s_i,\ldots\ldots,s_n^*$ ） is called the Nash equilibrium of game G.

Game between players is to find a Nash equilibrium, however, not all of the games have Nash equilibrium. We have obtained the existence of Nash equilibrium[13-14]. In the game G= ｛ $S_1,\ldots\ldots,S_n$; $u_1,\ldots\ldots,u_n$ ｝, the existence of Nash equilibrium satisfied the following two conditions: (1) for any of the player i, S_i is a non-empty convex set on European space; (2) and for any of the player i, u_i is a continuous concave function.

2.3 Stackelberg game

The Stackelberg leadership model is a strategic game in economics in which the leader firm moves first, after which the followers firms move sequentially[15-16]. If we consider the two-person game program, the leader has the right to make the first decision, and then the follower must optimize their performance within the leader's strategy. The leader and follower have their own decision variables and objective functions, and the leader can only influence (rather than dictate) the reactions of the follower through their own decision variables, while the follower has full authority to decide how to optimize their objective function in view of the decision of the leader.

The concept of Stackerberg equilibrium can be applied to network virtualization because InPs and SPs always play a strong or weak role. InPs want to maximize their profit, but SPs want to minimize their costs; therefor, the developed model consists of a decentralized planning system in which the upper level is the leader and the lower level is the objective of the follower[17].

There are two decision-making levels which include m leaders and n followers. Let M= （ 1,2,……,m） be the set of leaders and the set of followers are defined as N=（1,2,……,n）. Assume that the strategy combination of leaders is x=（x_1, $x_2,\ldots\ldots,x_m$）, strategy set is X, and the strategy combination of followers is y=（y_1, $y_2,\ldots\ldots,y_n$）, strategy set is Y. We have $x\in X, y\in Y$, the utility function of leader i is defined as $Cost_i$（x, y） while the utility function of follower j is Pro_j（x, y）. Then the above problem is a Stackelberg game with multiple leaders_followers.

Use N(x) to represent a set of the Nash equilibrium points of non-cooperative game of followers. Then the problem is expressed as

$$N(x)=\{y^*=\left(y_1^*\cdots y_j^*\cdots y_n^*\right): \quad \Pr o_i\left(x,y_j^*,y_{-j}^*\right)\geq \Pr o_i\left(x,y_j,y_{-j}^*\right) \tag{1}$$

Any strategy y*= （ $y_1^*\ldots\ldots y_j^*\ldots\ldots y_n^*$ ） satisfies the Eq.(1), then y* is called a Nash equilibrium points of non-cooperative game.

Let U= ｛x, y, $Cost_i$（x, y）, Pro_j（x, y）｝（

$i \in M, j \in N$) represent the two-stage Stackelberg game with master-slave problem, which can be expressed as: we have $\left(x^*, y^*\right) \in X \times Y$, making

$$Cost_i\left(x_i^*, x_{-i}^*, y^*\right) > Cost_i\left(x_i, x_{-i}^*, y^*\right) \qquad (2)$$

where $i \in M, y^* \in N(x)$.

If any strategy combination $\left(x^*, y^*\right) \in X \times Y$ satisfies Eq.(2), (x*, y*) is called equilibrium point of a master-slave Stackelberg game, also known as sub-game perfect Nash equilibrium.

3 Network Model

3.1 Stackelberg game model

In network virtualization environment, each of the substrate network is defined as a leader, the leader is a owner of network resources and a strategy maker. Leader make pricing strategy according to market information, and influence requirement of followers through pricing strategy, to maximize its own utility. Each virtual network is defined as a follower, which is the demander of network resources, obtain and purchase network resources depending on different prices the leaders made. Substrate network and virtual network play Stackelberg game to get the equilibrium results. In the state of equilibrium, regardless of the substrate network or virtual network will no longer change their strategy, then their revenue have reached the maximum at this time.

The set of leader is M=(1,2,……,m), while the follower's is N=(1,2,……,n). The price strategy combination of leader i is c=(c_1, c_2,……,c_m), where c_j ($j \in M$) is a unit price formulated for physical link j by substrate network. Assume that x_{ij} represents a virtual network i on the physical link j allocated resources. Define xi=(x_{i1}, x_{i2},……,x_{ij},……,x_{im}), $i \in N, j \in M$, the strategy combination of follower is x=(x_1, x_2,……,x_n). Utility function of substrate network is $Cost_j$(c_j, x), while the virtual network's is Pro_i(c, x_i, x_{-i}). We use S={x, c, Cost(c,x), Pro(c,x)} to represent the Stackelberg game problem in network virtualization environment.

Let x_i be a physical link resources of virtual network i obtained and c be a unit price formulated for physical link by substrate network. The price strategy of virtual network i is v_i. The amount of resources the unit price can buy is represented by q, then we can get $x_i=qv_i$. Total bandwidth requirements for all virtual network using the physical link is $Q = \sum_{i=1}^{n} x_{ij} = q\sum_{i=1}^{n} v_i$. R_j is defined as the capacity of the physical link. Then, x_{ij} is the amount of resources allocated from virtual network i on the jth physical link for $0 \le x_{ij} \le R_j$. The load of entire link j satisfies the formula $\sum_{i=1}^{n} x_{ij} \le R_j$.

Competition model between the substrate network and virtual network consists of two stages. First of all, the substrate network state price strategy c, and inform all the virtual network of this information. Virtual network deploys its own bandwidth scheme x according to the received price strategy c. After determining the price strategy, virtual network competition for network resources becomes a non-cooperative game, and Nash equilibrium is the solution of this game. Secondly, after learning the bandwidth strategy of virtual network, the substrate network will adjust their prices to obtain further optimal utility. Thereinto, (c, x) stands for the strategy distribution of substrate network and virtual network, which is a solution of Stackelberg game.

3.2 Utility function of virtual network

In network virtualization environment, for virtual network, they constitute a non-cooperative game relationship. Each virtual network does not know the information of others and rents physical resources independently to establish network. The utility of virtual network includes two part: revenue and cost. Assume that,there is a virtual network i, Pro_i(c, v_i, v_{-i}) is the utility function which is expressed as:

Pro_i(c, v_i, v_{-i}) =U_i(x_{ij}) -P_i(x_{ij}) -D_i(x_{ij}) (3)

Where, U_i is the revenue function for virtual network i, P_i represents the cost of the virtual network i paid to the substrate network. D_i is the time delay for virtual network brought by substrate network.

In practical applications, the bandwidth resources of virtual network requirements is a very important index. Only consider the performance index of bandwidth to measure the revenue function $U_i=c_vv_iq$ of virtual network, where c_v is a price with the resources on sale by the virtual network. And that, c_v=a-bQ is a linear decreasing function of the total bandwidth requirements Q, where α, β are constants which is greater than zero. Thereupon,

$$U_i(x_{ij}) = (a\text{-}bQ)\, v_i q = \left(\alpha - \beta\sum_{i=1}^{n} x_{ij}\right) x_{ij} \qquad (4)$$

Each virtual network adjusts its bandwidth requirement according to the current pricing strategy c. The ultimate goal for each virtual network is to choose its own optimal strategy x_i^* to maximize its own utility function Pro_i . Namely, satisfy the formula max｛Pro_i（c^*, x_i, x_{-i}^*）｝, where c^* and x_{-i}^* represent that all of other players in game select the optimal price strategy and bandwidth requirement strategy respectively. Using linear price scheme based on bandwidth to measure the cost of the bandwidth of virtual network purchased, so

$$P_i(x_{ij}) = \sum_{j=1}^{m} c_j x_{ij} \qquad (5)$$

Where, j represents a physical link associated with the current virtual network i, x_{ij} is the bandwidth that the virtual network i requests.

The function of delay cost is a polynomial delay function based on the network load[18]. Polynomial delay function can ensure that the bandwidth requirement and pricing strategy of each virtual network both are predictable effective values, and enable the Nash equilibrium of non-cooperative game of virtual network to be existing. If and only if the total load of virtual network satisfy the formula $Q_j \geq R_j$, network congestion will occur. Only when $Q_j < R_j$, the network can ensure the effective transmission of bandwidth. Specifically, when the load of physical link j is Q_j, the delay cost function of virtual network can be expressed as:

$$D_i(x_{ij}) = \begin{cases} \dfrac{\gamma_j}{R_j - Q_j}, & R_j > Q_j \\ \infty, & R_j \leq Q_j \end{cases} \qquad (6)$$

Where γ_j is a constant. So the utility function of virtual network can be expressed as:

$$Pro_i(c, x_i, x_{-i}) = \left(\alpha - \beta\sum_{i=1}^{n} x_{ij}\right) x_{ij} - \sum_{j=1}^{m} c_j x_{ij} - \sum_{j=1}^{m} \frac{\gamma_j}{R_j - Q_j} \qquad (7)$$

3.3 Utility function of substrate network

For the substrate network, regardless of the other costs, the obtained revenue is defined as the the utility of substrate network, so the utility function can be represented by function $Cost_j$（c_j, x）. The revenue of substrate network is the cost that obtains by selling bandwidth to the virtual network. $Cost_j$（c_j, x）$=Q_j c_j$ is the utility function of the j[th] substrate network. From the perspective of the substrate network to analyze that substrate network wants to choose the optimal price to maximize its own revenue. If the price is high, the virtual network may be switched to the hands of competitor's network, so that the load of current access network is low. If the price is very low, even if the substrate network makes the network tend to be saturate may also reduce the revenue. So substrate network must choose an appropriate price. Assume that the optimal pricing strategy of substrate network is c_j^*, its maximum revenue satisfy the formula max｛$Cost_j$（c_j, c_{-j}^*, q^*）｝, where c_{-j}^* and q^* represent that other players in the game all choose the pricing strategy and the bandwidth requirements strategy which meet their own utility best.

4 Game Analysis and Solution

4.1 Nash equilibrium of non-cooperative game

Nash equilibrium is the basic concept of non-cooperative game problem and is the solution of this game. After reaching the Nash equilibrium, the utility function of each player in non-cooperative game has reached the maximum value, and the players changing their strategies unilaterally does not increase their own revenue. But not every non-cooperative game has Nash equilibrium, even some of non-cooperative game has multiple Nash equilibrium.

Theorem 1 for a given pricing strategy c of the substrate network, the utility function of virtual network is non-cooperative game of Pro_i（c, x_i, x_{-i}）and has Nash equilibrium point.

Prove the bidding strategy ｛xi｝ of all the virtual networks is a convex set in Euclidean space. In addition, the utility function Pro_i（c, x_i, x_{-i}） of virtual network is continuous in the strategy space.

Calculate the first-order partial derivative of utility function Pro_i（c, x_i, x_{-i}） of an arbitrary virtual network i.We obtain the formula

$$\frac{\partial Pro_i(c, x_i, x_{-i})}{\partial x_{ij}} = \alpha - \beta x_{ij} - \beta\sum_{i=1}^{n} x_{ij} - c_j - \frac{\gamma_j}{(R_j - Q_j)^2} \qquad (8)$$

Then calculate the second-order partial derivative of the utility function Pro_i（c, x_i, x_{-i}）

$$\frac{\partial^2 \Pr o_i(c, x_i, x_{-i})}{\partial x_{ij}^2} = -2\beta - \frac{2\gamma_j}{(R_j - Q_j)^3} < 0 \quad (9)$$

Where β >0; γ_j >0. Thus it can be seen that, the utility function of virtual network is strictly concave, so, the solution of Nash equilibrium exists[19].

4.2 Solution of the Stackelberg game problem

The method for solving the problem of Stackelberg game is generally backward induction, but which method is a solution of complete information. In network virtualization environment, due to the existence of competition relationship between virtual networks which may not open their own information completely. Thus, this paper presents a distributed iterative algorithm so that each virtual network based on merely the price information of current physical link can get the Nash equilibrium.

We consider a simple dynamic model that the rate of bandwidth change for virtual network is proportional to the gradient of the utility function.

$$\frac{dx_{ij}}{d\tau} = \dot{x}_{ij} = \frac{\partial \Pr o_i(c, x_i, x_{-i})}{\partial x_{ij}} \quad (10)$$

Where τ is a variable of game phase. Then, the bidding change iterative equation of virtual network i in phase τ and τ +1 can be expressed as:

$$x_{ij}(\tau+1) = x_{ij}(\tau) + \theta_i \frac{\partial \Pr o_i(c, x_i, x_{-i})}{\partial x_{ij}} \quad (11)$$

Where θ_i is the bandwidth strategy adjustment parameter of virtual network i. When virtual network reaching the Nash equilibrium, substrate network adjust the price of physical link according to the bidding strategy of virtual network to maximize their revenue. The price iterative equation of any physical link j is:

$$c_j(t+1) = c_j(t) + \lambda c_j(t) \quad (12)$$

Where $\lambda > 0$ represents the adjustment step length of pricing strategy.

The round-robin algorithm of whole process of iteration is expressed as follows:

[1] Step 1: at every moment t, the substrate network develop pricing strategy according to Eq. (12).

[2] Step 2: after obtaining the pricing strategy of physical link, virtual network adjusts their strategies until earning largest revenue according to Eq. (11) at each time interval $\Delta\tau$, then the virtual network get into the optimal state, all the virtual networks reach the Nash equilibrium.

[3] Step 3: if the revenue of substrate network gets the maximum at this time, stop the iteration; otherwise, substrate network repeats step 1 according to the strategy of virtual network at time t+1.

5 Experimental results and analysis

5.1 Parameter setting

The simulation scenario takes two substrate networks and two types of virtual networks into consider. The bandwidths of the two substrate networks are M1=5000 and M2=1000 respectively. Each of the two types of virtual networks include 50 networks with the costs v1=2, v2=1.5.Their initial bid is 1, then change it gradually, and the request of the total bandwidth of virtual network is not more than the actual bandwidth of this link. Assume that in the initial conditions, the initial price of physical link is 1, and α =4.5, β =0.025, γ =1, $\theta = \lambda = 1$. Using MATLAB to simulate this algorithm, with Office 2010 and Visio drawing.

5.2 Simulation result

Analyzing the strategy process based on non-cooperative game between virtual networks after announcing the pricing strategy by substrate network. Fig. 1 analyzes Nash equilibrium of virtual network with different requirements in the iterative process. Fig. 2 is the influence factor affecting the utility function of virtual network v_1=2. Fig. 3 shows the process of reaching Nash equilibrium point of the game between different types of virtual networks.

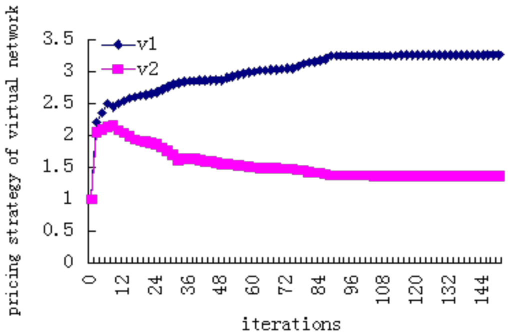

Fig.1 Nash equilibrium of different requirements of virtual network

Fig. 1 shows Nash equilibrium solution of two kinds of different requirements for virtual network. As can be seen in the iterative process that, the virtual network v_1=2 increases gradually, tends to be stable after reaching 5.23, while virtual network v2=1.5 decreases gradually, and tends to be stable when reached 3.17.

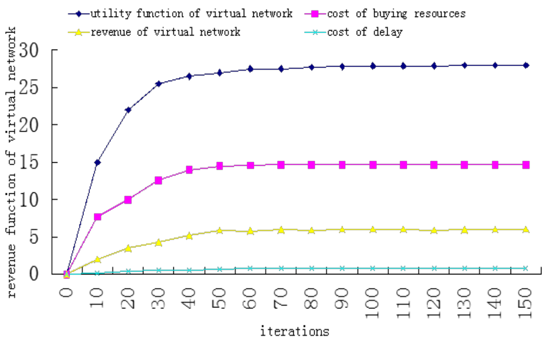

Fig.2 The influence factors of the virtual network v_1=2 revenue

Fig. 2 shows the effect of different factors on the utility function of virtual network v1=2. As can be seen in this figure, with the resources of virtual network requesting to substrate network increase gradually, the utility function and cost of virtual network increase either, despite fluctuations but finally stabilization. Moreover, the delay cost of virtual network has little influence on itself, finally reached a stable in Nash equilibrium.

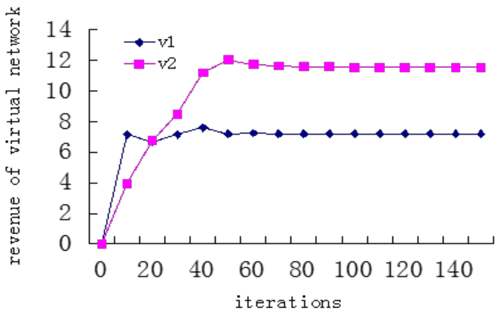

Fig.3 The changes of different requirements of utility function of virtual network in the process of iteration

Fig. 3 shows the changing curve of the two kinds of utility functions of virtual networks in iteration process. The curves are fluctuant, but after several iterations, we can see that the two kinds of utility functions of the virtual networks tend to be stable.

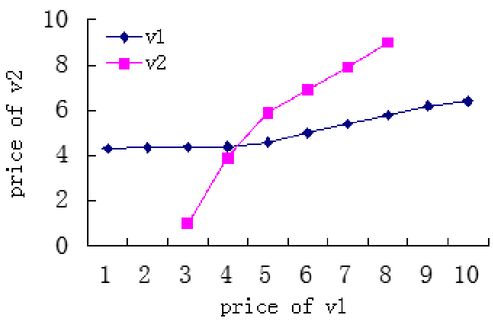

Fig.4 The Nash equilibrium of substrate networks

In Fig. 4, we get the sub-game perfect Nash equilibrium of Stackelberg game in network virtualization environment. The two curves in Fig. 4 is optimal pricing strategy of two substrate networks. The intersection of tow curves is the Nash equilibrium, because the pricing strategy at this pint can satisfy the maximum utility of two substrate network. If any one of substrate network unilateral changes the current price would reduce its revenue.

6 Conclusion

In this paper, under the environment of network virtualization, consider the interests of the InPs and SPs at the same time, use Stackelberg game to analyze the interaction relationship between them. Virtual network forms a non-cooperative problem when the pricing strategy of InP is determined. Verify the existence of Nash equilibrium in non-cooperative game between virtual network by showing that the utility function of SP is concave function. In addition, we put forward a distributed iterative algorithm to obtain the optimal pricing strategy and bandwidth strategy of players. On the premise of given pricing strategy by substrate network, this algorithm ensure that the game between virtual network can converge to Nash equilibrium point. What is more, the substrate network maximize its revenue through iterative, so that the Stackelberg game in entire network virtualization environment reaches sub-game perfect Nash equilibrium.

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