PTS with Alternate Odd-Even Subblock Weighting for PAPR Reduction in OFDM Systems

LINGYIN WANG
University of Jinan
School of Information Science and Engineering
West Road of Nan Xinzhuang 336, 250022 Jinan
People's Republic of China
andrewandpipi@hotmail.com

Abstract: Partial transmit sequence (PTS) is one of multiple signal representation schemes for peak-to-average power ratio (PAPR) reduction in orthogonal frequency division multiplexing (OFDM) systems, which provides good PAPR reduction performance without any signal distortion. However, for conventional PTS (CPTS), large numbers of candidate sequences are required to obtain the satisfactory PAPR reduction performance, which induces large computational complexity. In this paper, a novel PTS with alternate odd-even subblock weighting is proposed. In the proposed scheme, the whole phase weighting process is divided into two stages, i.e., the odd subblock weighting and the even subblock weighting. Moreover, in each stage of phase weighting process, the characteristic of phase weighting sequences is utilized for simplifying the computation of candidate sequences. Simulation results show that compared with CPTS, the proposed PTS scheme can reduce computational complexity clearly with similar PAPR reduction performance.

Key–Words: OFDM, PAPR, PTS, phase weighting

1 Introduction

Orthogonal frequency division multiplexing (OFDM) has been widely adopted for many wireless communication applications due to its high-data-rate transmission and good reliability [1], such as digital video broadcasting (DVB), digital audio broadcasting (DAB) and wireless local area networks (WLAN). However, one of the major shortcomings associated with OFDM systems is that the OFDM signals have large peak-to-average power ratio (PAPR), which reduces the efficiency of power consumption and induces undesired spectral spreading [2].

Recently, some schemes have been proposed for reducing the PAPR of OFDM signals [3, 4], such as clipping and filtering [5], coding [6], tone reservation and injection [7, 8], constellation extension [9], companding [10, 11] and multiple signal representation [12]. Partial transmit sequence (PTS) [13] is one of multiple signal representation schemes for PAPR reduction in OFDM systems. It can provide good PAPR reduction performance without any signal distortion. But this satisfactory PAPR reduction performance is achieved from the premise that a lot of candidate sequences must be generated. The use of sufficient candidate sequences requires large numbers of phase weighting sequences and makes the phase weighting process more complicated, which results in large computational complexity.

In this paper, a novel PTS scheme with alternate odd-even subblock weighting is proposed, which aims to obtain obvious computational complexity reduction and similar PAPR reduction performance compared with conventional PTS (CPTS). In the proposed scheme, the whole phase weighting process is divided into two stages, i.e., the odd subblock weighting and the even subblock weighting. It means that the allowed phase weighting factors are firstly used for weighting the odd subblocks, whereas the even ones remain unchanged; then, the even subblocks are weighted and the odd ones are kept unchanged. Moreover, in each stage of phase weighting process, the characteristic of phase weighting sequences is utilized for simplifying the computation of candidate sequences. Simulation results show that compared with CPTS, the proposed PTS scheme can reduce computational complexity clearly with similar PAPR reduction performance.

This paper is organized as follows. In Section 2 and Section 3, the OFDM system model and conventional PTS are introduced respectively. Section 4 gives the basic ideas of proposed PTS scheme and its computational complexity analysis. In Section 5, the simulation and numerical results are discussed to show the performance of proposed PTS scheme and it is followed by a brief conclusion in Section 6.

2 OFDM System Model

In an OFDM system with N subcarriers, $X_k, k = 0, 1, \cdots, N-1$, represents the kth modulated symbol by phase shift keying (PSK) or quadrature amplitude modulation (QAM). The output signal x_n can be obtained by employing inverse fast Fourier transform (IFFT), given as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, 0 \le n \le N-1 \quad (1)$$

where n is the discrete time index and $j^2 = -1$.

The PAPR of an OFDM signal is defined as the ratio of the maximum power to the average power of this OFDM signal, expressed by

$$\mathrm{PAPR}(x_n) = 10\log_{10} \frac{\max\limits_{0 \le n \le N-1} \{|x_n|^2\}}{E\{|x_n|^2\}} \mathrm{dB} \quad (2)$$

where $\max\limits_{0 \le n \le N-1} \{|x_n|^2\}$ represents the maximum power of an OFDM signal and $E\{|x_n|^2\}$ denotes the average value of the OFDM signal power.

Accordingly, the PAPR distribution of an OFDM system is usually evaluated by complementary cumulative distribution function (CCDF) [3], which can be used for evaluating PAPR reduction performance of any schemes, given as

$$\begin{aligned} CCDF(N, \mathrm{PAPR}_0) &= \Pr\{\mathrm{PAPR} > \mathrm{PAPR}_0\} \\ &= 1 - (1 - e^{-PAPR_0})^N \end{aligned} \quad (3)$$

where PAPR_0 is the fixed value of PAPR and N is the number of subcarriers in an OFDM system.

It is worth mentioning that the PAPR of a continuous time OFDM signal cannot be correctly described by N samples per signal period. That is to say, some signal peaks may be missed. To mitigate this problem, the oversampling is always employed. It can be obtained by LN-point IFFT with $(L-1)N$ zero-padding. According to ref. [14], an oversampling factor $L = 4$ is sufficient to approach the real PAPR results.

3 Conventional PTS

In conventional PTS, the input data sequence is firstly partitioned into several subblocks. Let the input data sequence be $\boldsymbol{X} = [X_0, X_1, \cdots, X_{N-1}]^T$, where N represents the number of subcarriers in an OFDM system. After the input data sequence is partitioned, V disjoint subblocks can be obtained, i.e., $\boldsymbol{X}_v, v = 1, 2, \cdots, V$. Thereupon, the input data sequence $\boldsymbol{X}$ can be expressed by

$$\boldsymbol{X} = \sum_{v=1}^{V} \boldsymbol{X}_v \quad (4)$$

Then, each subblock is multiplied by a phase weighting factor to complete the phase weighting process. Here, let $b_v = \exp(j\varphi_v), \varphi_v \in [0, 2\pi), v = 1, 2, \cdots, V$ be the phase weighting factor for the vth subblock. The candidate sequence $\boldsymbol{x}'$ can be achieved by

$$\begin{aligned} \boldsymbol{x}' &= \mathrm{IFFT}\left\{\sum_{v=1}^{V} b_v \boldsymbol{X}_v\right\} \\ &= \sum_{v=1}^{V} b_v \cdot \mathrm{IFFT}\{\boldsymbol{X}_v\} = \sum_{i=v}^{V} b_v \boldsymbol{x}_v \end{aligned} \quad (5)$$

By employing the exhaustive search, the objective is to obtain the optimal candidate sequence. In the practical application of CPTS, the number of allowed phase weighting factors is always fixed. Suppose there are W allowed phase weighting factors. Without any loss of PAPR reduction performance, the phase weighting factor for the first subblock can be set to one. Accordingly, in order to find the candidate sequence with the minimum PAPR, W^{V-1} possible candidate sequences should be generated. Finally, the one with the minimum PAPR is selected for transmitting. The block diagram of CPTS scheme is given in Figure 1.

Moreover, in order to recover the original data successfully, all the phase weighting sequences should be known at the receiver. Hence, the side information is required, which can be transmitted by accompanying with the OFDM signal. When W^{V-1} possible candidate sequences are generated, $\lceil \log_2 W^{V-1} \rceil$ bits should be required to represent this side information, where $\lceil \cdot \rceil$ denotes the element to the nearest integer toward infinity.

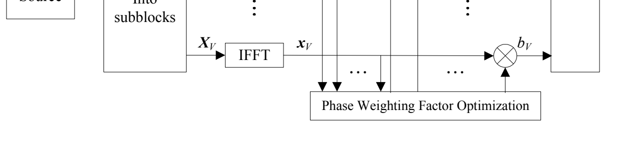

Figure 1: Block diagram of CPTS scheme

4 PTS with Alternate Odd-Even Subblock Weighting

As for CPTS, all the subblocks except the first one need to be weighted by the allowed phase weighting factors and the exhaustive search is adopted for obtaining the optimal candidate sequence, which results in large computational complexity. In this section, an alternate odd-even subblock weighting method is incorporated to reduce computational complexity of CPTS, which can obtain significant computational complexity reduction and similar PAPR reduction performance compared with CPTS.

4.1 Basic ideas of proposed PTS scheme

In the proposed PTS scheme, the whole phase weighting process is divided into two stages, i.e., the odd subblock weighting and the even subblock weighting. Firstly, the odd subblocks are weighted by the allowed phase weighting factors and the even ones remain unchanged. Then, the even subblocks are weighted and the odd ones are kept unchanged.

For example, when the number of subblocks $V = 4$ and the set of allowed phase weighting factors is $\{1, -1\}$ (i.e., $W = 2$), phase weighting sequences in the two stages of phase weighting process can be given as follows.

$$\begin{aligned} &First\ stage : [-1,1,1,1]^T, [1,1,-1,1]^T, \\ &\qquad [-1,1,-1,1]^T \\ &Second\ stage : [1,-1,1,1]^T, [1,1,1,-1]^T, \\ &\qquad [1,-1,1,-1]^T \end{aligned} \tag{6}$$

Besides the above phase weighting sequences, the original subblocks must be considered, i.e., the phase weighting sequence is $[1,1,1,1]^T$, which can be listed in either of the above two stages.

Moreover, for the phase weighting sequences in each stage, an important characteristic of phase weighting sequences can be utilized, i.e., there exist the same phase weighting factors in the corresponding positions of these phase weighting sequences. For example, for the phase weighting sequences in the first stage, the phase weighting factors for even subblocks are same; consider the phase weighting sequences in the second stage, they are going in the opposite direction, i.e., the phase weighting factors for odd subblocks are same. Thereupon, in the process of generating candidate sequences, the important characteristic of phase weighting sequences can be utilized for simplifying the computation of candidate sequences. For phase weighting sequences in the first stage of the above example, the corresponding candidate sequences can be given as

$$\begin{aligned} &[-1,1,1,1]^T : -\boldsymbol{x}_1 + \boldsymbol{x}_2 + \boldsymbol{x}_3 + \boldsymbol{x}_4 \\ &[1,1,-1,1]^T : \boldsymbol{x}_1 + \boldsymbol{x}_2 - \boldsymbol{x}_3 + \boldsymbol{x}_4 \\ &[-1,1,-1,1]^T : -\boldsymbol{x}_1 + \boldsymbol{x}_2 - \boldsymbol{x}_3 + \boldsymbol{x}_4 \end{aligned} \tag{7}$$

where the common term $\boldsymbol{x}_2 + \boldsymbol{x}_4$ can be computed only once. In this way, some additional calculations can be avoided, which reduces computational complexity. The same method can be adopted for simplifying the computation of candidate sequences in the second stage.

Based on the above discriptions, the proposed PTS scheme can be summarized as follows, where the number of subblocks must be bigger than three.

(1) Partition the input sequence into several subblocks and employ IFFT operations to obtain the time-domain subblocks;

(2) Generate phase weighting sequences in the first stage of phase weighting process; the rule is that the odd subblocks are weighted by the allowed phase weighting factors and the even ones remain unchanged;

(3) Generate phase weighting sequences in the second stage of phase weighting process; the rule is that the even subblocks are weighted and the odd ones are kept unchanded;

(4) Use the characteristic of phase weighting sequences to simplify the computation of candidate sequences and obtain all the candidate sequences;

(5) Calculate the PAPR values of all the candidate sequences and choose the one with the lowest PAPR for transmitting.

For easily understanding the algorithm of proposed PTS scheme, we can still take the above example. Because the number of subblocks is four, the time-domain subblocks $\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4$ can be obtained after Step (1). In terms of Step (2) and Step (3), phase weighting sequences in the two stage of phase weighting process can be achieved, shown in Eq. (6). Here, the phase weighting sequence corresponding to the original subblocks (i.e., $[1,1,1,1]^T$) is listed in the first stage of phase weighting process. All the phase weighting sequences are shown in Table 1.

Then, the corresponding candidate sequences can be achieved as follows.

$$\begin{cases} \boldsymbol{y}_1 = \boldsymbol{x}_1 + \boldsymbol{x}_2 + \boldsymbol{x}_3 + \boldsymbol{x}_4 = \boldsymbol{x}_1 + \boldsymbol{x}_3 + \boldsymbol{C} \\ \boldsymbol{y}_2 = -\boldsymbol{x}_1 + \boldsymbol{x}_2 + \boldsymbol{x}_3 + \boldsymbol{x}_4 = -\boldsymbol{x}_1 + \boldsymbol{x}_3 + \boldsymbol{C} \\ \boldsymbol{y}_3 = \boldsymbol{x}_1 + \boldsymbol{x}_2 - \boldsymbol{x}_3 + \boldsymbol{x}_4 = \boldsymbol{x}_1 - \boldsymbol{x}_3 + \boldsymbol{C} \\ \boldsymbol{y}_4 = -\boldsymbol{x}_1 + \boldsymbol{x}_2 - \boldsymbol{x}_3 + \boldsymbol{x}_4 = -\boldsymbol{x}_1 - \boldsymbol{x}_3 + \boldsymbol{C} \end{cases} \tag{8}$$

Table 1: All the phase weighting sequences with *W*=2 and *V*=4

First Stage		Second Stage	
$\boldsymbol{B}_1$	$[1,\ 1,\ 1,\ 1]^T$	$\boldsymbol{B}_5$	$[1, -1,\ 1,\ 1]^T$
$\boldsymbol{B}_2$	$[-1,\ 1,\ 1,\ 1]^T$	$\boldsymbol{B}_6$	$[1,\ 1,\ 1, -1]^T$
$\boldsymbol{B}_3$	$[1,\ 1, -1,\ 1]^T$	$\boldsymbol{B}_7$	$[1, -1,\ 1, -1]^T$
$\boldsymbol{B}_4$	$[-1,\ 1, -1,\ 1]^T$		

$$\left\{ \begin{array}{l} \boldsymbol{y}_5 = \boldsymbol{x}_1 - \boldsymbol{x}_2 + \boldsymbol{x}_3 + \boldsymbol{x}_4 = -\boldsymbol{x}_2 + \boldsymbol{x}_4 + \boldsymbol{C}' \\ \boldsymbol{y}_6 = \boldsymbol{x}_1 + \boldsymbol{x}_2 + \boldsymbol{x}_3 - \boldsymbol{x}_4 = \boldsymbol{x}_2 - \boldsymbol{x}_4 + \boldsymbol{C}' \\ \boldsymbol{y}_7 = \boldsymbol{x}_1 - \boldsymbol{x}_2 + \boldsymbol{x}_3 - \boldsymbol{x}_4 = -\boldsymbol{x}_2 - \boldsymbol{x}_4 + \boldsymbol{C}' \end{array} \right. \quad (9)$$

where the common terms $\boldsymbol{C} = \boldsymbol{x}_2 + \boldsymbol{x}_4$ and $\boldsymbol{C}' = \boldsymbol{x}_1 + \boldsymbol{x}_3$ are calculated only once.

Finally, among all the candidate sequences, the one with the minimum PAPR is chosen for transmitting.

Moreover, for the proposed PTS scheme, the side information is still required. Since the number of candidate sequences in proposed scheme should be considered in the following two cases,

$$\left\{ \begin{array}{ll} 2^{\frac{V+1}{2}} + 2^{\frac{V-1}{2}} - 1 & V \text{ is odd} \\ 2^{\frac{V}{2}+1} - 1 & V \text{ is even} \end{array} \right. \quad (10)$$

the side information in proposed PTS scheme can be shown as

$$\left\{ \begin{array}{ll} \left\lceil \log_2 \left(2^{\frac{V+1}{2}} + 2^{\frac{V-1}{2}} - 1 \right) \right\rceil bits & V \text{ is odd} \\ \left\lceil \log_2 \left(2^{\frac{V}{2}+1} - 1 \right) \right\rceil bits & V \text{ is even} \end{array} \right. \quad (11)$$

4.2 Computational complexity analysis

For CPTS and proposed PTS, the number of IFFT operations is decided by the number of subblocks. Hence, when the number of subblocks is same, these two schemes need the same number of IFFT operations to obtain the time-domain subblocks. Based on this, the computational complexity for searching the optimal candidate sequence is mainly taken into account. Suppose the number of allowed phase weighting factors is *W* and *V* subblocks are generated.

For CPTS, W^{V-1} candidate sequences need to be generated, which results in $LN\left(V-1\right)W^{V-1}$ complex multiplications and $LN\left(V-1\right)W^{V-1}$ complex additions.

In proposed PTS scheme, for each stage of phase weighting process, there exist the common terms for simplifying the computation of candidate sequences, which reduces computational complexity. Thus, the computational complexity of proposed PTS scheme can be given by

$$ComplexMul.: \left\{ \begin{array}{ll} \frac{LN}{2}\left[\left(V+1\right)\left(2^{\frac{V+1}{2}}+1\right)+\left(V-1\right)2^{\frac{V-1}{2}}\right] & V \text{ is odd} \\ LNV\left(\frac{1}{2}+2^{\frac{V}{2}}\right) & V \text{ is even} \end{array} \right. \quad (12)$$

$$ComplexAdd.: \left\{ \begin{array}{ll} \frac{LN}{2}\left[\left(V+1\right)2^{\frac{V+1}{2}}+\left(V-1\right)2^{\frac{V-1}{2}}+V-3\right] & V \text{ is odd} \\ LN\left(V2^{\frac{V}{2}}+\frac{V}{2}-2\right) & V \text{ is even} \end{array} \right. \quad (13)$$

To show the advantage in computational complexity against CPTS, computational complexity reduction ratio (CCRR) [10] is usually employed, expressed as

$$\text{CCRR} = \left(1 - \frac{\text{Complexity of proposed scheme}}{\text{Complexity of CPTS}}\right) \times 100\% \quad (14)$$

In terms of the CCRR, the detailed numerical results will be given in the following section.

5 Simulation and Numerical Results

To investigate the performance of proposed PTS scheme, the simulation and numerical results are incorporated. Here, 10^5 random OFDM signals are generated for achieving the PAPR reduction performance. The other parameters are the number of subcarriers $N = 256$, QPSK modulation and the oversampling factor $L = 4$. For CPTS and proposed PTS, the set of allowed phase weighting factors $\{1, -1\}$ (i.e., $W = 2$) is adopted and all the simulation results are based on adjacent subblock partition method.

Figure 2 and Figure 3 gives the PAPR reduction performance comparisons between proposed PTS and CPTS in terms of CCDFs.

It can be seen from Figure 2 and Figure 3 that compared with CPTS, the proposed PTS scheme can obtain the similar PAPR reduction performance. For the proposed PTS scheme, the negligible degradation in PAPR reduction performance is the price for significant reduction in computational complexity.

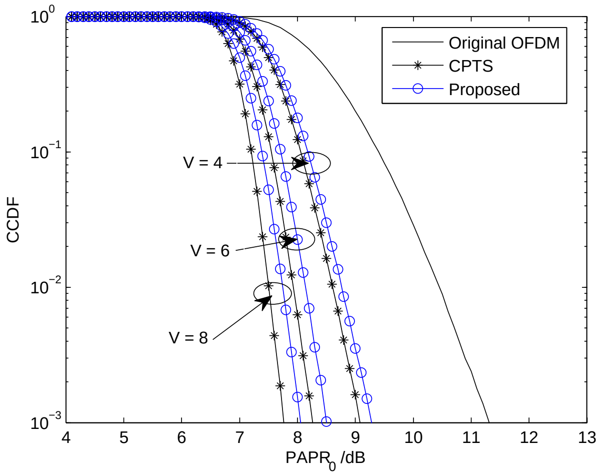

Figure 2: Comparisons in PAPR reduction performance between proposed PTS and CPTS when the number of subblocks is even

For computational complexity, based on the definition of CCRR, we can achieve multiplicative CCRR (i.e., $\mathrm{CCRR}^{\times}$) and additive CCRR (i.e., CCRR^{+}). Table 2 shows multiplicative CCRRs and additive CCRRs of proposed PTS over CPTS.

It is shown in Table 2 that compared with CPTS, the proposed PTS scheme can obtain dramatic reduction in computational complexity. As shown in Table 2, when the number of allowed phase weighting factors *W* is fixed, more and more reduction in computational complexity can be achieved with the number of subblocks *V* increasing.

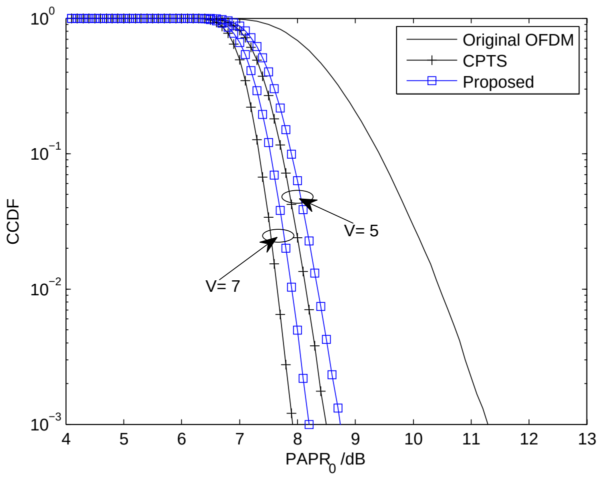

Figure 3: Comparisons in PAPR reduction performance between proposed PTS and CPTS when the number of subblocks is odd

Table 2: Multiplicative CCRRs and additive CCRRs of proposed PTS over CPTS

	$W=2$				
	$V=4$	$V=5$	$V=6$	$V=7$	$V=8$
$\mathrm{CCRR}^{\times}$	25 %	45.3%	68.1 %	76%	85.3%
CCRR^{+}	33.3 %	48.4 %	69.4%	76.6%	85.5%

6 Conclusion

In this paper, a novel PTS with alternate odd-even subblock weighting is proposed. In the proposed scheme, the whole phase weighting process is divided into two stages, i.e., the odd subblock weighting and the even subblock weighting. Moreover, in each stage of phase weighting process, the characteristic of phase weighting sequences is utilized for simplifying the computation of candidate sequences. Simulation results show that compared with CPTS, the proposed PTS scheme can reduce computational complexity clearly with similar PAPR reduction performance.

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