### QoS-Aware Power Control Methods for Two-Way Amplify-and-Forward Relay Systems

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*Abstract:* - We consider power control methods for two-way amplify-and-forward (AF) relay systems. Traffics with different quality-of-service (QoS) requirements are exchanged between two transmit nodes where one of the traffics is a rate-constrained (RC) traffic and the other is a best-effort (BE) traffic. Based on the QoS feasibility of RC traffic, we propose two types of power control methods to maximize the transmission rate of BE traffic. In case QoS requirement of RC traffic is satisfied, we derive a power control method that maximizes the transmission rate of BE traffic. On the contrary, when QoS requirement of RC traffic is not feasible, we propose a power control method to maximize the transmission rate of BE traffic considering the automatic-repeat-request (ARQ) protocols. Through the analysis and numerical results, we verify the performance improvement of the proposed method. Compared with the equal power allocation method, the overall transmission rate of the BE traffic is increased up to 30 % depending on the relay location and the QoS requirement of the RC traffic.

Key-Words: - Amplify-and-Forward, quality of service, power control, relay, outage.

#### **1** Introduction

Two-way channel introduced first by Shannon [1] has drawn a renewed interest due to the improvement of the spectral efficiency when it is deployed in relay communication systems [2], [3]. With the knowledge of terminals' signals and broadcast nature of wireless medium, two-way relay systems can compensate the 1/2 pre-log factor of one-way relay systems under half-duplex constraint. Recently, various types of two-way relaying protocols are developed for both amplify-and-forward (AF) and decode-and-forward (DF) relaying systems [4]–[10].

Compared with the DF scheme, AF relayig has less processing burden on the relay [11]. Achievable rate region of AF two-way relay systems is found in [2] and [8] where it was assumed that the transmit powers of the transmit nodes are equal and fixed. However, it is well known that the system performance can be improved by applying power control methods [12]–[16]: the improvement of the channel estimation performance by means of the power control is shown in [12], the maximization of the instantaneous sum rate and the average sum rate with sum power constraint is shown in [13], [14], the maximization of the instantaneous sum rate in OFDM system with individual node power constraint is shown in [15], and the minimization of the sum error probability in the distributed space-time coding system with sum power constraint is shown in [16]. Most of the existing power control strategies in AF two-way relay systems have focused on homogeneous users with the same service and demand. Meanwhile, future wireless networks are expected to support various communication services with diverse quality of service (QoS) requirements [17].

In this paper, we propose a power control method for AF two-way relay systems when traffics with different QoS requirements are bi-directionally exchanged where one of traffic types is rateconstrained (RC) traffic and the other is best-effort (BE) traffic. In the proposed method, different power control strategies are applied depending on whether the power resources are enough to satisfy the QoS requirement of RC traffic or not. In case when the QoS requirement of RC traffic is not satisfied, the proposed method is designed to improve the transmission rate of the BE traffic.

# 2 System Model and Problem Formulation

We consider a two-way relay system shown in Fig.1, where user node  $T_1$  and  $T_2$  exchange messages through the AF relay node T<sub>3</sub> over two time slots. Suppose that user node  $T_1$  transmits RC traffic, and user node T<sub>2</sub> transmits BE traffic. During the first time slot, called multiple access (MAC) phase, user nodes  $T_1$  and  $T_2$  transmit its information symbols to the relay node  $T_3$ , and then, the relay node broadcasts a received signal to both  $T_1$  and  $T_2$  during the following time slot, called broadcast (BC) phase. Channels are assumed to experience block fading, in other words, channels remain unchanged over consecutive two time slots and change randomly every two time slots. The channel response between the user node  $T_i$  and the relay node  $T_3$  is denoted by  $h_i$ , where  $i \in \{1, 2\}$  is user index. During the MAC phase, the relay node receives signals from both  $T_1$  and  $T_2$  and the received signal can be written as

$$y_3 = \sqrt{p_1}h_1x_1 + \sqrt{p_2}h_2x_2 + w_3, \tag{1}$$

where  $p_i$  and  $x_i \sim CN(0, 1)$ , i = 1, 2, denote the transmit power and information symbol of the user node  $T_i$ , respectively, and  $w_3 \sim CN(0, \sigma_w^2)$  means background noise at the relay node. We assume a complex channel gain  $h_i \sim CN(0, \sigma_h^2/d_i^\alpha)$ , where  $d_i$  is the distance between user node  $T_i$  and relay node  $T_3$ , and  $\alpha$  is the path loss exponent having a value between 1 and 3. Without loss of generality, we assume  $\sigma_h^2 = 1$  and  $\sigma_w^2 = 1$ . During the following BC phase, user node *i* receives signal from the relay node and the received signal is given by

$$y_i = \sqrt{p_1} h_i g h_1 x_1 + \sqrt{p_2} h_i g h_2 x_2 + h_i g w_3 + w_i, \quad (2)$$

where  $g \triangleq \sqrt{p_3 / (p_1 |h_1|^2 + p_2 |h_2|^2 + 1)}$  is the relay gain. Throughout this paper, it is assumed that  $T_i$ knows the full channel state information (CSI). When i = 1, the first term on the right-hand side of (2) is the self-interference and it can be eliminated with the knowledge of self-interference channel,  $\sqrt{p_1}h_1gh_1$ . After cancellation of the self-interference, the received signal at user node 1 becomes

$$\tilde{y}_1 = \sqrt{p_2} h_1 g h_2 x_2 + h_1 g w_3 + w_1.$$
 (3)



Fig. 1. AF two-way relaying system model (solid arrows mean MAC-phase transmissions and dotted arrows mean BC-phase transmissions); best-effort traffics are for user node  $T_1$  and rate-constrained traffics are for user node  $T_2$ .

If we denote the signal to noise ratio (SNR) of  $\tilde{y}_1$  by  $\gamma_{BE}$ , then  $\gamma_{BE}$  can be written as

$$\gamma_{BE} = \frac{p_2 p_3 |h_1|^2 |h_2|^2}{(p_1 + p_3) |h_1|^2 + p_2 |h_2|^2 + 1}.$$
 (4)

Similarly, if we denote the SNR at user node T2 by  $\gamma_{RC}$ , then  $\gamma_{RC}$  can be written as

$$\gamma_{RC} = \frac{p_1 p_3 |h_1|^2 |h_2|^2}{p_1 |h_1|^2 + (p_2 + p_3) |h_2|^2 + 1}.$$
 (5)

#### **3** Proposed Power Control Method

In this section, we study a power control problem with individual node transmit power constraints when RC and BE traffics are exchanged through AF two-way relaying system shown in Fig.1. Because RC traffic has higher priority than BE traffic, we separate the power control methods depending on whether the power resources are enough to meet the QoS requirement of RC traffic or not.

## 3.1 When power resources are sufficient to satisfy the QoS requirement of the RC traffic

If power resources are sufficient, both RC and BE traffics are transmitted so that the QoS requirement of RC traffic is satisfied. Therefore, the power allocation problem is represented by

$$(p_1^*, p_2^*, p_3^*) = \arg \max_{p_1, p_2, p_3} \frac{1}{2} \log_2(1 + \gamma_{BE}),$$
 (6)  
subject to  $\gamma_{RC} \ge \gamma_{th}$ , and  $0 \le p_i \le p_{i,\max}$  for all *i*

where  $p_{i,\text{max}}$  is the maximum transmit power at node *i*,  $\gamma_{th} \triangleq 2^{(2R_{RC}-1)}$ , and  $R_{RC}$  is the rate constraint of RC traffic.

In order to maximize the transmission rate of BE traffic while satisfying  $\gamma_{RC} \ge \gamma_{th}$ , it is required to set  $\gamma_{RC} = \gamma_{th}$  because power resources are sufficient. Then, (6) can be rewritten as

$$(p_1^*, p_2^*, p_3^*) = \arg \max_{p_1, p_2, p_3} \frac{1}{2} \log_2(1 + \gamma'_{BE}),$$
 (7)

subject to  $0 \le p_1 = \frac{\gamma_{th}((p_2 + p_3) | h_2 |^2 + 1)}{|h_1|^2 (p_3 | h_2 |^2 - \gamma_{th})} \le p_{1,\max},$ and  $0 \le p_i \le p_{i,\max}$  for  $i \in \{2, 3\}$ 

where  $\gamma'_{BE}$  is a different representation of  $\gamma_{BE}$ eliminating  $p_1$  and it is represented by

$$\gamma_{BE}' = \frac{p_2 |h_1|^2 |h_2|^2 (p_3 |h_2|^2 - \gamma_{th})}{|h_1|^2 (p_3 |h_2|^2 - \gamma_{th}) + p_3 |h_2|^4 + |h_2|^2 + \gamma_{th} |h_2|^2}.$$
(8)

The solution of (7) is obtained by an exhaustive search over 2-dimensional space  $(p_1, p_2)$ , not over 3-dimensional space  $(p_1, p_2, p_3)$ . We can further reduce the computational complexity of finding the solution of (7) as follows: Because  $\log(1 + \gamma'_{BE})$  is an increasing function over  $p_3$  and  $p_1$  is increasing over  $p_3$ , the solution of (7) can be found by setting  $p_3 = p_{3,\max}$  and searching over  $p_2$  only so as to satisfy  $0 \le p_1 \le p_{1,\max}$ .

### **3.2** When power resources are insufficient to satisfy the QoS requirement of the RC traffic

If power resources are not enough to satisfy the QoS requirement of RC traffic, *i.e.*,  $\gamma_{RC} < \gamma_{th}$ , then  $\gamma_{RC}$  should be maximized because RC traffic has a higher priority than BE traffic. Maximum of  $\gamma_{RC}$  is written by

$$\overline{\gamma}_{RC} \triangleq \frac{p_{1,\max} p_{3,\max} |h_1|^2 |h_2|^2}{p_{1,\max} |h_1|^2 + p_{3,\max} |h_2|^2 + 1},$$
(9)

and it is obtained by sacrificing the transmission rate of the BE traffic, *i.e.*,  $p_2 = 0$ . For given channel condition and power constraints, if  $\overline{\gamma}_{RC} < \gamma_{th}$ , the outage of RC traffic is inevitable and packets in those time instances (or time slots in block fading channels) cannot be delivered without errors. Usually, in communication standard such as 3rd generation partnership project (3GPP) long term evolution (LTE), those erroneous packets are discarded and the retransmission protocols such as Repeat Request Automatic (ARQ) are performed <sup>1</sup>[18]. Therefore, during time instance (or time slot) in which  $\overline{\gamma}_{RC} < \gamma_{th}$ , we propose a power control method to maximize the transmission rate of the BE traffic, instead of transmitting RC traffic which would be discarded and retransmitted. Similarly to  $\overline{\gamma}_{RC}$ , maximum of  $\overline{\gamma}_{BE}$  for the BE traffic is obtained by not allocating any power to the RC traffic, *i.e.*,  $p_1 = 0$ . Then, the corresponding transmission rate is represented by

$$\frac{1}{2} \log(1 + \overline{\gamma}_{BE})$$

where 
$$\overline{\gamma}_{BE} \triangleq \frac{p_{2,\max} p_{3,\max} |h_1|^2 |h_2|^2}{p_{3,\max} |h_1|^2 + p_{2,\max} |h_2|^2 + 1}$$

In [17], when power resources are not enough to render QoS requirements feasible, a connection admission control policy is presented. The admission control policy in [17] drops the lower priority traffic even when QoS requirement of traffic with higher priority cannot be satisfied. The proposed method, however, drops traffic with higher priority. Thereby, instead of abusing the power resource for improving higher priority traffic of which infeasible QoS is inevitable, the proposed method uses the power resource for improving the transmission rate of traffic with lower priority.

#### **4** Performance Analysis

To analyze the performance of the proposed power control method, we derive the outage probability of the RC traffic and the upper bound of overall average transmission rate of BE traffic including the cases of both  $\overline{\gamma}_{RC} \ge \gamma_{th}$  and  $\overline{\gamma}_{RC} < \gamma_{th}$ .

#### 4.1 Outage probability of RC traffic

The outage probability of RC traffic is represented by

$$\frac{\Pr(\overline{\gamma}_{RC} < \gamma_{th}) = 1 - 2\exp(1 - (\lambda + \mu)\gamma_{th}) \times}{\sqrt{\lambda\mu\gamma_{th}(\gamma_{th} + 1)}K_1(2\sqrt{\lambda\mu\gamma_{th}(\gamma_{th} + 1)})}$$
(10)

<sup>&</sup>lt;sup>1</sup> We considered sliding window type ARQ protocols such as 'Stop-and-wait', 'Go-Back-N', and 'Selective-Repeat-ARQ', that tells the transmitter to determine which (if any) packets need to be retransmitted.

where  $\lambda \triangleq d_1^{\alpha} / p_{1,\max}$ ,  $\mu \triangleq d_2^{\alpha} / p_{3,\max}$ ,  $K_{\nu}(\cdot)$  is the  $\nu$  th order modified Bessel function of the second kind [19].

*Proof:* Let  $p_{1,\max} |h_1|^2 = X_1$  and  $p_{2,\max} |h_2|^2 = X_2$ , then  $X_1 \sim EXP(\lambda)$  and  $X_2 \sim EXP(\mu)$  are statistically independent exponential random variable, where EXP(a) denotes an exponential distribution with a hazard rate *a*. From the Definition 1 in [20],  $\overline{\gamma}_{RC}$  is the MacDonald variable with parameters  $\lambda$  and  $\mu$ , and its cumulative distribution function (CDF), which is equivalent to the outage probability, is presented at Theorem 1 in [20]. This completes the proof.

### 4.1 Upper bound of BE traffic's overall average transmission rate

The overall average transmission rate of BE traffic which is composed of average transmission rates when RC traffic is in outage  $\overline{\gamma}_{RC} < \gamma_{th}$  or in non-outage  $\overline{\gamma}_{RC} \ge \gamma_{th}$  is represented by

$$E\left(\frac{1}{2}\log(1+\gamma_{BE})\right) = \Pr(\overline{\gamma}_{RC} < \gamma_{th})E\left(\frac{1}{2}\log(1+\overline{\gamma}_{BE})\right) + \Pr(\overline{\gamma}_{RC} \ge \gamma_{th})E\left(\frac{1}{2}\log(1+\overline{\gamma}_{BE}')\right).$$
(11)

In order to calculate (11), first we need to compute the average rate of BE traffic in case  $\overline{\gamma}_{RT} < \gamma_{th}$  and it can be written as

$$E_{\overline{\gamma}_{BE}|\overline{\gamma}_{RC}<\gamma_{th}}\left(\frac{1}{2}\log(1+\overline{\gamma}_{BE})\right).$$
 (12)

It is very difficult to derive the expectation in (12) due to unwieldy conditional probability density function (pdf). Thus, in order to make the derivation easier, we assume that all transmit power constraints are equal, *i.e.*,  $p_{i,\text{max}} = p_{\text{max}}$ . Then, (12) is derived as follows:

$$E_{\overline{\gamma}_{BE}|\overline{\gamma}_{RC} < \gamma_{th}} \left(\frac{1}{2}\log(1+\overline{\gamma}_{BE})\right)$$

$$= \frac{1}{\Pr(\gamma_{RC} < \gamma_{th})} \left(\Phi_{0}(\gamma_{th}) + \Phi_{1}(\gamma_{th})\right)$$
(13)

where

$$\Phi_0(\gamma_{th}) \triangleq \int_0^{\gamma_{th}} \log(1+z) \exp(-(\lambda+\mu)z) \lambda \mu(2z+1) \times$$

 $K_0(2\sqrt{\lambda\mu z(z+1)})dz$  and  $\Phi_1(\gamma_{th}) \triangleq \int_0^{\gamma_{th}} \log(1+z) \times \exp(-(\lambda+\mu)z)(\lambda+\mu)\sqrt{\lambda\mu z(z+1)}K_1(2\sqrt{\lambda\mu z(z+1)})dz$ , and a detailed derivation is given in Appendix 1.

Secondly, we need to derive the average transmission rate of BE traffic when  $\overline{\gamma}_{RC} \ge \gamma_{th}$  which is given by

$$E_{\overline{\gamma}'_{BE}|\overline{\gamma}_{RC} \geq \gamma_{th}}\left(\frac{1}{2}\log(1+\overline{\gamma}'_{BE})\right).$$
 (14)

It is also difficult to derive the expectation in (14) due to the intractable conditional pdf. To make the derivation mathematically tractable, the assumption of  $p_{i,\max} = p_{\max}$  is also applied and furthermore the assumption of  $\gamma_{th} \rightarrow 0$  is added.  $\gamma_{th} \rightarrow 0$  provides the upper bound of transmission rate. Resultantly, (14) is upper bounded as follows:

$$E_{\overline{\gamma}_{BE}|\overline{\gamma}_{RC} \geq \gamma_{th}}\left(\frac{1}{2}\log(1+\overline{\gamma}_{BE}')\right) \leq \frac{1}{1-\Pr(\overline{\gamma}_{RC} < \gamma_{th})} \left(\Phi_{0}'(\gamma_{th}) + \Phi_{1}'(\gamma_{th})\right),$$
(15)

where

$$\begin{split} \Phi_0'(\gamma_{th}) &\triangleq \int_{\gamma_{th}}^{\infty} \log(1+z) \exp(-(\lambda+\mu)z) \lambda \mu(2z+1) \times \\ K_0(2\sqrt{\lambda\mu z(z+1)}) dz \text{ and } \Phi_1'(\gamma_{th}) &\triangleq \int_{\gamma_{th}}^{\infty} \log(1+z) \times \\ \exp(-(\lambda+\mu)z)(\lambda+\mu)\sqrt{\lambda\mu z(z+1)} K_1(2\sqrt{\lambda\mu z(z+1)}) dz \text{ see Appendix 2 for detailed derivation. Then, with } \\ (10), (13) \text{ and } (15), \text{ the upper bound of overall average transmission rate of BE traffic (11) can be calculated.} \end{split}$$

#### **5** Simulation results

In order to evaluate the proposed power control method, we simulated for randomly generated  $10^5$  channels. For simplicity, we set the total distance between user node  $T_1$  and  $T_2$  to be 1, *i.e.*,  $d_1 + d_2 = 1$  and evaluate the outage probability of RC traffic and the average transmission rate of BE traffic. Also, we assume *i.i.d.* complex channels  $h_1$ ,

 $h_2$  with  $\alpha = 3$ ,  $\sigma_w^2 = 1$  and  $p_{i,\max} = p_{\max} = 10$ . Because conventional power control methods for AF two-way relaying systems do not discriminate the QoS of traffic, we simulate the equal power allocation scheme for comparison, where maximum transmit powers are allocated at each node.



Fig. 2. Outage probability of RC traffic.



Fig. 3. The average transmission rate of BE traffic when RC traffic is in outage.

Additionally, to verify the impact of outage probability of RC traffic, simulations are conducted at  $\gamma_{th} = 3$  and 8.

Fig. 2 shows the outage probability of RC traffic. From the figure, we can observe that the outage probability of proposed power control method is smaller (or better) than that of the equal power allocation method, because  $\overline{\gamma}_{RC}$  is higher than the SNR of the equal power method,

$$p_{\max}^2 |h_1|^2 |h_2|^2 / (p_{\max} |h_1|^2 + 2p_{\max} |h_2|^2 + 1)).$$

Figs. 3 and 4 show the average transmission rates of BE traffic when RC traffic is in outage or in nonoutage, respectively. In Fig. 4, when  $\gamma_{th} = 8$ , the difference between simulation and analytical result is not negligible. This is due to the assumption of  $\gamma_{th} \rightarrow 0$  in (15). From the Fig. 5, which shows the *overall* average transmission rate of BE traffic in



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Fig. 4. Average transmission rate of BE traffic when RC traffic is in non-outage.



Fig. 5. Overall average transmission rate of BE traffic in (11).

(11), we can observe the performance improvement of proposed power control method. Specifically, 30% rate improvement over the equal power method (from 1.25 to 1.63) is observed when  $\gamma_{th} = 10$  and  $d_1 = 0.1$ . Interestingly, as  $\gamma_{th}$  decreases, the *overall* average transmission rate of BE traffic increases. This coincides with the intuition that the QoS of RC traffic is paid for the improvement of BE traffic transmission rate.

#### 6 Conclusion

We proposed the power allocation method for two-way AF relay systems where traffics with different QoS requirements are exchanged. Based on the QoS feasibility of RC traffic, we proposed two types of power control strategies. First, when the QoS requirement of RC traffic is satisfied, we proposed a power control method to maximize the transmission rate of BE traffic so that the QoS constraint of RC traffic is satisfied. Secondly, when QoS requirement of RC traffic is not feasible, we proposed a power control method to maximize the rate of BE traffic with sacrificing the transmission rate of RC traffic by considering ARQ protocol. From the analysis and simulation results, we have confirmed the effectiveness of the proposed power control methods. Specifically, when  $\gamma_{th} = 8$  [dB] and relay is located at the center of two transmit nodes, overall transmission rate of BE traffic is improved 12.4 % compared with the conventional equal power allocation method.

#### Appendix 1

To take expectation, it is required to derive the following conditional pdf:

$$f_{\overline{\gamma}_{BE}|\overline{\gamma}_{RC} < \gamma_{th}}(z) = \frac{d}{dz} \left\{ \frac{\Pr(\overline{\gamma}_{BE} < z, \overline{\gamma}_{RC} < \gamma_{th})}{\Pr(\overline{\gamma}_{RC} < \gamma_{th})} \right\}$$

$$= \frac{d}{dz} \left\{ \frac{\Pr(\overline{\gamma}_{RC} < z, \overline{\gamma}_{RC} < \gamma_{th})}{\Pr(\overline{\gamma}_{RC} < \gamma_{th})} \right\}$$

$$= \left\{ \frac{1}{\Pr(\overline{\gamma}_{RC} < \gamma_{th})} \frac{d}{dz} \left( \Pr(\overline{\gamma}_{RC} < z) \right) \text{ if } z < \gamma_{th} \\ 0 \text{ otherwise} \right\}$$

$$= \left\{ \frac{1}{\Pr(\overline{\gamma}_{RC} < \gamma_{th})} f_{\overline{\gamma}_{RC}}(z) \text{ if } z < \gamma_{th} \\ 0 \text{ otherwise.} \right\}$$
(16)

To obtain the above tractable pdf, it is assumed that  $p_{i,\max} = p_{\max}$ . This assumption makes  $\overline{\gamma}_{BE} = \overline{\gamma}_{RC}$  and applies to the second equality. The pdf of Macdonald variable  $\overline{\gamma}_{RC}$ , presented at Theorem 1 in [20], is represented:

$$f_{\overline{\gamma}_{RC}}(z) = 2\exp(-(\lambda + \mu)z) \times \left[ \begin{array}{c} \lambda\mu(2z+1)K_0(2\sqrt{\lambda\mu z(z+1)}) \\ +(\lambda + \mu)\sqrt{\lambda\mu z(z+1)}K_1(\sqrt{\lambda\mu z(z+1)}) \end{array} \right].$$

With this pdf, (12) is derived as follows:

$$\begin{split} E_{\overline{\gamma}_{BE}|\overline{\gamma}_{RC} < \gamma_{th}} & \left(\frac{1}{2}\log(1+\overline{\gamma}_{BE})\right) \\ = \int_{0}^{\infty} \frac{1}{2}\log(1+z)f_{\overline{\gamma}_{BE}|\overline{\gamma}_{RC} < \gamma_{th}}(z)dz \\ = \frac{1}{\Pr(\overline{\gamma}_{RC} < \gamma_{th})} \int_{0}^{\gamma_{th}} \frac{1}{2}\log(1+z)f_{\overline{\gamma}_{RC}}(z)dz \\ = \frac{1}{\Pr(\overline{\gamma}_{RC} < \gamma_{th})} \left(\Phi_{0}(\gamma_{th}) + \Phi_{1}(\gamma_{th})\right), \end{split}$$
(17)

where the definitions of  $\Phi_0(\gamma_{th})$  and  $\Phi_1(\gamma_{th})$  are given in (13).

#### **Appendix 2**

The conditional pdf is defined as follows:

$$f_{\gamma'_{BE}|\overline{\gamma}_{RC} \ge \gamma_{th}}(z) = \frac{d}{dz} \left\{ \frac{\Pr(\gamma'_{BE} < z, \ \overline{\gamma}_{RC} \ge \gamma_{th})}{\Pr(\overline{\gamma}_{RC} \ge \gamma_{th})} \right\}.$$
 (18)

For tractable derivation, we assume that  $p_{i,\max} = p_{\max}$  and  $\gamma_{ih} \rightarrow 0$ , then  $\gamma'_{BE} = \overline{\gamma}_{RC}$ . With these assumptions, (18) can be simplified:

$$f_{\gamma_{BE}'|\overline{\gamma}_{RC} \geq \gamma_{th}}(z) = \frac{d}{dz} \left\{ \frac{\Pr(\overline{\gamma}_{RC} < z, \ \overline{\gamma}_{RC} \geq \gamma_{th})}{\Pr(\overline{\gamma}_{RC} \geq \gamma_{th})} \right\}$$

$$= \begin{cases} \frac{1}{1 - \Pr(\overline{\gamma}_{RC} < \gamma_{th})} f_{\overline{\gamma}_{RC}}(z) & \text{if } z \geq \gamma_{th} \\ 0 & \text{otherwise} \end{cases}$$
(19)

With (19), the upper bound of (14) is derived as follows:

$$E_{\gamma_{BE}'|\overline{\gamma}_{RC} \geq \gamma_{th}}\left(\frac{1}{2}\log(1+\gamma_{BE}')\right)$$

$$\leq \int_{0}^{\infty} \frac{1}{2}\log(1+z)f_{\gamma_{BE}'|\overline{\gamma}_{RC} \geq \gamma_{th}}(z)dz$$

$$= \frac{1}{1-\Pr(\overline{\gamma}_{RC} < \gamma_{th})}\int_{\gamma_{th}}^{\infty} \frac{1}{2}\log(1+z)f_{\overline{\gamma}_{RC}}(z)dz$$

$$= \frac{1}{1-\Pr(\overline{\gamma}_{RC} < \gamma_{th})}(\Phi_{0}'(\gamma_{th}) + \Phi_{1}'(\gamma_{th})),$$
(20)

and the definitions of  $\Phi'_0(\gamma_{th})$  and  $\Phi'_1(\gamma_{th})$  are given in (15).

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