Abstract: - Since the performance metrics of digital communication systems over composite multipath/lognormal fading channels is intractable, a mixture gamma (MG) fading model is used to approximate Nakagami-lognormal (NL) fading model. By using MG fading model, the ergodic capacity of dual-hop semi-blind amplify-and-forward relaying systems is investigated over independent non-identical composite NL fading channels. First, we derived an exact closed-form expression of the ergodic capacity for the considered system based on the elementary probability transformation. Then, several capacity bounds and one approximate expression are obtained over MG fading channels for the purpose of comparison with the above exact analysis. Finally, numerical and simulation results are shown to verify the accuracy of the analytical results under different conditions, such as varying average signal to noise ratio, fading parameters per hop and the choice of the semi-blind gain.

Key-Words: - Dual-hop relaying, Nakagami-lognormal, Mixture Gamma distribution, Ergodic capacity, Fixed gain, Capacity bound

1 Introduction

Multi-hop relaying transmissions have emerged as a promising technique to provide the high data-rate coverage and mitigate channel impairments required in the wireless communication networks. In such networks, the key idea is that one source node communicates with one destination node through one or several intermediate nodes called relays. In the past few years, as an important relay scheme, amplify-and-forward (AF) fixed gain systems have been paid considerable attention in term of outage probability, average bit/symbol error rate for various system models and fading channel models, such as [1]-[5] and the references therein. Generally, there are two main categories of fixed gain relaying systems: blind relay and semi-blind relay. The former exploits the fixed relay gain regardless of the fading amplitude of the first-hop link, and the latter required only statistical channel state information (CSI) of the first-hop link. Compared with the variable gain AF relaying systems, the latter are low complexity and ease of deployment, and make them more attractive from a practical viewpoint.

Recently, as an important performance metric, the ergodic capacity of various AF relaying systems with fixed gain has also been an active field of research over different fading channels, such as [6]-[10] and the references therein. In [6], several tight bounds of the ergodic capacity for dual-hop fixed gain systems is presented based on Steffensen’s, Chebyshev’s, and Jensen’s inequality only for Rayleigh fading channels. The authors in [7] provided a tight closed-form approximation of the ergodic capacity exploiting a Taylor series expansion for dual-hop fixed gain systems with partial relay selection over Nakagami-m fading channels. By using the same method as [7], the authors in [3] obtained the ergodic capacity for dual-hop semi-blind AF relaying over arbitrary Nakagami-m fading channels. Wu et al. [8] derived an upper bound of ergodic capacity for dual-hop fixed gain systems using Jensen’s inequality over Nakagami-lognormal (NL) fading channels approximated using generalized-K (KG) distribution. KG distribution is the mixture of Nakagami-m and gamma distribution, where gamma distribution approximates lognormal distribution. The authors in [9] studied generic ergodic capacity bounds for dual-hop AF systems with fixed gain relay over various fading channels including Nakagami-m, Weibull, and KG distributions. Based on the works in [9], the authors further investigated the upper and
lower bounds of the ergodic capacity for dual-hop AF relaying systems with fixed gain relaying over NL fading channels by using $\mathcal{G}$ distribution in [10]. $\mathcal{G}$ distribution is the mixture of Nakagami-m and inverse-Gaussian (IG) distribution, where the lognormal distribution is approximated by IG distribution.

To the best of our knowledge, no exact closed-form expression of the ergodic capacity for the dual-hop AF fixed gain relaying has so far been derived over any fading channels. Furthermore, some expressions of capacity bounds still keep complicated and intractable in the KG and $\mathcal{G}$ fading models since their probability density functions (PDFs) of average signal to noise ratio (SNR) include modified Bessel functions.

In [11], the authors developed one new approach to approximate the NL model by using the mixture gamma (MG) distribution. This distribution is composed of a weighted sum of gamma distribution, and can obtain some exact results by adjusting the number of gamma distribution. In [12], we compared the end-to-end performance of dual-hop variable gain relay systems over NL fading channels by using MG and KG distribution, and found it is more precise and amenable to approximate the NL distribution by using the former than the latter.

In this paper, we focus on a dual-hop AF fixed gain relaying system with semi-blind and analyse its ergodic capacity performance over independent non-identical composite NL fading channels using MG distribution. The main contribution of this paper is to derive an exact and novel closed-form expression of ergodic capacity and several simple expressions for the ergodic capacity performance over independent non-identical composite NL fading channels. Due to assuming that the dual-hop fixed gain relaying system with semi-blind and analyse its ergodic capacity performance over independent non-identical composite NL fading channels, we can obtain some exact results by adjusting the number of gamma distribution. In [12], we compared the end-to-end performance of dual-hop variable gain relay systems over NL fading channels by using MG and KG distribution, and found it is more precise and amenable to approximate the NL distribution by using the former than the latter.

In section 3, some expressions of the ergodic capacity for the dual-hop fixed gain system are obtained. The performance evaluations and conclusions are presented in section 4 and 5, respectively.

2 System and Channel Models
We consider a wireless dual-hop AF fixed gain relaying system over composite NL fading channels. The source node (S) communicates with the destination node (D) via a relaying node (R). The whole transmission is divided into two phases. In the first phase, S only transmits its signals to R. In the second phase, R amplifies the received signals by a gain factor $\beta$ and then forwards their amplified versions to D. Without loss of generality, we assume that the average powers of S and R are normalized to unity. If $\beta$ is selected according to the fixed relay gain, which is defined as $\beta^2 = 1/N_0$ as in [1]. Thus, the instantaneous end-to-end SNR, $\gamma_{SRD}$, at the destination can be expressed as in [1]

$$\gamma_{SRD} = \gamma_i \gamma_S (\gamma_S + Z),$$

where $\gamma_i = \rho |h_i|^2$ is the instantaneous SNR of the $i$-th-hop link, $|h_i|$ is the fading amplitude of the $i$-th-hop link, $\rho = 1/N_0$ denotes the un-faded SNR, $N_0$ is the power of the additive white Gaussian noise (AWGN) component, $Z$ is a constant for a fixed gain $\beta$. Then, $\bar{\gamma}_i = \rho \mathbb{E}(|h_i|^2) = \rho \Omega_i$ denotes the average SNR of the $i$-th-hop link, $\mathbb{E}[^*]$ is the statistical expectation, $\Omega_i$ denotes the average power of the fading channels. Due to assuming that the $i$-th-hop link experiences NL fading, $\gamma_i$ is a composite Gamma-lognormal distribution variable with the PDF approximated by [11], as

$$f_{\gamma_i}(x) = \sum_{j=1}^{N} T_j x^{m_j-1} \exp(-M_j x),$$

where $T_j = c_j a_j / 2 \rho m_j$, $M_j = b_j / \rho$, $c_j = \sqrt{\pi} / \Sigma_{j=1}^{N} w_j$ is the normalization factor, $b_j = m_j \exp(-\{\sqrt{2} \lambda_i t_j + \mu_i\})$, $a_j = 2m_j w_j \exp(-m_j (\sqrt{2} \lambda_i t_j + \mu_i)) / \sqrt{\pi} \Gamma(m_j)$, $m_j$ is fading parameter in Nakagami-m fading and is integer, $\mu_i$ and $\lambda_i$ are the mean and the standard deviation of lognormal shadowing, respectively, $\mu_i = \ln \Omega_i$, $\lambda_i = (\ln 10/10) \sigma_i$, $\sigma_i$ denotes the standard deviation in dB, $w_j$ and $t_j$ are abscissas and weight factors for Gaussian-Hermite integration, $w_j$ and $t_j$ for different $N$ are available in [13, Table(25.10)]. Eq.(2) can describe various fading and shadowing models by using different value of $m$ and/or $\sigma$. For example, for $m=1$, it can represent Rayleigh-lognormal fading. For $\sigma=0$, it reduces to the well-known Nakagami-m or Rayleigh ($m=1$) fading.

3 Ergodic Capacity Analysis
3.1 Exact Analysis
For a dual-hop AF fixed gain system, the ergodic capacity can be obtained as

$$\bar{C} = \Delta \mathbb{E} [\ln(1 + \gamma_{SRD})],$$

where $\Delta=1/2\ln 2$. The reason of the 1/2 factor is that we need two orthogonal channels or two time slots for transmitting the data in a dual-hop system.
Since an exact closed-form expression in (3) over MG fading channels is not mathematically tractable by directly using a traditional approach (i.e., finding the PDF of $\gamma_{\text{SRD}}$), we thus restructure (3) as in [6]

$$\bar{C} = \Delta\{E[\ln(1+(1+\gamma_1)\gamma_2/Z)] - E[\ln(1+\gamma_2/Z)]\}. \quad (4)$$

In order to find the closed-form expression of (4), we must obtain the closed-form expressions of $I_1$ and $I_2$. Here, we let $X=\gamma_2/Z$, $Y=1+\gamma_1$, $U=XY$. Thus, the key problem is to find the PDFs of variables $X$, $Y$ and $U$, respectively. With the help of (2) and using the variable transform method, the PDFs of $X$ and $Y$ can be obtained, respectively, as

$$f_X(x) = \sum_{n=1}^{N} T_n Z^n x^{n-1} \exp(-Z MX) x, \quad (5)$$

$$f_Y(y) = \sum_{n=1}^{N} T_n \exp(My) (y-1)^{n-1} \exp(-MY). \quad (6)$$

Then, by using (5), the closed-form expression of $I_2$ can be written as

$$I_2 = E[\ln(1+X)] = \int_{0}^{\infty} \ln(1+x) f_X(x) dx$$

$$= \sum_{n=1}^{N} T_n z^n x^{n-1} \exp(-Z MX) dx. \quad (7)$$

By expressing $\ln(1+x) = G_{22}^2(x|1,2)$ and $\exp(-x) = G_{21}^2(x|0,2)$ in (7) as a Meijer’s G function defined in [14, Eq.(01.04.26.0003.01)] and [14, Eq.(01.03.26.0004.01)], respectively, and using Eq.(07.34.21.011.01) in [14], eq.(7) can be rewritten as

$$I_2 = \sum_{n=1}^{N} R_n G_{22}^2(ZM|1^{0.1})_{m_{n,0.0}}, \quad (8)$$

where $R_n = c_n / 2k_n$, $G_{22}^2[•,•]$ is the Meijer’s G-function.

In the following, for the sake of finding the PDF of $U$, we let $V=X$ as an auxiliary variable. Due to the fact of the independence between $\gamma_1$ and $\gamma_2$, $X$ and $Y$ are also independent each other. Thus, by using Jacobian determinant, we can obtain the composite PDF of variables $V$ and $U$ as

$$f_U(u,v) = f_X(v) f_U(u/v) v. \quad (9)$$

By using (5) and (6), and the binomial expansion defined in [15, Eq.(1.111)] when $m_i$ is integer, and with the aid of Eq.(6.621.3) in [15], the PDF of $U$ can be written as

$$f_U(u) = \int_{0}^{\infty} f_U(u,v) dv$$

$$= \sum_{i=0}^{N} \sum_{j=0}^{N-m_i} 2C^{-1}_i Z^{(m_i+m_j+1/2)} \exp(-(m_i+m_j+1/2)Z)$$

$$\times U^{m_i-m_j+1} K_{m_i-m_j+1/2}(2\sqrt{ZM},M,\mu), \quad (10)$$

where $C' = ((i-j)!)(i-j)!$, $K_\alpha(*)$ is the second kind modified Bessel function of order $\alpha$.

Then, by expressing $K_\alpha((2\sqrt{ZM})/\alpha) = G_{0,2}^2(x|1,2)/2$ as a Meijer’s G function defined in [14, Eq.(01.04.26.0009.01)], and similar as (8), the closed-form expression of $I_1$ can be obtained as

$$I_1 = \sum_{i=0}^{N} \sum_{j=0}^{N-m_i} (-1)^i C^{-1}_i R_i M^{-1}(\exp(M))^{-1} G_{22}^2(ZM,M|1^{0.1})_{m_{i,0,0.0}} \quad (11)$$

Finally, by substituting (11) and (8) into (4), we can obtain the exact closed-form expression of the ergodic capacity for the dual-hop fixed gain system over MG fading channels. To the best of our knowledge, this result is novel.

As special cases, when the dual-hop system experiences Nakagami-m fading, the exact closed-form expressions of the ergodic capacity for the dual-hop fixed gain system can be obtained with the aid of the above analysis as

$$\bar{C}_N = \Delta\{\sum_{i=0}^{N-m_i} C^{-1}_i \exp(m_i/\gamma_1)(-1)^i G_{22}^2(Zm_i,\gamma_2/\gamma_1|1^{0.1,0.1,0.0})$$

$$- G_{22}^2[Zm_i,\gamma_2/\gamma_1|1^{0.0}] / \Gamma(m_i) \}. \quad (12)$$

When $m_i=1$, eq.(12) can be reduced to the case of Rayleigh, as

$$\bar{C}_R = \Delta\{\exp(1/\gamma_1) G_{22}^2[Z,\gamma_2/\gamma_1|1^{0.1,0.1,0.0}] - G_{22}^2[Z,\gamma_2/\gamma_1|1^{0.0}])\}. \quad (13)$$

### 3.2 Capacity Bounds Analysis

Recently, the ergodic capacity bounds of dual-hop system have been widely investigated over different fading channels. For the purpose of comparison with the above exact analysis, we will give the ergodic capacity bounds analysis for the considered system over MG fading channels in this section.

#### 3.2.1 The lower bound

From [9], the lower ergodic capacity bound for dual-hop AF fixed gain system can be given by

$$\bar{C}_L = \Delta\{\ln[1 + \exp(E[\ln\gamma_1] + E[\ln\gamma_2] - E[\ln(Z + \gamma_2)])]\}. \quad (14)$$
To evaluate the lower bound in (14), we need to derive closed-form expressions for $E[\ln \gamma_i]$ and $E[\ln(Z+\gamma_i)]$ over MG fading channels, respectively. To this purpose, the first term $E[\ln \gamma_i]$ can be computed by using (2) as

$$E[\ln \gamma_i] = \sum_{j=1}^{N} \int_{0}^{\infty} x^{n_i-1}\ln x \exp(-M_j x)dx.$$  \hfill (15)

With the help of the integration relationship [15, Eq.(4.352.1)] and some simple algebraic manipulations, (15) can rewritten as

$$E[\ln \gamma_i] = \psi(m_i) - \sum_{j=1}^{N} R_i\Gamma(m_i)\ln M_j,$$  \hfill (16)

where $\psi(*)$ is the psi function.

Additionally, with the help of the integration relationship [15, Eq.(4.352.1) and similar as (7), the second term $E[\ln(Z+\gamma_i)]$ can be computed by using (2) as

$$E[\ln(Z+\gamma_i)] = \sum_{j=1}^{N} \int_{0}^{\infty} x^{m_j-1}\ln(Z+x)\exp(-M_j x)dx$$  

$$= \ln Z + \sum_{j=1}^{N} R_i G_{ij}^2[Z M_j^{m_j}].$$  \hfill (17)

By substituting (16) and (17) into (14), we can obtain the lower bound of the ergodic capacity for the dual-hop fixed gain system over MG fading channels.

### 3.2.2 The upper bound

From [8] and [9], the upper ergodic capacity bound for dual-hop AF fixed gain system by using Jensen’s inequality can be given by

$$\bar{C}_{ul} = \Delta \ln[1+E[\gamma_1]E[\gamma_2/(Z+\gamma_2)]].$$  \hfill (18)

To evaluate the upper bound in (18), we need to derive closed-form expressions for $E[\gamma_i]$ and $E[\gamma_i/(Z+\gamma_i)]$ over MG fading channels. To this purpose, the first term $E[\gamma_i]$ can be easily obtained by using (2) as

$$E[\gamma_i] = \sum_{j=1}^{N} T_i\Gamma(m_i+1)/M_j^{m_i+1}.$$  \hfill (19)

Similarly, with the aid of [15, Eq.(3.838.10)], the second term $E[\gamma_i/(Z+\gamma_i)]$ can be expressed as

$$E[\gamma_i/(Z+\gamma_i)] = \sum_{j=1}^{N} T_i\int_{0}^{\infty} x^{m_j} (z+x)^{-1}\exp(-M_j x)dx$$  

$$= \sum_{j=1}^{N} T_i Z^{m_j} \exp(ZM_j) \Gamma(m_j+1)\Gamma(-m_j,ZM_j),$$  \hfill (20)

where $\Gamma(*,*)$ is the incomplete gamma function.

Then, by substituting (19) and (20) into (18), we can obtain the first upper bound of the ergodic capacity for the dual-hop AF fixed gain system by using Jensen’s inequality over MG fading channels.

Additionally, in order to improve the poor performance in (18) in the high SNR regions, a new alternative upper bound is given as in [9]

$$\bar{C}_{ul} = \Delta \ln[1+ZE[\gamma_1^{-1}]E[\gamma_2^{-1}]+E[\gamma_1^{-1}]] + \Delta E[\ln \gamma_1 + \ln \gamma_2 - \ln(Z+\gamma_2)].$$  \hfill (21)

From (21), we only need to obtain the closed-form expression of $E[\gamma_i^{-1}]$. Similar as (19), the first negative moment of $\gamma_i$ is given by

$$E[\gamma_i^{-1}] = \sum_{j=1}^{N} T_i\Gamma(m_i-1)/M_j^{m_i-1}.$$  \hfill (22)

Noted that (22) is unavailable when $m_i=1$. As a special case, we have to loose the integral limit and obtain the approximate expression of (22). This approximate result is proved to be correct in high SNR regions in section 4. When $m_i=1$, eq.(22) can be approximated by using eq.(3.351.5) in [15] as

$$E[\gamma_i^{-1}] > \sum_{j=1}^{N} T_i\int_{0}^{\infty} x^{-1}\exp(-M_j x)dx$$  

$$> \sum_{j=1}^{N} T_i \Gamma(\exp(-Ei(-M_j))).$$  \hfill (23)

where $Ei(*)$ is the exponential integral function.

Finally, by using (16), (17) and (22), and substituting them into (21), we can obtain the second upper bound of the ergodic capacity for the dual-hop AF fixed gain system over MG fading channels.

### 3.3 Capacity Approximation Analysis

In [3] and [7], a tight approximation of the ergodic capacity for dual-hop fixed gain systems are adopted based on a Taylor series expansion of $\log_2(1+\gamma)$. It can be expressed as in [7]

$$\bar{C}_{app} = \Delta \sqrt{\ln[1+E[\gamma_{SRD}]] - E^2[\gamma_{SRD}]/2(1+E[\gamma_{SRD}])^2}.$$  \hfill (24)

For (24), we need to find the first and the second moments of $\gamma_{SRD}$. Then, the $n$th moment of $\gamma_{SRD}$ must be obtained. Using (1), it can be expressed as

$$E[\gamma_{SRD}^{n}] = \int_{0}^{\infty} \int_{0}^{\infty} \left(\frac{\gamma_2}{\gamma_2+Z}\right)^{n} x f_{\gamma_1}(x) f_{\gamma_2}(y)dydx.$$  \hfill (25)
Due to the independence between $\gamma_1$ and $\gamma_2$, two integrals in (25) can be calculated, respectively. By using (2) and after some algebraic manipulations, eq.(25) can be rewritten in closed-form as

$$
E[Y_{b,2}^o] = \sum_{j=1}^{N} T_j \sum_{n=0}^{\infty} \frac{T_j \Gamma(m_1+n)}{M_j^{m_1+n}} \Gamma(n)Z^{-1} \left[ M_{j-1}^{2} \right]^{-1}.
$$

Thus, the closed-form expression of $C_{app}$ can be obtained by setting $n=1$ and 2 in (26) and substituting them into (24).

### 3.4 The choice of the semi-blind gain

For the dual-hop fixed gain system, the semi-blind gain is determined by the channel statistics at the first hop. In general, there are two major schemes to calculate the relay gain as in [3]. In the first scheme, the fixed-gain relaying factor $\beta$ is chosen equal to the average of channel state information assisted gain as $\beta^2 = pe^{-E[\gamma_1]+1}$. Since the first-hop link undergoes MG fading, the constant $Z$ is given by

$$
Z_1 = \left( \sum_{j=1}^{N} T_j \exp(M_j) \Gamma(m_j) \Gamma(1-m_j,1) \right)^{-1}.
$$

In the second scheme, the fixed-gain relaying factor $\beta$ is chosen as $\beta^2 = \rho (E[\gamma_1]+1)$. Thus, the constant $Z$ is determined by the average SNR of the first-hop, and obtained as

$$
Z_2 = 1 + \frac{1}{\rho} - 1 + \sum_{j=1}^{N} R_j \Gamma(m_j+1) / M_j.
$$

### 4 Numerical and Simulation Results

In this section, we present some numerical and simulation results to evaluate the ergodic capacity of the dual hop AF fixed gain system by using the MG distribution.

Fig.1 and Fig.2 illustrate the ergodic capacity for the dual-hop AF fixed gain system versus the unfaded SNR ($\rho$) under different fading scenarios. Without loss of generality, we assume $\Omega_1=\Omega_2=5$, $N=10$ for MG distribution. As expected, the ergodic capacity increases with increasing $\rho$ from these figures. Fig.1 shows the impact of multipath parameters ($m$) on ergodic capacity. It can be seen that the ergodic capacity increases with increasing $m_i$ ($i=1,2$), whereas, the effect of $m_i$ on capacity becomes weaker when $m_i$ becomes larger. Fig.2 shows the impact of shadowing parameters ($\sigma$) on ergodic capacity. As expected, the ergodic capacity decreases with increasing $\sigma$. Compared with the exact analysis, the lower bound in (14) and the upper bound in (21) get tighter in medium and high SNR regions, and the upper bound in (18) keeps loose in entire range of SNR. However, the upper bound in (18) and the approximation in (24) show tighter in light fading environments, for example, smaller $\sigma$ or larger $m$. At the same time, it is clear that they match well between our exact analytical expression and simulations over entire range of $\rho$.

In Fig.3, we show the impact of the semi-blind gain factor ($Z$) and the fading parameter ($\Omega$) on the ergodic capacity for the dual-hop system, where $\Omega_2=10$. For the choice of semi-blind gain factor, the case using $Z_1$ in (27) shows better performance than the case using $Z_2$ in (28). For the change of fading parameter ($\Omega$), it shows better performance when the value of $\Omega$ in the first hop is larger than that in the second hop. These results can be explained from two points. One is the fact that for the dual-hop fixed gain system, the end-to-end performance is dominated by the first-hop channel conditions. The
other is the fact that the calculation of the fixed-gain in (27) and (28) differs in the position of the expectation operator. The case using $Z_1$ averages the received signal-plus-noise power, and the case using $Z_2$ only averages the received signal power, such that $Z_2$ is always greater than $Z_1$.

Fig. 3 Ergodic capacity for the dual-hop AF fixed gain system versus the unfaded SNR ($\rho$) with $m_1=m_2=2$ and $\sigma_1=\sigma_2=4$dB

5 Conclusion

In this paper, we investigated the ergodic capacity of a dual-hop AF wireless communication system with fixed gain relays over the composite NL fading channels approximated by using MG distribution. Based on MG fading models, an exact closed-form expression and several bounds of ergodic capacity for the dual-hop AF fixed gain system are derived, respectively. Then, we showed numerical and simulation results to verify the accuracy of the analytical results, and discussed the effect of the fading parameters and the semi-blind gain factor on the ergodic capacity of the dual-hop system. These works in this paper can be helpful to analyse the performance of cooperative relaying systems over composite fading channels in the future.

References:


