Level Crossing Rate of Ratio of Product of Two *α-k-μ* Random Variables and *α-k-μ* Random Variable

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Abstract: - In this paper, the ratio of product of two *α-k-μ* random variables and *α-k-μ* random variable is analyzed. The closed form expression for average level crossing rate (LCR) of the ratio of product of two *α-k-μ* random variables and *α-k-μ* random variable is determined. The expression for level crossing rate of the ratio of product of two α-k-μ random variables and α-k-μ random variable can be used for calculation of average fade duration of wireless communication system operating over composite α-k-μ multipath fading environment in the presence of cochannel interference subjected to α-k-μ multipath fading. The *α-k-μ* distribution can be used to describe small scale signal envelope variations in non linear line-of-sight multipath fading environments.

Key-Words: *α-k-μ* random variable; co-channel interference; level crossing rate (LCR), multipath fading, wireless communication systems

1 Introduction

The short term fading and long term fading or shadowing can seriously degrade and limit performance and capacity of wireless communication systems. Received signal suffer multipath fading and shadowing which resulting in signal envelope variation. There are some distributions using for describing signal envelope variation in fading environments depend on propagation channels and communication scenario [1], [2].

For small scale signal envelope variation in linear non line-of-sight multipath fading environments Rayleigh and Nakagami-*m* distributions are used. Nakagami-*m* distribution is determined by parameter *m*. If parameter *m* is equal to 1, Nakagami-*m* distribution moved in Rayleigh distribution. The parameter *m* is fading severity. Nakagami-*m* fading is more severe for lower values of parameter *m*.

The Rician distribution is used in linear line-of-sight multipath fading environments. It describes small scale signal envelope variation. It is characterized by factor *k*, which can be calculated as ratio of dominant component power and scattering components power. Rice fading is more severe for smaller values of Rice factor *k*.

The Rayleigh distribution can be obtained from Rician distribution by setting for parameter k=0. As Rice factor increases, power of scattering component decreases and fading severity decreases also.

The *α-μ* and Weibull distributions can be used to analyze small scale signal variations in non linear, non line-of-sight multipath fading environments.

The *α-μ* distribution has two parameters where parameter *α* is related to non linearity of communication channel and parameter *μ* is tied to the number of clusters in propagation environments. The *α-μ* distribution is general distribution [3]. Rayleigh, Weibull and Nakagami-*m* distributions can be obtained from *α-μ* distribution as special cases. By setting for α=2, the *α-μ* distribution reduces to Nakagami-*m* distribution. Weibull distribution can be derived from *α-μ* distribution by setting for μ =1. For α=2 and μ =1, the *α-μ* distribution becomes Weibull distribution.

In this paper, *α-k-μ* distribution is observed. This distribution is used to describe signal envelope variation in multipath fading channels. The closed form expressions for average level crossing rate of product of two *α-k-μ* random variables are derived. The results obtained in this paper can be used for calculation of average fade duration of wireless systems operating over α-k-μ multipath fading channels.

The average level crossing rate (LCR) and average fade duration (AFD) are the second order performance measures of wireless communication system. The average level crossing rate can be calculated as average value of the first derivative of random process. It is necessary to determine the joint probability density function of random variable and first derivative of random variable for determination of average value of the first derivative of random process. The average fade duration of communication system can be calculated as a ratio of outage probability and average level crossing rate of system's output signal envelope.

In interference limited environment, the outage probability is equal to probability that ratio of the useful signal's envelope and co-channel envelope less than the threshold. In noise limited environment, the outage probability is defined as the probability that ratio of signal envelope power and noise power is below the defined threshold. When the noise power is equal in both branches, the outage probability is defined as probability that signal power is less than the threshold. In interference and noise limited environment, the outage probability is equal to probability that the ratio of the useful signal's power and sum of co-channel and noise powers is less than the threshold. The average level crossing rate and average fade duration show the influence of correlation to system performance.

The important functions should to be considered are: sum of two random variables, product of two random variables, ratio of two random variables, maximum of two random variables.

The output signal from dual equal gain combiner (EGC) is equal to the sum of signals from its inputs. When two fading affect together at the combiner inputs, the equivalent envelope is equal to product of two random variables.

The selection combiner (SC) output signal is equal to the maximum of two random variables. For determining the probability of relay system envelope, the probability of minimum of two random variables is needed to be determined.

Also, important functions are: product of three random variables, sum of three random variables, quotient of one random variable and the product of two random variables and quotient of two products of two random variables. For all these functions, it is necessary to designate the probability density, cumulative probability density, characteristic function, the average level crossing rate and moments.

2 Related Works

There are more papers in the literature considering the second order statistic analysis of wireless communication system with SC receiver in the presence of multipath fading with different distributions (Rayleigh, Rician, Weibull or Nakagami-*m*) [4]-[7].

The second order statistic analysis of selection macro-diversity combining over Gama shadowed Rayleigh fading environments is given in [4] and the second order statistics of the signal in Ricean-lognormal fading channel with selection combining in [5].

The wireless communication system with SIR based dual branches selection combining (SC) diversity receiver operating over correlated Rician multipath fading channels in the presence of cochannel interference subjected to multipath Rayleigh fading is analyzed in [6]. Average level crossing rate of such system is determined and results are presented to highlight the effects of branch correlation and fading severity on the average level crossing rate.

The average fade duration of dual selection diversity combiner over correlated unbalanced Nakagami-m fading channels in the presence of cochannel interference is calculated in [7].

Average LCR and AFD for SC diversity over correlated Weibull fading channels are investigate in [8]. The two formulae for the average LCR and AFD at the output of dual-branch selection diversity receivers are performed and some earlier published results given in a more general and compared.

Some expressions for average LCR and AFD for dual-branch maximum ratio combining (MRC) and selection combining (SC) schemes operating in the correlated fading channel are derived in [9]. It is supposed that channel model of the diversity branches is correlated small scale with Nakagami-*m* statistics. The numerical results point out that the average LCR and AFD of MRC and SC schemes are significantly affected by the correlation between each branch when they are working in the correlated environments.

Spectral efficiency comparison of TDMA and DS-CDMA in cellular mobile radio systems in a Rayleigh fading environment is given in [10], and statistics of the channel capacity for a DS/FFH-CDMA system in [11] by Varzakas. The performance evaluation for the cooperative communication systems in decode-and-forward mode with a Maximal Ratio Combining scheme are presented in [12].

The formulation and derivation of the α-k-μ/gamma distribution which corresponds to a physical fading model is shown in [13]. It is composite and constituted by the α-k-μ non-linear generalized multi-path model. That represents the basis for deriving the α-k-μ extreme /gamma model which accounts for non-linear severe multipath and shadowing effects and also includes the more widely known α-μ and k-μ models which includes as special cases the Rice, Weibull, Nakagami-m and Rayleigh distributions. This is achieved thanks to the significant flexibility of their parameters which have been shown to make them capable to provide good concurrence to experimental data associated with realistic communication scenarios.

The paper [14] presents two newer fading distributions, the *α-η-μ* and the *α-k-μ*. The *α-η-μ* distribution includes *α-μ*, Nakagami-*m*, Nakagami-*q*, Weibull, Hoyt, Rayleigh, Exponential, and the One-Sided Gaussian distributions as special cases. The *α-k-μ* distribution includes *α-μ*, Nakagami-*m*, Weibull, Rice, Rayleigh, Exponential, and the One-Sided Gaussian distributions as special cases. Furthermore, it proposes estimators for the involved parameters and uses field measurements to validate the distributions. The performance analysis of wireless communication system in α-k-μ environment subjected to shadowing is done in [15]. Second-order statistics for the envelope of α-κ-μ fading channels are derived in [16] and second order statistics of SC receiver output SIR in the presence of α-k-μ multipath fading and co-channel interference in [17].

In [18] the level crossing rate *α-k-μ* multipath fading at combiner inputs is determined. The expression for level crossing rate of product of two *α-k-μ* random variables is derived. In this paper, the LCR for the ratio of product of two *α-k-μ* random variables and *α-k-μ* random variable will be derived. Numerical results will be presented to show the influence of *α-k-μ* fading parameters on average level crossing rate. To the best author knowledge the results obtained in this paper are not published in the open technical literature up to now.

3 Product of two α-k-μ random variables

The *α-k-μ* distribution can be used to describe small scale signal envelope variation in non linear, line-of-sight multipath fading environment. This distribution has three parameters. The parameter α is related to nonlinearity of environment. The parameter *k* is related to ratio of dominant components power and scattering components power. The parameter *μ* is associated to the number of clusters in propagation environment. The *α-k-μ* multipath fading is more severe for lower value of parameter *k*, lower values of parameter μ, and higher values of parameter α [19].

The *α-k-μ* distribution is general distribution. Rayleigh, Nakagami-*m*, Rician, Weibull, *α- μ* and *α-k* distributions can be obtained from *α-k-μ* distribution as special cases. By setting for α=2, *α-k-μ* distribution reduces to *k-μ* distribution; by setting for *k*=0, *α-μ* distribution can be derived from *α-k-μ* distribution and for *μ=1* Weibull distribution can be obtained from *α-k-μ* distribution. By setting α=2 and *k*=0, *α-k-μ* distribution approximates Nakagami-*m* distribution, α=2 and *μ* =1, *α-k-μ* distribution reduces to Rician distribution. Finaly, for *k*=0 and *μ*=1, Weibull distribution can be acquired from *α-k-μ* distribution.

The product of two *α-k-μ* random variables *x* and *y* is:

$$z = x \cdot y = \left(x_1 \cdot y_1\right)^{\frac{2}{\alpha}} \quad \text{or}$$

$$z^{\alpha} = x_1^2 \cdot y_1^2, \; z^{\frac{\alpha}{2}} = x_1 \cdot y_1 \tag{1}$$

where x_1 and y_1 are squared *k-μ* random variables. Squared *k-μ* random variables x_1 and y_1 are:

$$x_1^2 = x_{11}^2 + x_{12}^2 + \cdots + x_{12\mu}^2$$

$$y_1^2 = x_{11}^2 + x_{12}^2 + \cdots + x_{12\mu}^2 \tag{2}$$

Squared *k-μ* random variable is equal to the sum of 2μ independent Gaussian random variables with the same variances. The first derivative of the product of two *α-k-μ* random variables is:

$$\dot{z} = \frac{2}{\alpha z^{\frac{\alpha}{2}-1}}\left(\dot{x}_1 y_1 + x_1 \dot{y}_1\right) \tag{3}$$

The first derivatives of *α-k-μ* random variables $\dot{x}_1$ and $\dot{y}_1$ are:

$$\dot{x}_1 = \frac{1}{x_1}\left(x_{11}\dot{x}_{11} + x_{12}\dot{x}_{12} + \cdots + x_{12\mu}\dot{x}_{12\mu}\right)$$

$$\dot{y}_1 = \frac{1}{y_1}\left(y_{11}\dot{y}_{11} + y_{12}\dot{y}_{12} + \cdots + y_{12\mu}\dot{y}_{12\mu}\right) \tag{4}$$

After substituting (4) in (3), the expression for the first derivative of product of two *α-k-μ* random variables becomes:

$$\dot{z} = \frac{2}{\alpha z^{\frac{\alpha}{2}-1}}\left(\frac{y_1}{x_1}\left(x_{11}\dot{x}_{11} + x_{12}\dot{x}_{12} + \cdots + x_{12\mu}\dot{x}_{12\mu}\right) + \right.$$

$$\frac{x_1}{y_1}\left(y_{11}\dot{y}_{11}+y_{12}\dot{y}_{12}+\cdots+y_{12\mu}\dot{y}_{12\mu}\right)\Big) \quad (5)$$

The first derivative of Gaussian random variable is Gaussian random variable and linear transformation of Gaussian random variable is also Gaussian random variable.

Thus, $\dot{x}_{11},\dot{x}_{12},...,\dot{x}_{12\mu}$, $\dot{y}_{11},\dot{y}_{12},...,\dot{y}_{12\mu}$ are independent zero mean Gaussian random variable. Therefore, product of two *α-k-μ* random variables follows conditional Gaussian distribution. The mean of $\dot{z}$ is:

$$\bar{\dot{z}}=0, \quad (6)$$

since

$$\bar{\dot{x}}_{11}=\bar{\dot{x}}_{12}=\cdots=\bar{\dot{x}}_{12\mu}=0, \quad (7)$$

and

$$\bar{\dot{y}}_{11}=\bar{\dot{y}}_{12}=\cdots=\bar{\dot{y}}_{12\mu}=0 \quad (8)$$

The variance of $\dot{z}$ is:

$$\sigma_{\dot{z}}^2=\frac{4}{\alpha^2 z^{\alpha-2}}\left(\frac{y_1^2}{x_1^2}\left(x_{11}^2\sigma_{\dot{x}_{11}}{}^2+x_{12}^2\sigma_{\dot{x}_{12}}{}^2+\cdots+x_{12\mu}^2\sigma_{\dot{x}_{12\mu}}{}^2\right)+\right.$$

$$\left.+\frac{x_1^2}{y_1^2}\left(y_{11}^2\sigma_{\dot{y}_{11}}{}^2+y_{12}^2\sigma_{\dot{y}_{12}}{}^2+\cdots+y_{12\mu}^2\sigma_{\dot{y}_{12\mu}}{}^2\right)\right) \quad (9)$$

where

$$\sigma_{\dot{x}_{11}}{}^2=\sigma_{\dot{x}_{12}}{}^2=\cdots=\sigma_{\dot{x}_{12\mu}}{}^2=\pi^2 f_m^2\frac{\Omega_1}{\mu(k+1)}=f_1^2$$

$$\sigma_{\dot{y}_{11}}{}^2=\sigma_{\dot{y}_{12}}{}^2=\cdots=\sigma_{\dot{y}_{12\mu}}{}^2=\pi^2 f_m^2\frac{\Omega_2}{\mu(k+1)}=f_2^2 \quad (10)$$

After substituting (10) in (9), the expression for variance of the first derivative of product of two *α-k-μ* random variables becomes:

$$\sigma_{\dot{z}}^2=\frac{4}{\alpha^2 z^{\alpha-2}}\left(\frac{y_1^2}{x_1^2}f_1^2\left(x_{11}^2+x_{12}^2+\cdots+x_{12\mu}^2\right)+\right.$$

$$\left.+\frac{x_1^2}{y_1^2}f_2^2\left(y_{11}^2+y_{12}^2+\cdots+y_{12\mu}^2\right)\right)=$$

$$=\frac{4}{\alpha^2 z^{\alpha-2}}\left(\frac{y_1^2 f_1^2}{x_1^2}\cdot x_1^2+\frac{x_1^2 f_2^2}{y_1^2}\cdot y_1^2\right)=$$

$$=\frac{4}{\alpha^2 z^{\alpha-2}}\left(y_1^2 f_1^2+x_1^2 f_2^2\right)=$$

$$=\frac{4}{\alpha^2 z^{\alpha-2}}\left(y_1^2 f_1^2+\frac{z^\alpha}{y_1^2}f_2^2\right)=$$

$$=\frac{4}{\alpha^2 z^{\alpha-2}y_1^2}\left(y_1^4 f_1^2+z^\alpha f_2^2\right) \quad (11)$$

The joint probability density function of *z*, $\dot{z}$ and *y₁* is:

$$p_{z\dot{z}y_1}(z\dot{z}y_1)=p_{\dot{z}}(\dot{z}/zy_1)\cdot p_{zy_1}(zy_1)=$$

$$=p_{\dot{z}}(\dot{z}/zy_1)\cdot p_z(z/y_1)\cdot p_{y_1}(y_1) \quad (12)$$

where conditional probability density function of z is:

$$p_z(z/y_1)=\left|\frac{dx_1}{dz}\right|p_{x_1}\left(z^{\frac{\alpha}{2}}\cdot\frac{1}{y_1}\right) \quad (13)$$

where

$$\frac{dx_1}{dz}=\frac{1}{y_1}\frac{\alpha}{2}z^{\frac{\alpha}{2}-1} \quad (14)$$

After substituting, $p_{z\dot{z}y_1}(z\dot{z}y_1)$ gives the form:

$$p_{z\dot{z}y_1}(z\dot{z}y_1)=p_{\dot{z}}(\dot{z}/zy_1)\cdot p_{y_1}(y_1)\cdot$$

$$\cdot\frac{\alpha}{2}\frac{1}{y_1}z^{\frac{\alpha}{2}-1}\cdot p_{x_1}\left(z^{\frac{\alpha}{2}}\cdot\frac{1}{y_1}\right) \quad (15)$$

The joint probability density function of *α-k-μ* random variable and the first derivative of *α-k-μ* random variable can be calculated as integral of previous expression with respect to y_1:

$$p_{z\dot{z}_1}(z\dot{z})=\int_0^\infty dy_1\, p_{z\dot{z}}(z\dot{z}y_1)\cdot p_{y_1}(y_1)=$$

$$=\frac{\alpha}{2}z^{\frac{\alpha}{2}-1}\cdot\int_0^\infty dy_1\frac{1}{y_1}p_{\dot{z}}(\dot{z}/zy_1)\cdot p_{y_1}(y_1)\cdot p_{x_1}\left(z^{\frac{\alpha}{2}}\cdot\frac{1}{y_1}\right) \quad (16)$$

The first derivative of the *α-k-μ* random variable can be calculated as average value of the first derivative of *α-k-μ* random variable:

$$N_z=\int_0^\infty d\dot{z}\,\dot{z}\,p_{z\dot{z}}(z\dot{z})=\frac{\alpha}{2}z^{\frac{\alpha}{2}-1}\cdot$$

$$\cdot\int_0^\infty dy_1\frac{1}{y_1}\cdot p_{y_1}(y_1)\cdot p_{x_1}\left(z^{\frac{\alpha}{2}}\cdot\frac{1}{y_1}\right)\cdot\int_0^\infty d\dot{z}\,\dot{z}\,p_{\dot{z}}(\dot{z}/zy_1)=$$

$$=\frac{\alpha}{2}z^{\frac{\alpha}{2}-1}\cdot\int_0^\infty dy_1\frac{1}{y_1}p_{y_1}(y_1)\cdot p_{x_1}\left(z^{\frac{\alpha}{2}}\cdot\frac{1}{y_1}\right)\cdot\frac{\sigma_{\dot{z}}}{\sqrt{2\pi}}=$$

$$=\frac{\alpha}{2}z^{\frac{\alpha}{2}-1}\frac{1}{\sqrt{2\pi}}\cdot\int_0^\infty dy_1\frac{1}{y_1}p_{y_1}(y_1)\cdot p_{x_1}\left(z^{\frac{\alpha}{2}}\cdot\frac{1}{y_1}\right)\cdot$$

$$\cdot\frac{2}{\alpha z^{\frac{\alpha}{2}-1}y_1}\sqrt{y_1^4 f_1^2+z^\alpha f_2^2}=$$

$$=\frac{1}{\sqrt{2\pi}}\cdot\int_0^\infty dy_1\frac{1}{y_1^2}\sqrt{y_1^4 f_1^2+z^\alpha f_2^2}\cdot p_{y_1}(y_1)\cdot p_{x_1}\left(z^{\frac{\alpha}{2}}\cdot\frac{1}{y_1}\right) \quad (17)$$

The random variable x_1 and y_1 are k-μ distributed:

$$p_{x_1}(x_1)=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{\mu k}\Omega_1^{\mu+1}}x_1^{\mu}e^{-\frac{\mu(k+1)x_1^2}{\Omega_1}}\cdot$$

$$\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{\Omega_1}}x_1\right), \tag{18}$$

$$p_{y_1}(y_1)=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{\mu k}\Omega_2^{\mu+1}}y_1^{\mu}e^{-\frac{\mu(k+1)y_1^2}{\Omega_2}}\cdot$$

$$\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{\Omega_2}}y_1\right). \tag{19}$$

After substituting (19) in (18), the expression for average level crossing rate of the product of two α-k-μ random variables becomes:

$$N_z=\frac{1}{\sqrt{2\pi}}\cdot\frac{\left(2\mu(k+1)^{\frac{\mu+1}{2}}\right)^2}{k^{\frac{\mu-1}{2}}e^{2\mu k}\Omega_1^{\mu+1}\Omega_2^{\mu+1}}\cdot$$

$$\cdot\int_0^{\infty}dy_1\frac{1}{y_1^2}\sqrt{y_1^4f_1^2+z^{\alpha}f_2^2}\cdot\cdot$$

$$\cdot z^{\frac{\alpha\mu}{2}}\frac{1}{y_1\mu}\cdot e^{-\frac{\mu(k+1)x_1^2}{\Omega_1}}\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{\Omega_2}}z^{\frac{\alpha}{2}}\cdot\frac{1}{y_1}\right)$$

$$\cdot y_1^{\mu}\cdot e^{-\frac{\mu(k+1)y_1^2}{\Omega_2}}\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{\Omega_2}}y_1\right) \tag{20}$$

4 Ratio of product of two α-k-μ random variables and α-k-μ random variable

Ratio of product of two α-k-μ random variables and α-k-μ random variable is:

$$w=\frac{x_1y_1}{z_1}=\frac{x^{\frac{2}{\alpha}}y^{\frac{2}{\alpha}}}{z^{\frac{2}{\alpha}}}\qquad\text{or}$$

$$w^{\frac{\alpha}{2}}=\frac{xy}{z},\quad x^2=w^{\alpha}\frac{z^2}{y^2} \tag{21}$$

where x_1, y_1 and z_1 are α-k-μ random variables and x, y and z are k-μ random variables. The first derivative of w is:

$$\dot{w}=\frac{2}{\alpha w^{\frac{\alpha}{2}-1}}\left(\frac{\dot{x}y}{z}+\frac{\dot{y}x}{z}-\frac{xy\dot{z}}{z^2}\right) \tag{22}$$

Squared k-μ random variables x, y and z are:

$$x^2=x_1^2+x_2^2+\cdots+x_{2\mu}^2$$

$$y^2=x_1^2+x_2^2+\cdots+x_{2\mu}^2$$

$$z^2=z_1^2+z_2^2+\cdots+z_{2\mu}^2 \tag{23}$$

where x_1, $x_2, \ldots, x_{2\mu}$, are independent Gaussian random variables with variances σ_1^2, y_1, $y_2, \ldots, y_{2\mu}$, are independent Gaussian random variables with variances σ_2^2, and z_1, $z_2, \ldots, z_{2\mu}$, are independent Gaussian random variables with variances σ_3^2. The first derivative of x, y and z are:

$$\dot{x}=\frac{1}{x}\left(x_1\dot{x}_1+x_2\dot{x}_2+\cdots+x_{2\mu}\dot{x}_{2\mu}\right)$$

$$\dot{y}=\frac{1}{y}\left(y_1\dot{y}_1+y_2\dot{y}_2+\cdots+y_{2\mu}\dot{y}_{2\mu}\right)$$

$$\dot{z}=\frac{1}{z}\left(z_1\dot{z}_1+z_2\dot{z}_2+\cdots+z_{2\mu}\dot{z}_{2\mu}\right) \tag{24}$$

After substituting (24) in (22), the expression for $\dot{w}$ becomes:

$$\dot{w}=\frac{2}{\alpha w^{\frac{\alpha}{2}-1}}\left(\frac{y}{z}\frac{1}{x}\left(x_1\dot{x}_1+x_2\dot{x}_2+\cdots+x_{2\mu}\dot{x}_{2\mu}\right)+\right.$$

$$+\frac{x}{z}\frac{1}{y}\left(y_1\dot{y}_1+y_2\dot{y}_2+\cdots+y_{2\mu}\dot{y}_{2\mu}\right)+$$

$$\left.+\frac{xy}{z^2}\frac{1}{z}\left(z_1\dot{z}_1+z_2\dot{z}_2+\cdots+z_{2\mu}\dot{z}_{2\mu}\right)\right) \tag{25}$$

The first derivative of ratio of product of two α-k-μ random variables and α-k-μ random variable follows conditional Gaussian distribution. The main of $\dot{w}$ is zero due to $\bar{\dot{x}}_1=\bar{\dot{x}}_2=\cdots=\bar{\dot{x}}_{2\mu}=0$, $\bar{\dot{y}}_1=\bar{\dot{y}}_2=\cdots=\bar{\dot{y}}_{2\mu}=0$ and $\bar{\dot{z}}_1=\bar{\dot{z}}_2=\cdots=\bar{\dot{z}}_{2\mu}=0$. The variance of $\dot{w}$ is

$$\sigma_{\dot{w}}^2=\frac{4}{\alpha^2w^{\alpha-2}}\left(\frac{y^2}{z^2x^2}\left(x_1^2\sigma_{\dot{x}_1}^2+x_2^2\sigma_{\dot{x}_2}^2+\cdots+x_{2\mu}^2\sigma_{\dot{x}_{2\mu}}^2\right)+\right.$$

$$+\frac{x^2}{z^2y^2}\left(y_1^2\sigma_{\dot{y}_1}^2+y_2^2\sigma_{\dot{y}_2}^2+\cdots+y_{2\mu}^2\sigma_{\dot{y}_{2\mu}}^2\right)+$$

$$\left.+\frac{x^2y^2}{z^6}\left(z_1^2\sigma_{\dot{z}_1}^2+z_2^2\sigma_{\dot{z}_2}^2+\cdots+z_{2\mu}^2\sigma_{\dot{z}_{2\mu}}^2\right)\right) \tag{26}$$

where

$$\sigma_{\dot{x}_1}^2=\sigma_{\dot{x}_2}^2=\cdots=\sigma_{\dot{x}_{2\mu}}^2=\pi^2f_m^2\sigma_1^2$$

$$\sigma_{\dot{y}_1}^2=\sigma_{\dot{y}_2}^2=\cdots=\sigma_{\dot{y}_{2\mu}}^2=\pi^2f_m^2\sigma_2^2$$

$$\sigma_{\dot{z}_1}^{\ 2}=\sigma_{\dot{z}_2}^{\ 2}=\cdots=\sigma_{\dot{z}_{2\mu}}^{\ 2}=\pi^2 f_m^2\sigma_3^2 \qquad (27)$$

After substituting (27) in (26), the expression for variance of the first derivative of ratio of product of two α-k-μ random variables and α-k-μ random variable becomes:

$$\sigma_{\dot{w}}^2=\frac{4}{\alpha^2 w^{\alpha-2}}\left(\frac{y^2\pi^2 f_m^2\sigma_1^2}{z^2x^2}\left(x_1^2+x_2^2+\cdots+x_{2\mu}^2\right)+\right.$$

$$+\frac{x^2\pi^2 f_m^2\sigma_2^2}{z^2y^2}\left(y_1^2+y_2^2+\cdots+y_{2\mu}^2\right)+$$

$$\left.+\frac{x^2y^2\pi^2 f_m^2\sigma_3^2}{z^6}\left(z_1^2+z_2^2+\cdots+z_{2\mu}^2\right)\right)=$$

$$=\frac{4\pi^2 f_m^2}{\alpha^2 w^{\alpha-2}}\left(\frac{y^2}{z^2}\sigma_1^2+\frac{w^\alpha}{y^2}\sigma_2^2+\frac{w^\alpha}{z^2}\sigma_3^2\right) \qquad (28)$$

The joint probability density function of w, $\dot{w}$, y and z is

$$p_{w\dot{w}yz}(w\dot{w}yz)=p_{\dot{w}}(\dot{w}/w)\cdot p_{wyz}(wyz)=\cdot$$

$$=p_{\dot{w}}(\dot{w}/w)\cdot p_{w/yz}\,p_{yz}(yz)=\cdot$$

$$=p_{\dot{w}}(\dot{w}/w)\cdot p_{w/yz}(w/yz)p_{yz}(y\dot{z}) \qquad (29)$$

where conditional probability density function of w is:

$$p_w(w/yz)=\left|\frac{dx}{dw}\right|p_x\left(\frac{w^{\frac{\alpha}{2}}}{y}\right)=$$

$$=\frac{\alpha}{2}w^{\frac{\alpha}{2}-1}\,\frac{z}{y}\cdot p_x\left(\frac{w^{\frac{\alpha}{2}}z}{y}\right) \qquad (30)$$

After substituting, the joint probability density function of w and $\dot{w}$ becomes:

$$p_{w\dot{w}}(w\dot{w})=$$

$$=\int_0^\infty dy\int_0^\infty dz\,\frac{\alpha}{2}w^{\frac{\alpha}{2}-1}\,\frac{z}{y}\cdot p_x\left(\frac{w^{\frac{\alpha}{2}}z}{y}\right)p_y(y)\cdot p_z(z)p_{\dot{w}}(\dot{w}/yzw) \qquad (31)$$

The level crossing rate of the ratio of product of two α-k-μ random variables and α-k-μ random variable is:

$$N_w=\int_0^\infty d\dot{w}\,\dot{w}\,p_{w\dot{w}}(w\dot{w})=$$

$$\cdot\int_0^\infty dy\cdot\int_0^\infty dz\,\frac{\alpha}{2}\cdot w^{\frac{\alpha}{2}-1}\,\frac{z}{y}\,p_x(y_1)\cdot p_{x_1}\left(\cdot\frac{w^{\frac{\alpha}{2}}z}{y}\right)p_y(y)p_z(z)\cdot\frac{\sigma_{\dot{w}}}{\sqrt{2\pi}}$$

The random variables x, y and z follows k-μ distribution.

$$p_x(x)=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{\mu k}\Omega_1^{\mu+1}}x^\mu e^{-\frac{\mu(k+1)x^2}{\Omega_1}}\cdot$$

$$\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\,x\right), \qquad (18)$$

$$p_y(y)=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{\mu k}\Omega_2^{\mu+1}}y^\mu e^{-\frac{\mu(k+1)y^2}{\Omega_2}}\cdot$$

$$\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\,y\right)$$

$$p_z(z)=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{\mu k}\Omega_2^{\mu+1}}z^\mu e^{-\frac{\mu(k+1)z^2}{\Omega_3}}\cdot$$

$$\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\,z\right)$$

where $\Omega_1=2\mu\sigma_1^2$, $\Omega_2=2\mu\sigma_2^2$ and $\Omega_3=2\mu\sigma_3^2$.

$$N_w=\frac{\alpha}{2}w^{\frac{\alpha}{2}-1}\left(\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{\mu k}}\right)^3\frac{1}{\Omega_1^{\mu+1}\Omega_2^{\mu+1}\Omega_3^{\mu+1}}\cdot\frac{2\pi f_m}{\sqrt{2\pi}}\cdot$$

$$\cdot\int_0^\infty dy\cdot\int_0^\infty dz\,\frac{z}{y}\sqrt{\left(\frac{y^2}{z^2}\sigma_1^2+\frac{w^\alpha}{y^2}\sigma_2^2+\frac{w^\alpha}{z^2}\sigma_3^2\right)}\cdot\left(\cdot\frac{w^{\frac{\alpha}{2}}z}{y}\right)^\mu y^\mu z^\mu\cdot$$

$$e^{-\frac{\mu(k+1)}{\Omega_1}\frac{w^\alpha z^2}{y^2}-\frac{\mu(k+1)}{\Omega_2}y^2-\frac{\mu(k+1)}{\Omega_2}z^2}\cdot$$

$$\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\,x\right)\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\,y\right)\cdot$$

$$\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\,z\right).$$

5 Numerical results

In Fig. 1, the level crossing rate of product of two *α-k-μ* random variables versus signal envelope is presented for different values of Rice factor *k*, the number of clusters in propagation environment μ (*m* in the figures) and signal power Ω (*r* in the figures). Parameter α is constant for all curves and equal to 3.

It can be seen from Fig. 2. that an increasing of parameter α affects the shapes of LCR curves. They are not approximately constant any more, but the LCR growth for small amplitude, reaches a maximum and begins to decline slowly. For bigger values of parameter *α*, the LCR tends to zero.

One can also see the influence of distribution's parameters on the level crossing rate of *α-k-μ* random variable versus envelope *z*, Rician factor *k*, parameter α, and *y*.

The parameter *m* has greater influence on average level crossing rate for lower values of parameter *k*. The level crossing rate increases as parameter *k* decreases. The system performances are better for lower values of average level crossing rate.

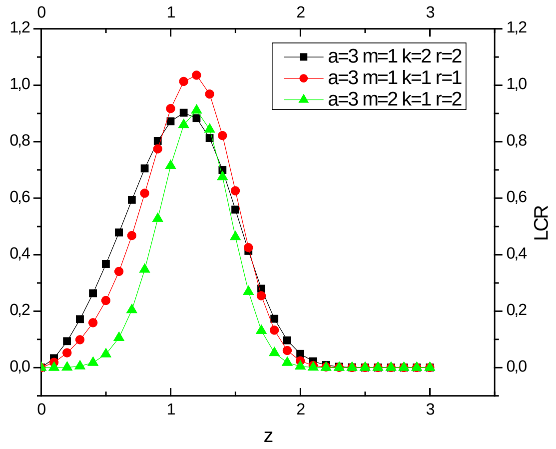

Fig. 1. The level crossing rate (LCR) of *α-k-μ* random variable versus signal envelope *z*

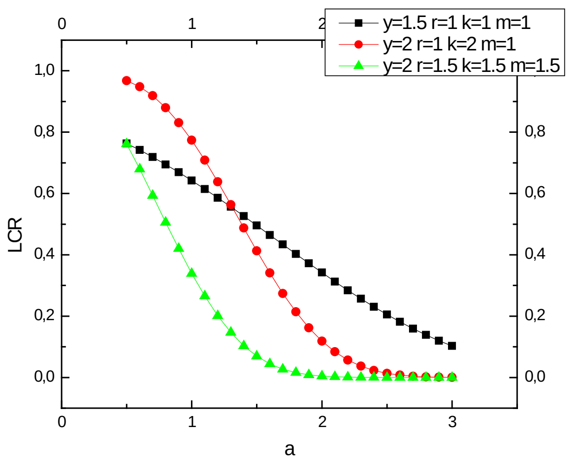

Fig. 2. The LCR versus parameter α, for *r* =1 and some values of parameters *m*, *k* and *y*

As parameter *α* increases, the average level crossing rate decreases. The parameter *α* has greater influence on average level crossing rate for lower values of parameter *α*. The parameter *k* has greater influence on average level crossing rate for lower values of parameter *k* and parameter α.

The system performances are better for lower values of average level crossing rate. The outage probability is better for lower values of parameter *α* and higher values of parameters *k* and *m*.

6 Conclusion

In this paper, *α-k-μ* multipath fading is considered. Rayleigh, Nakagami-*m*, Weibull, Rician and *α-μ* distributions can be performed from *α-k-μ* distribution as special cases. The parameter *α* is connected to nonlinearity of environments. The parameter *μ* is referred to the number of clusters. The Rice factor *k* is a ratio of dominant components power and scattering components power.

The closed form expression for level crossing rate of product of two *α-k-μ* random variables is determined. Then, the expression for level crossing rate of the ratio of product of two *α-k-μ* random variables and *α-k-μ* random variable is calculated.

The expression for level crossing rate of the ratio of product of two α-k-μ random variables and α-k-μ random variable can be used for calculation of average fade duration of wireless communication system operating over composite α-k-μ multipath fading environment in the presence of cochannel interference subjected to α-k-μ multipath fading.

The numerical results are presented graphically to pointed out the influence of fading parameters on average level crossing rate.

Acknowledgment

This paper has been funded by the Ministry of Education, Science and Technological Development of Republic of Serbia under projects TR-33035 and III-44006.

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