Implementation of Generalized Detector in MIMO Radar Systems

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Abstract: - In this paper, we consider the problem of multiple-input multiple-output (MIMO) radars employing the generalized detector (GD) based on the generalized approach to signal processing in noise (GASP) and using the space-time coding to achieve a desired diversity. To that end, we derive a suitable GD structure after briefly outlining the model of the received target return signal. GD performance is expressed in closed form as a function of the clutter statistical properties and of the space-time code matrix. We investigate a particular case when GD requires a priori knowledge of the clutter covariance, i.e., the decision statistics under the null hypothesis of “a no” target is an ancillary statistic in the sense that it depends on the actual clutter covariance matrix but its probability density function (pdf) is functionally independent of such a matrix. Therefore, threshold setting is feasible with no a priori knowledge as to the clutter power spectrum. As to the detection performance, a general integral form of the probability of detection is provided, holding independent of the searched object fluctuation model. The formula is not analytically manageable, nor does it appear to admit general approximate expressions, which allow giving an insightful look in the MIMO radar system behaviour. We thus restrict our attention to the case of Rayleigh-distributed target attenuation (Swerling-1 model). To code construction we use an information-theoretic approach and compare conditions for code optimality with ones for classical Chernoff bound. This approach offers a methodological framework for space-time coding in MIMO radar systems constructed based on GASP, as well as simple and intuitive bounds for performance prediction.

Key-Words: - Generalized detector, multiple-input multiple-output (MIMO), Rayleigh fading, Chernoff bound, generalized approach to signal processing, Swerling-1 model.

1 Introduction
Multiple-input multiple-output (MIMO) radar systems are widely used, for example, under weather observations, whose output is often seen on the television weather report. There also exist MIMO radar systems that determine wind speed and direction as a function of altitude, by detecting the very weak radar echo from the clear air. MIMO radar systems located around airports warn of dangerous wind shear produced by the weather effect known as downburst that can accompany severe storms. There is usually specially designed weather avoidance MIMO radar system in the nose of small as well as large aircraft to warn of dangerous or uncomfortable weather in flight.

Another successful MIMO radar systems were the downward-looking space borne altimeter radar that measured worldwide the geoids, for example, the mean sea level, which is not the same all over the world, with exceptionally high accuracy. There have been attempts in the past to use MIMO radar system for determining soil moisture and for assessing the status of agriculture crops, but these attempts have not provided sufficient accuracy. Imaging MIMO radar systems in satellite or aircraft are used to help ships efficiently navigate northern seas coated with ice because the radar can tell which types of ice are easier for a ship to penetrate.

MIMO radar systems have received a great attention owing to the following viewpoints:

- MIMO systems have been deemed as efficient spatial multiplexers;
- MIMO systems have been deemed as a suitable strategy to ensure high-rate communications on wireless channels [1].

Space-time coding has been largely investigated as a viable means to achieve spatial diversity, and thus to
contrast the effect of fading [2] and [3]. We apply the generalized approach to signal processing (GASP) in noise [4]–[9] to the design and implementation of MIMO radars, which use space-time coding technique.

Theoretical principles of MIMO radars were discussed in [10], while in [11] and [12] the potential advantages of MIMO radars are thoroughly considered. MIMO radar system architecture is able to provide independent diversity paths, thus yielding remarkable performance improvements over conventional radar systems in the medium-high range of the probability of detection. As was shown in [13], the MIMO mode can be conceived as a means of boot-strapping to obtain greater coherent gain. Some practical issues concerning implementation, namely, equipment specifications, dynamic range, phase noise, system stability, isolation and spurs, of MIMO radars are discussed in [14]. The waveform design for MIMO beamforming is an object of [15]. Experimental investigations concerning MIMO radars are presented in [16]. MIMO imaging and the related resolution issues are investigated in [17].

MIMO radar system can be represented by $N_T$ transmit antennas, spaced several wavelengths apart, and $N_R$ receive antennas, not necessarily collocated, and possibly forwarding, through a wired link, the received echoes to a fusion center, whose task is to make the final decision as to the presence of a searched object in the coverage area. If the spacing between the transmit antennas is large enough and so is the spacing between the receive antennas, a rich scattering environment is generated, and each receive antenna processes $l$ statistically independent copies of a target return echo. The concept of rich scattering environment is borrowed from communication theory, and models a situation where the MIMO architecture yields target return scattering under a number of different angles, eventually resulting into a number of independent random channels.

Unlike a conventional radar array system, which attempts to maximize the coherent processing, MIMO radar system resorts to the diversity of target return scattering in order to improve the detection performance. Indeed, it is well known that, in conventional radar systems, fluctuations of the order of 10 dB in the reflected energy may arise by changing the target return signal aspect angle by as little as one mrad [18]. This effect leads to severe degradations in the radar detection performance, due to the high signal correlation at the array elements. This drawback might be partially circumvented under the use of MIMO radar, which exploits the spatial diversity resulting from the target return signal angular spread. Otherwise, uncorrelated signals at the array elements are available. Based on mentioned above statements, it was shown in [10]–[12] that in the case of additive white Gaussian noise (AWGN), transmitting orthogonal waveforms result into increasingly constrained fluctuation of the back-scattered energy.

Our approach is based on GASP implementation [4]–[9] and use some key results from communication theory, and in particular, the well-known concept that, upon suitably space-time encoding the transmitted waveforms, a maximum diversity order given by $N_T \times N_R$ can be achieved. Importing these results in a radar scenario poses a number of problems, which form the object of the present study, and in particular:

- The issue of waveform design, which exploits the available knowledge as to space-time codes, adapting it to the radar context.
- The issue of designing a suitable detection structure based on GASP, also in the light of the fact that the disturbance can no longer be considered as AWGN, due to the presence of clutter returns;
- At the performance assessment level, the issue of evaluating the maximum diversity order that can be achieved and the space-time coding ensuring it under different instances of clutter and/or searched object.

In the present paper, the first and third tasks are merged in the unified problem of determining the space-time coding achieving maximum diversity order in target return signal detection, for the constrained probability of false alarm, and for the given clutter covariance. As to the second task, the decision-making criterion exploiting by GASP is employed. Unlike [10]–[12], no assumption is made on either the target return signal fluctuation model or the disturbance covariance. Thus, a family of detection structures is derived, depending upon the number of transmitting and receiving antennas and the disturbance covariance. A side result, which paves the way to further investigations on the feasibility of fully adaptive MIMO architectures based on GASP is that the decision statistic, under the null hypothesis of “a no” the target return signal, is an ancillary statistic, in the sense that it depends on the actual clutter covariance matrix, but its probability density function (pdf) is functionally independent of such a matrix.
Therefore, the threshold setting is feasible with no prior knowledge as to the clutter power spectrum.

As to the detection performance, a general integral form of the probability of detection is provided, holding independent of the target return signal fluctuation model. The formula is not analytically manageable, nor does it appear to admit general approximate expressions, which allow giving an insightful look in the system behaviour. We thus restrict our attention to the case of Rayleigh-distributed target attenuation (Swerling-1 model), and use an information-theoretic approach to code construction discussed in [19] and [20] and compare conditions for code optimality with ones for classical Chernoff bound.

The rest of the paper is organized as follows. Section II deals with the statement of problem and model of MIMO radar system. Section III represents the GD design for MIMO radar systems. The performance analysis is discussed in Section IV. Section V is devoted to the code design problems based on the information-theoretic approach and comparative analysis with the classical Chernoff bounds. Simulation results are presented in Section VI and illustrate definite conditions allowing an achieving the maximum diversity order. Finally, Section VII represents conclusions and possible ways for further research.

2 System Model

We consider MIMO radar composed of \( N_T \) fixed transmitters and \( N_R \) fixed receivers (see Fig.1) and assume that the antennas as the two ends of the system are sufficiently spaced such that a possible searched object and/or clutter provides uncorrelated reflection coefficients between each transmit/receive pair of sensors. Denote by \( s_m(t) \) the baseband equivalent of the coherent pulse train transmitted by the \( m \)th antenna, for example,

\[
s_m(t) = \sum_{j=1}^{M} a_{m,j} p[t - (j - 1)T_p], \quad m = 1, \ldots, N_T
\]

where \( p(t) \) is the signature of each transmitted pulse, which we assume, without loss of generality, with unit energy and duration \( T_p \); \( T_p \) is the pulse repetition time;

\[
a_m = [a_{m,1}, \ldots, a_{m,M}]^T,
\]

is an \( M \)-dimensional column vector whose entries are complex numbers which modulate both in amplitude and in phase the \( N \) pulses of the train, where \((\cdot)^T\) denotes transpose. In the sequel, we refer to \( a_m \) as the code word of the \( i \)th antenna.

The baseband equivalent of the signal received by the \( i \)th sensor, from a searched object with two-way time delay \( \tau \), can be presented in the following form

\[
x_i(t) = \sum_{m=1}^{N_T} \alpha_{i,m} \sum_{j=1}^{M} a_{m,j} p[t - \tau - (j - 1)T_p] + w_i(t),
\]

\[
i = 1, \ldots, N_R
\]

where \( \alpha_{i,m} \), \( i = 1, \ldots, N_R \) and \( m = 1, \ldots, N_T \), are complex numbers accounting for both the searched object back-scattering and the channel propagation effects between the \( m \)th transmitter and the \( i \)th receiver; \( w_i(t), \ i = 1, \ldots, N_R \), are zero-mean, spatially uncorrelated, complex Gaussian random processes accounting for both the external and the internal disturbance.

For simplicity, we assume a zero-Doppler searched object, but all the derivations can be easily extended to account for a possible known Doppler shift. We explicitly point out that the validity of the above model requires the narrowband assumption

\[
\frac{d_{N_T}^N + d_{N_R}^N}{c} \ll \frac{1}{W},
\]

where \( W \) is the bandwidth of the transmitted pulse, \( d_{N_T}^N \) and \( d_{N_R}^N \) denotes the maximum spacing between two sensors at the transmitter and the receiver end, respectively. The signal \( x_i(t) \), at each of the receive elements, is matched filtered to the pulse \( p(t) \) by preliminary filter (PF) of GD and the PF output is sampled at the time instants \( \tau + (j - 1)T_p, \ j = 1, \ldots, M \).

Before further analysis, there is a need to recall the main GD functioning principles discussed in [4] and [5]. The simple model of GD in form of block diagram is represented in Fig.2. In this model, we use the following notations: MSG is the model signal generator (local oscillator), the AF is the additional filter (the linear system) and the PF is the preliminary filter (the linear system, too). A detailed discussi-
Consider briefly the main statements regarding the AF and PF. There are two linear systems at the GD front end that can be presented, for example, as bandpass filters, namely, the preliminary filter or PF with the impulse response $h_{PF}(\tau)$ and the additional filter or AF with the impulse response $h_{AF}(\tau)$. For simplicity of analysis,
we consider that these filters have the same impulse responses and bandwidths by value. Moreover, the AF resonant frequency is detuned relative to the PF one on such a value that the incoming signal cannot pass through the AF. Thus, the received signal and noise can be appeared at the PF output and the only noise is appeared at the AF output.

It is well known fact that if a value of detuning between the AF and PF resonant frequencies is more than \(4 \div 5\Delta f_a\), where \(\Delta f_a\) is the signal bandwidth, the processes forming at the AF and PF outputs can be considered as independent and uncorrelated processes between each other. In practice, the coefficient of correlation is not more than 0.05. In the case of signal absence in the input process, the statistical parameters is coming in at the AF and PF inputs. We may think that the AF and PF do not change the statistical parameters of input process, since they are the linear front-end systems of GD. By this reason, the AF can be considered as a generator of reference signal forming at the AF output.

There is a need to make some comments regarding the noise forming at the PF and AF outputs. If the mentioned above Gaussian noise comes in at the AF and PF inputs, the noise forming at the AF and PF outputs is Gaussian, too, because the same noise with the same statistical parameters is coming in at the AF and PF inputs. We may think that the AF and PF do not change the statistical parameters between the AF and PF resonant frequencies is more than \(4 \div 5\Delta f_a\).

Moreover, the AF resonant frequency is detuned relative to the PF one on such a value that the incoming signal cannot pass through the AF. Thus, the received signal and noise can be appeared at the PF output and the only noise is appeared at the AF output.

If, for example, the AWGN with zero mean and two-sided power spectral density \(N_0/2\) is coming in at the AF and PF inputs (GD linear system front-end), then the noise forming at the AF and PF outputs is Gaussian with zero mean and variance given by [5, pp.264–269]

\[
\sigma_n^2 = \frac{N_0\omega_0}{8\Delta_f},
\]

where, in the case if the AF or PF is the RLC oscillatory circuit, then the AF or PF bandwidth \(\Delta_f\) and resonance frequency \(\omega_0\) are defined in the following manner:

\[
\Delta_f = \pi\beta, \quad \omega_0 = \frac{1}{\sqrt{LC}}, \quad \text{where} \quad \beta = \frac{R}{2L}.
\]

The main functioning condition of GD is the equality over the whole range of parameters between the coherent pulse transmitted by the \(m\)th antenna (the model signal \(s^*_m(t)\) at the GD MSG output in the receiver) and the signal received by the \(i\)th sensor from the target forming at the GD input liner system (the PF) output, i.e.

\[
s^*_m(t) = \sum_{j=1}^{M} a_{m,j} p^{*}\left[t - (j-1)T_p\right] = s_i(t). \quad (8)
\]

How we can satisfy this condition in practice is discussed in detail in [4] and [5, pp.669-695]. More detailed discussion about a choice of PF and AF and their impulse responses is given also in [4] and [5]. Also see additionally the following link http://www.sciencedirect.com/science/journal/10512004, clicking “Volume 8, 1998”, “Volume 8, Issue 3”, and “A new approach to signal detection theory”.

Thus, denote by \(x_i(k)\) the \(k\)-th sample, i.e.,

\[
x_i(k) = \sum_{m=1}^{M} \alpha_{i,m} a_{m,k} + \xi_i(k) , \quad (9)
\]

where \(\xi_i(k)\) is the filtered noise sample. Define the \(N\)-dimensional column vectors

\[
x_i = [x_i(1), \ldots, x_i(N)]^T \quad (10)
\]

and rewrite them as

\[
x_i = A_i \alpha_i + \xi_{PF_i}, \quad i = 1, \ldots, N_R \quad (11)
\]

where

\[
x_{PF_i} = [\xi_{PF_i}(1), \ldots, \xi_{PF_i}(M)]^T, \quad \alpha_i = [\alpha_{i,1}, \ldots, \alpha_{i,N_R}]^T, \quad (12, 13)\]
and the \((N \times N_T)\)-dimensional matrix \(A\) defined in the following form

\[
A = [a_1, \ldots, a_{N_T}]
\]

(14)

has the code words as columns. This last matrix is referred to as the code matrix. We assume that \(A\) is a full rank matrix. It is worth underlining that the model given by (9) applies also to the case that space-time coding is performed according to [19] and [20], namely, by dividing a single pulse in \(N\) subpulses. The code matrix \(A\) thus defines \(N_T\) different code words of length \(M\), which can be received by a single receive antenna, thus defining the multiple-input single-output (MISO) structure of [21], as well as by a set of \(N_R\) receive antennas, as in the present study.

3 GD for MIMO Radar Systems

The problem of detecting a target with a MIMO radar system can be formulated in terms of the following binary hypothesis test

\[
\begin{align*}
H_0 & \quad \Rightarrow \quad x_i = \xi_{PF_i}, & i = 1, \ldots, N_R \\
H_1 & \quad \Rightarrow \quad x_i = Aa_i + \xi_{PF_i}, & i = 1, \ldots, N_R
\end{align*}
\]

(15)

where \(\xi_{PF_i}, i = 1, \ldots, N_R\), are statistically independent and identically distributed (i.i.d.) zero-mean complex Gaussian vectors with covariance matrix

\[
E[\xi_{PF_i} \xi_{PF_i}^*] = E[\xi_{AF_i} \xi_{AF_i}^*] = M. \tag{16}
\]

Here \(E[\cdot]\) denotes the statistical mathematical expectation and \((*)\) denotes the conjugate transpose. The covariance matrix given by (16) is assumed positive definite and known.

According to the Neyman-Pearson criterion, the optimum solution to the hypotheses testing problem in (15) must be the likelihood ratio test. However, for the case at hand, it cannot be implemented since total ignorance of the parameters \(a_i\) is assumed. One possible way to circumvent this drawback is to resort to the generalized likelihood ratio test (GLRT) [22], which is tantamount to replacing the unknown parameters with their maximum likelihood (ML) estimates under each hypothesis. Applying the GLRT to the GASP [4]–[9], we obtain the following decision-making rule

\[
\max_{a_{i_1}, \ldots, a_{i_{N_R}}} \frac{f(x_1, \ldots, x_{N_R} \mid H_1, M, a_1, \ldots, a_{N_R})}{f(\xi_{AF_1}, \ldots, \xi_{AF_{N_R}} \mid H_0, M)} \geq \frac{1}{K_\epsilon} \tag{17}
\]

where \(f(x_1, \ldots, x_{N_R} \mid H_1, M, a_1, \ldots, a_{N_R})\) is the pdf of data under the hypothesis \(H_1\) and \(f(\xi_{AF_1}, \ldots, \xi_{AF_{N_R}} \mid H_0, M)\) is the pdf of data under the hypothesis \(H_0\), respectively; \(K_\epsilon\) is a suitable modification of the original threshold in the case of GD. Previous assumptions imply that the aforementioned pdfs can be written in the following form:

\[
\begin{align*}
&f(\xi_{AF_1}, \ldots, \xi_{AF_{N_R}} \mid H_0, M) = \\
&= \frac{1}{\pi^{MN_R} \det(M)} \exp \left[ - \sum_{i=1}^{N_R} \xi_{AF_i}^* M^{-1} \xi_{AF_i} \right] \tag{18}
\end{align*}
\]

at the hypothesis \(H_0\) and

\[
\begin{align*}
&f(x_1, \ldots, x_{N_R} \mid H_1, M, a_1, \ldots, a_{N_R}) = \\
&= \frac{1}{\pi^{MN_R} \det(M)} \exp \left[ - \sum_{i=1}^{N_R} (x_j - Aa_j)^* M^{-1} (x_j - Aa_j) \right] \tag{19}
\end{align*}
\]

under the hypothesis \(H_1\), where \(\det(\cdot)\) denotes the determinant of a square matrix. Substituting (18) and (19) in (17), we can recast the GLRT based on the GASP [4]–[9], after some mathematical transformations, in the following form

\[
\sum_{i=1}^{N_R} \xi_{AF_i}^* M^{-1} \xi_{AF_i} - \sum_{i=1}^{N_R} \min_{a_i}(x_j - Aa_j)^* M^{-1} (x_j - Aa_j) \geq \frac{1}{K_\epsilon} \tag{20}
\]

In order to solve the \(N_R\) minimization problems in (20) we have to distinguish between two different cases.

Case 1: \(N_R > N_T\). In this case, the quadratic forms in (20) achieve the minimum at

\[
\hat{a}_i = (A^* M^{-1} A)^{-1} A^* M^{-1} x_i, \quad i = 1, \ldots, N_R \tag{21}
\]

and, as a consequence, the GLRT based on the GASP [4]–[9] at the main condition of GD functioning, i.e., equality in whole range of parameters bet-
ween the transmitted information signal and reference signal (signal model) in the receiver part (see (8)), becomes

\[ 2 \sum_{i=1}^{N_k} x_i^* M^{-1} A (A' M^{-1} A)^{-1} A' M^{-1} x_i \]
\[ - \sum_{i=1}^{N_k} x_i^* M^{-1} A A' M^{-1} x_i + \sum_{i=1}^{N_k} \xi_{iM}^* M^{-1} \xi_{iM} - \sum_{i=1}^{N_k} \xi_{iM}^* M^{-1} A A' M^{-1} x_i \leq_{H_0} K_g. \]

(22)

Case 2: \( N_R \leq N_Tr \). In this case, the minimum of the quadratic forms in equation (20) is zero, since each linear system

\[ A \hat{a}_i = x_i, \quad i = 1, \ldots, N_R \]

(23)
is underdetermined. Consequently, the GLRT based on the GASP [4]–[9], at the main condition of GD functioning, i.e., equality in whole range of parameters between the transmitted information signal and reference signal (signal model) in the receiver part, becomes

\[ \sum_{i=1}^{N_k} \xi_{iM}^* M^{-1} \xi_{iM} = \sum_{i=1}^{N_k} x_i^* M^{-1} A A' M^{-1} x_i \leq_{H_0} K_g. \]

(24)

4 Performance Analysis

In order to define possible design criteria for the space-time coding, it is useful to establish a direct relationship between the detection performance and the transmitted waveform, which is thus the main goal of the present section. Under the hypothesis \( H_0 \), the left hand side of the GLRT based on the GASP [4]–[9] can be written in the following form

\[ \sum_{i=1}^{N_k} \xi_{iM}^* M^{-1} \xi_{iM} - \sum_{i=1}^{N_k} \xi_{iM}^* M^{-1} A A' M^{-1} x_i \leq_{H_0} K_g. \]

(25)

and, represents the GD background noise. It follows from [7] that the decision statistic is defined by the modified second-order Bessel function of an imaginary argument or, as it is also called, McDonald’s function with \( N_Tr \times N_R \) degrees of freedom.

Thus, the decision statistic is independent of dimensionality \( M \) of the column vector given by equation (2) whose entries are complex numbers, which modulate both in amplitude and in phase the \( M \) pulses of the train. Consequently, the probability of false alarm \( P_{FA} \) can be evaluated in the following form [6] and [23]:

\[ P_{FA} = \frac{1}{4 \sigma_n^2} \exp \left\{ \frac{-T_p \beta K_g^2}{8 \sigma_n^4} \sum_{j=1}^{2j+1} \left[ \frac{T_p \beta}{V_{\sigma_n^2} K_g} \right]^{j+1} \right\}. \]

(26)

This last expression allows us to note the following observations:

- The decision statistic is ancillary, in the sense that it depends on the actual clutter covariance matrix, but its pdf is functionally independent of such a matrix;
- The threshold setting is feasible with no prior knowledge as to the clutter power spectrum, namely, the GLRT based on the GASP [4]–[9] ensures the constant false alarm rate (CFAR) property.

Under the hypothesis \( H_1 \), given \( a_i \), the vectors \( x_i \), \( i = 1, \ldots, N_R \) are statistically independent complex Gaussian vectors with the mean value \( M^{-1} A a_i \) and identity covariance matrix. It follows that, given \( a_i \), the GLRT based on the GASP [4]–[9] is no the central distributed modified second-order Bessel function of an imaginary argument, with the no centrality parameter \( \sum_{i=1}^{N_k} a_i^* A' M^{-1} A a_i \), and degrees of freedom \( N_R \times N_Tr \).

Consequently, the conditional probability of detection \( P_D \) based on statements in [24] and discussion in [6] and [7] can be represented in the following form

\[ P_D = Q_{N_Tr \times N_k} \left( \sqrt{2q}, \frac{T_p \beta}{2 \sigma_n^2} K_g \right), \]

(27)

where

\[ q = \sum_{i=1}^{N_k} a_i^* A' M^{-1} A a_i \]

(28)

and \( Q_k (\cdot) \) denotes the generalized Marcum Q function of order \( k \).

An alternative expression for the conditional probability of detection \( P_D \), in terms of an infinite series, can be also written in the following form:

\[ P_D = \sum_{k=0}^{\infty} \frac{(-q)^k}{k!} \left[ 1 - \Gamma_{\infty} (K_g, k + N_Tr \times N_k) \right], \]

(29)
where
\[
\Gamma_{\nu}(p, r) = \frac{1}{\Gamma(r)} \int_0^\nu \exp(-z) z^{r-1} dz
\]
(30)
is the incomplete Gamma function. Finally, the unconditional probability of detection \(P_D\) can be obtained averaging the last expression over the pdf of \(a_i, i = 1, \ldots, N_R\).

5 Code Design Principles

In principle, the basic criterion for code design should be the maximization of the probability of detection \(P_D\) given by (27) over the set of admissible code matrices, i.e.,
\[
\arg \max \mathbb{E} \left[ Q_{N_R \times N_R} \left( \left[ \sqrt{2 q_i} \right] \right) \right] = \arg \max \mathbb{E} \left[ Q_{N_R \times N_R} \left( \sqrt{2 \sum_{i=1}^{N_R} a_i^H A^H M^{-1} A a_i \sqrt{2 K}} \right) \right]
\]
(31)
where \(\arg \max \chi(\cdot)\) denotes the value of \(A\), which maximizes the argument and the statistical average is over \(a_i, i = 1, \ldots, N_R\). Unfortunately, the above maximization problem does not appear to admit a closed-form solution, valid independent of the fading law, whereby we prefer here to resort to the information-theoretic criterion supposed in [19].

Another way is based on the optimization of the Chernoff bound over the code matrix \(A\). As was shown below, these ways lead to the same solution, which subsumes some well-known space-time coding, such as Alamouti code and, more generally, the class of space-time coding from orthogonal design presented in [2], [20], [21] and [25], which have been shown to be optimum in the framework of communication theory. In subsequent derivations, we assume that \(a_i, i = 1, \ldots, N_R\) are i.i.d. zero-mean complex Gaussian vectors with scalar covariance matrix, i.e.,
\[
E[a_i a_i^H] = \sigma_a^2 I
\]
(32)
where \(\sigma_a^2\) is a real factor accounting for the backscattered useful power, and the matrix \(I\) denotes the identity matrix.

Roughly speaking, the GLRT strategy overcomes the prior uncertainty as to the searched object fluctuations by ML estimation (MLE) of the target return complex amplitude, and plugging the estimated value into the conditional likelihood ratio in place of the true value. Also, it is well known that, under general consistency conditions, the GLRT converges towards the said conditional likelihood, thus achieving a performance closer and closer to the perfect measurement bound, i.e., the performance of an optimum test operating in the presence of known target parameters.

Diversity, on the other hand, can be interpreted as a means to transform an amplitude fluctuation in an increasingly constrained one. It is well known, for example that, upon suitable receiver design, exponentially distributed square amplitude of searched object may be transformed into a central chi-square fluctuation with \(d\) degrees of freedom through a diversity of order \(d\) in any domain. More generally, a central chi-square distributed random variable with \(2N_p\) degrees of freedom may be transformed into a central chi-square distributed random variable with \(2N_p \times d\) degrees of freedom. In this framework, a reasonable design criterion for the space-time coding is the maximization of the mutual information between the signals received from the various diversity branches and the fading amplitudes experienced thereupon.

Thus, denoting by \(I(a, X)\) the mutual information according to [19] between the random matrices
\[
a = [a_1, \ldots, a_{N_R}]
\]
(33)
and
\[
X = [x_1, \ldots, x_M] = A a + \Xi
\]
(34)
the quantity to be maximized is
\[
I(a, X) = H(X) - H(X | a)
\]
(35)
where
\[
\Xi = [\xi_1, \ldots, \xi_{N_R}]
\]
(36)
\(H(X)\) is the entropy of the random matrix \(\Xi\), and \(H(X | a)\) is the conditional entropy of \(X\) given \(a\) [19].

Exploiting the statistical independence between \(a\) and \(X\), we can write (35) in the following form
\[
I(a, X) = H(X) - H(X | a) = H(X) - H(\Xi)
\]
(37)
where $H(Ξ)$ is the entropy of the random matrix $Ξ$. Assuming that the columns of $a$ are i.i.d. zero-mean complex Gaussian vectors with covariance matrix $σ^2_a I$, we can write $H(X)$ and $H(Ξ)$, respectively, in the following form:

$$H(X) = x \log [(αe)^M \det(M + σ^2_a AA^*)]$$

and

$$H(Ξ) = x \log [(αe)^M \det(M)] .$$

As design criterion, we adopt the maximization of the minimum probability of detection $P_d$, which can be determined as the lower Chernoff bound, under an equality constraint for the average signal-to-clutter power ratio (SCR) given by

$$SCR = \frac{1}{MNTr} E \left[ \sum_{i=1}^{N_t} a_i^* A^* M^{-1} A a_i \right] = \frac{σ^2_a}{MNTr} \text{tr}(A^* A M^{-1} A A^*) = \frac{σ^2_a}{MNTr} \sum_{m=1}^{N_t} λ_m ,$$

where $\text{tr}(\cdot)$ denotes the trace of a square matrix and $λ_m$ are the elements or corresponding ordered (in decreasing order) eigenvalues of the diagonal matrix $Λ$ defined by the eigenvalue decomposition $V^* Λ V$ of the matrix $M^{-1} AA^* M^{-1}$, where $V$ is a $M × M$ unitary matrix. The considered design criterion relies on the maximization of the mutual information given by (37) under equality constraint defined in (40) for SCR.

This is tantamount to solving the following constrained minimization problem since $H(Ξ)$ does not exhibit any functional dependence on $A$.

$$\left\{ \begin{array}{l}
\min_{λ_1, \ldots, λ_{Nt}} \prod_{m=1}^{N_t} \left[ \frac{1}{1 + γ(λ_m σ^2_a + 1)} \right]^{N_t} \\
\frac{σ^2_a}{MNTr} \sum_{m=1}^{N_t} λ_m = μ \, ,
\end{array} \right. \quad (41)$$

which, taking the logarithm, is equivalent

$$\left\{ \begin{array}{l}
\min_{λ_1, \ldots, λ_{Nt}} \sum_{m=1}^{N_t} \log [1 + γ(λ_m σ^2_a + 1)] \\
\frac{N_t}{σ^2_a} \sum_{m=1}^{N_t} λ_m = μMNTr ,
\end{array} \right. \quad (42)$$

where $γ$ is the variable defining the upper Chernoff bound [19] and [21].

Since $\log[1 + γ(σ^2_a y + 1)]$ is a concave function of $y$, we can apply Jensen’s inequality [19], [20], and [21] to obtain

$$\sum_{m=1}^{N_t} \log[1 + γ(λ_m σ^2_a + 1)] \leq N_{Tr} \log[1 + γ(μM + 1)] .$$

Moreover, forcing in the right hand side of (43), the constraint of (42), we obtain

$$\sum_{m=1}^{N_t} \log[1 + γ(λ_m σ^2_a + 1)] \leq N_{Tr} \log[1 + γ(μM + 1)] .$$

The equality in (44) is achieved if

$$\lambda_m = \frac{μM}{σ^2_a} , \quad m = 1, \ldots, N_{Tr}$$

implying that an optimum code must comply with the condition

$$M^{-1} AA^* M^{-1} = \left\{ \begin{array}{ll}
\frac{μM}{σ^2_a} I & \Rightarrow \text{Case 1} , \\
\frac{μM}{4σ^2_a} I & \Rightarrow \text{Case 2} .
\end{array} \right. \quad (46)$$

In particular, if the additive disturbance is white Gaussian, i.e., $M = σ^2_a I$, the above equation reduces to

$$A A^* = \left\{ \begin{array}{ll}
\frac{σ^2_a μM}{2σ^2_a σ^2 - μN} A (A^* A)^{-1} A^* & \Rightarrow \text{Case 1} , \\
\frac{μM}{4σ^2_a} I & \Rightarrow \text{Case 2} .
\end{array} \right. \quad (47)$$

The last equation subsumes, as a relevant case, the set of orthogonal space-time codes. Indeed, assuming $M = N_R = N_{Tr}$, the condition given by (47) yields, for the optimum code matrix,

$$A A^* = \frac{μM}{4σ^2_a} I .$$

(48)
i.e., the code matrix $A$ should be proportional to any unitary $M \times M$ matrix.

Thus, any orthonormal basis of $\mathbb{F}^M$ can be exploited to construct an optimum code under the Case 2 and white Gaussian noise. If, instead, we restrict our attention to code matrices built upon Galois Fields (GF), there might be limitations to the existing number of optimal codes. Refer to [2] and [20] and to the Urvitz-Radon condition exploited therein, we just remind here that, under the constraint of binary codes, unitary matrices exist only for limited values of $M$: for $2 \times 2$ coding, we find the normalized Alamouti code [25], which is an orthonormal basis, with elements in GF given by (2), for $\mathbb{F}^2$.

Now, make some comments. First notice, that under the white Gaussian noise, both performance measures considered above are invariant under unitary transformations of the code matrix, while at the correlated clutter they are invariant with respect to right multiplication of $A$ by a unitary matrix. Probably, these degrees of freedom might be exploited for further optimization in different radar functions. Moreover, (42) represents the optimum solution for the case that no constraint is forced upon the code alphabet; indeed, the code matrices turn out in general to be built upon the completely complex field. If, instead, the code alphabet is constrained to be finite, then the optimum solution in (42) may be no longer achievable for arbitrary clutter covariance.

In fact, while for the special case of white clutter and binary alphabet the results discussed in [2] may be directly applied for given values of $N_p$ and $N_R$, for arbitrary clutter covariance and/or transmit/receive antennas number, a code matrix constructed on GF ($q$) and fulfilling the conditions given by (42) is no longer ensured to exist. In these situations, which however form the object of current investigations, a brute-force approach could consist of selecting the optimum code through an exhaustive search aimed at solving the equation (31), which would obviously entail a computational burden $O(q^{mn})$ floating point operations. Herein we use the usual Landau notation $O(n)$; hence, an algorithm is $O(n)$ if its implementation requires a number of floating point operations proportional to $N_R$ [26].

Fortunately, the exhaustive search has to be performed off line. The drawback is that the code matrix would inevitably depend on the searched object fluctuation law; moreover, if one would account for possible non-stationarities of the received clutter, a computationally acceptable code updating procedure should be envisaged so, as to optimally track the channel and clutter variations.

### 6 Simulation Results

The present section is aimed at illustrating the validity of the proposed encoding and detection schemes under diverse scenarios. In particular, we first assume uncorrelated disturbance, whereby orthogonal space-time codes are optimal. In this scenario, simulations have been run, and the results have been compared to the Chernoff bounds of the conventional GLRT receiver discussed in [23] and to the GD performance achievable through a single-input single-output (SISO) radar system.

Next, the effect of the disturbance correlation is considered, and the impact of an optimal code choice is studied under different values of transmit/receiver antenna numbers. In all cases, the behavior of the mutual information between the observations and the searched object replicas can be also represented, showing that such a measure is itself a useful tool for radar system design and assessment, but this analysis is outside a scope of the present paper.

Figure 3 represents the white Gaussian disturbance and assesses the performance of the GLRT GD. To elicit the advantage of waveform optimization, we consider both the optimum coded radar system and the uncoded one, corresponding to pulses with equal amplitudes and phases. The probability of detection $P_D$ is plotted versus SCR assuming $P_{FA} = 10^{-4}$ and $M = N_R = N_T = 2$. This simulation setup implies that the Alamouti code is optimum in the sense specified by (47). For comparison purposes, we also plot the performance of the uncoded SISO GD. We presented the performance of the conventional GLRT [23] to underline a superiority of GD implementation.

The curves highlight that the optimum coded radar system employing the GD and exploiting the Alamouti code, achieves a significant performance gain with respect to both the uncoded and the SISO radar systems. Precisely, for $P_D = 0.9$, the performance gain that can be read as the horizontal displacement of the curves corresponding to the analyzed radar systems, is about 1 dB with reference to the uncoded GLRT GD radar system and 5 dB regarding SISO GD. Superiority of GD employment with respect to the conventional GLRT radar systems achieves 6 dB
for the optimum coded radar systems, 8 dB for the uncoded radar systems, and 12 dB for SISO radar systems.

It is worth pointing out that the uncoded system performs slightly better the coded one for the low probability of detection. This is a general trend in detection theory, which predicts that less and less constrained fluctuations are detrimental in the high SCR region, while being beneficial in the low SCR region. On the other hand, the code optimization results in a more constrained fluctuation, which, for the low SCRs, leads to slight performance degradation as compared with uncoded systems. The effect of disturbance correlation is elicited in Fig. 3, too, where the analysis is produced assuming an overall disturbance with exponentially shaped covariance matrix, whose one-lag correlation coefficient $\rho$ is set to 0.95.

In this case, the Alamouti code is no longer optimum. The plots show that the performance gain of the optimum coded GLRT GD radar system over both the uncoded and the SISO GD detector is almost equal to that resulting when the disturbance is white. On the other hand, setting $M = N_R = N_{tr} = 2$ in (46), shows that, under correlated disturbance, the optimum code matrix is proportional to $M$: namely, an optimal code tends to restore the “white disturbance condition.” This also explains why the conventional Alamouti code follows rather closely the performance of the uncoded GLRT GD radar system.

The effect of number $N_R$ of receive antennas on the performance is shown in Fig. 4, where $P_D$ is plotted versus SCR for $M = N_{tr} = 8$, exponentially shaped clutter covariance matrix with $\rho = 0.95$, and several values of $N_R$. The curves highlight that the higher $N_R$, namely, the higher the diversity order, the better the performance. Specifically, the performance gap between the case $N_R = 8$ and the case of SIMO GLRT GD radar system (i.e., $N_R = 1$) is about 3.0 dB, while, in the case of the conventional GLRT radar systems [23], is about 8.0 dB for $P_D = 0.9$. A great superiority between the radar systems employing GLRT GD and conventional GLRT is evident and estimated at the level of 3.0 dB at $N_R = 8$ and 5.0 dB in the case of a SIMO ($N_R = 1$) for $P_D = 0.9$. Notice that this performance trend is also in accordance with the expression of the mutual information that exhibits a linear, monotonically increasing, dependence on $N_R$. The same qualitative, but not quantitative, performance can be presented under study of the number $N_{tr}$ of available transmit antennas on the GLRT GD radar system performance.

7 Conclusions

In this paper, we have addressed the synthesis and analysis of MIMO radar systems employing GD and exploiting the space-time coding. To this end, after a short description of the MIMO radar signal model, we have devised the GLRT GD under the assumpti-
of the AWGN. Remarkably, the decision statistic is ancillary and, consequently CFAR property is ensured, namely, the detection threshold can be set independent of the disturbance spectral properties. We have also assessed the performance of the GLRT GD providing closed-form expressions for both $P_d$ and $P_{fa}$. Lacking a manageable expression for $P_d$ under arbitrary searched object fluctuation models, we restricted our attention to the case of Rayleigh distributed amplitude fluctuation. The performance assessment that has been undertaken under several instances of number of receive and transmit antennas, and of clutter covariance, has confirmed that MIMO GD radar systems with a suitable space-time coding achieve significant performance gains over SIMO, MISO, SISO, or conventional SISO radar systems employing the conventional GLRT detector [23].

In addition, these MIMO GD radar systems outperform the listed above radar systems employing the conventional GD. Future research might concern the extension of the proposed framework to the case of an unknown clutter covariance matrix, in order to come up with a fully adaptive detection system. Moreover, another degree of freedom, represented by the shapes of the transmitted pulses could be exploited to further optimize the performance. More generally, the impact of space-time coding in MIMO CD radar remote sensing systems to estimate the target return signal parameters is undoubtedly a topic of primary concern. Finally, the design of GD and space-time coding strategies might be of interest under the very common situation of non-Gaussian radar clutter.

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References:


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