Design of Optimal Waveforms in MIMO Radar Systems Based on the Generalized Approach to Signal Processing

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Abstract: - We consider the problem of waveform design for multiple-input multiple-output (MIMO) radar systems employing the generalized detector that is constructed based on the generalized approach to signal processing in noise. We investigate the case of an extended target and without limiting ourselves to orthogonal waveforms. Instead, we develop a procedure to design the optimal waveform that maximizes the signal-to-interference plus-noise ratio (SINR) at the generalized detector output. The optimal waveform requires a knowledge of both target and clutter statistics. We also develop several suboptimal waveforms requiring knowledge of target statistics only, clutter statistics only, or both. Thus, the transmit waveforms are adjusted based on target and clutter statistics. A model for the radar returns that incorporates the transmit waveforms is developed. The target detection problem is formulated for that model. Optimal and suboptimal algorithms are derived for designing the transmit waveforms under different assumptions regarding the statistical information available to the generalized detector. The performance of these algorithms is illustrated by computer simulation.

Key-Words: - Generalized detector, additive white Gaussian noise, detection performance, multiple-input multiple-output, signal-to-interference plus-noise ratio (SINR).

1 Introduction

Recent advances in linear amplifier and arbitrary waveform generation technology, and the ever-increasing processing power, have spawned interest in the development of radar systems that attempt to make full use of the spatial and temporal degrees of freedom available to the radar transmitter. These technological advances make it possible to consider the design of radar systems that allow the transmitter full flexibility in selecting the transmitted waveform within the given bandwidth and power constraints on a pulse-by-pulse and antenna-by-antenna basis. The flexibility to use a multiplicity of transmitted waveforms and of adaptively adjusting these waveforms offers significant performance advantages. For example, the technique based on employment of probing signal transmission in multiple-input multiple-output (MIMO) radar system is discussed in [1]

Fundamentally, the additional degrees of freedom afforded by the ability to vary the transmit wave-form can be used to optimize a desired performance criterion. For example, the waveform can be adapted to the target signature to enhance detectability, to in-crease clutter or interference rejection, or to improve robustness to multipath. To put these works into per-spective we note that the radar design is driven by the assumed models of the target and the interferen-ce-plus-noise environment. Targets are often model-led, as a rule, as point scatterers. However, since the resolution of radar systems increases, the better mo-del is that of an extended target that is spread in ran-ge, azimuth, and Doppler. The target model can be deterministic or statistical: the former assumes that the target characteristics are fixed and known, pos-sibly up to some unknown parameters, which can be estimated, while the latter treats the target as a rand-om variable and attempts to characterize its statistics Similarly, different models can be used for the interference environment, for example, clutter, jamming, nearby targets, etc.

The work on optimum transmit-receive design in [2]–[4], for example, assumes a deterministic target model with a range spread, using a single transmit antenna, or an antenna with multiple polarization modes [5]. Optimal waveform design for single antenna radar is studied in [6]. A signal subspace framework that allowed the derivation of the optimal radar waveform for a given scenario and to evaluate the corresponding radar performance is discussed in [6]. Recently there has been considerable interest in radar systems employing multiple antennas at both the transmitter and receiver and performing space-time processing on both, commonly referred to as MIMO radar. The concept of MIMO radar allows each transmitting antenna element to transmit an arbitrary waveform. This provides extra degrees of freedom compared to the traditional transmit beamforming approach. Joint optimization of waveforms and receiving filters in the MIMO radar systems for the case of extended target in clutter is discussed in [7]. The case of the signal dependent noise (clutter) for two scenarios, namely, the first is based on assumption that different antennas see uncorrelated aspects of the target and the second is based on the correlated target with dependence between the clutter and signal is considered in [8]

This work has focused almost entirely on the point target model, and assumes transmission of orthogonal signals on the different antennas. This make it possible to separate the signals arriving from the different transmit antenna at the receiver, and to perform any transmit array processing functions on the receive side “after the fact.” For example, one can scan the transmit beam across the illuminated area within a single dwell time, or perform adaptive beamforming to reduce interference and improve resolution [9]–[15]. Note however that the coherent transmitter array gain is lost when doing the transmit beamforming after, rather than during, transmission. Employing adaptive processing it is possible to improve clutter rejection in ways that are not possible in conventional radar [16], [17]. MIMO radar can also provide angular diversity, which is useful in some scenarios [18]–[20]. MIMO radar systems with widely separated antennas provide spatial diversity by viewing the targets from different angles. A novel approach to accurately estimate properties, for instance, the position and velocity of multiple targets using such systems by employing sparse modelling is discussed in [21]. New metric to analyze the performance of the radar system and an adaptive mechanism for optimal energy allocation at the different transmit antennas are proposed in [21], also. In this case, the adaptive energy allocation mechanism significantly improves in performance over MIMO radar systems that transmit fixed equal energy across all the antennas.

In this paper, we investigate the waveform designing problem for MIMO radar systems employing the generalized detector (GD), which is constructed based on the generalized approach to signal processing in noise [22]–[27]. We study the case of an extended target, and without limiting ourselves to orthogonal waveforms. Instead, we use a procedure discussed in [28] to design the optimal waveform, which maximizes the signal-to-interference plus-noise ratio (SINR) at the GD output. The optimal waveform requires knowledge of both the target and clutter statistics. We also study a development of several suboptimal waveforms for radar systems employing the GD. This development requires a knowledge of target statistics only, clutter statistics only, or both.

In this paper, we present in some detail a model for the radar signal, which incorporates the waveforms transmitted by the antenna array elements. We derive the optimal GD for the received signals assuming, that statistics of the radar return are completely known. We study how to maximize the SINR at the GD output and derive iterative algorithms for computing the SINR maximizing the transmit waveform. We also investigate several suboptimal waveform design algorithms. Theoretical study is strengthened by simulation results illustrating the performance gains achievable by adaptive waveform design compared with conventional radar waveforms and compare obtained results with radar systems employing the well-known generalized minimum variance distortionless response detector [28].

2 Problem Formulation

We consider MIMO radar employing M antennas at the transmitter and N antennas at the receiver. Assume that the radar operates at a bandwidth B, and all the baseband signals of interest are sampled at a rate f_s greater than the Nyquist rate, i.e., $f_s > 2B$. If the radar operates using a pulse repetition rate f_{prf}, we need $N_s = \frac{f_s}{f_{prf}}$ samples to fully represent the signal over a one pulse-to-pulse interval. Thus, all of the signals of interest will be represented by vectors of length N_f .

It is more convenient to use a frequency domain representation. All the signals discussed below are the Fourier transforms of the collected samples. Let $\mathbf{s}_k$ be the $N_f \times 1$ vector representing the signal transmitted at the k-th antenna, where $k = 1,\ldots,M$. These vectors can be stacked into a single transmit vector $\mathbf{s}$ of size $MN \times 1$, i.e.,

$$\mathbf{s}_n = [\mathbf{s}_1^T,\ldots,\mathbf{s}_M^T]^T , \tag{1}$$

The radar illuminates a set of l scatterers located at coordinates (x_i, y_i, z_i) where $i = 1,\ldots,l$ which have complex amplitudes h_i. These amplitudes can be assembled into $l \times 1$ vector **h.** The radar illumination at the location of the ith scatterer is denoted by g_i. Thus, the radar signal reflected from this scatterer towards the receiver takes the form $h_i g_i$. In the vec-or form we can write that the vector **g** is the $l \times 1$ illumination vector and the reflected signal is $\mathbf{h} \otimes \mathbf{g}$, where $\otimes$ denotes the element-by-element vector or matrix multiplication.

Assume each target can be considered of so many points. A point target is assumed between each pair of transmitter-receiver antennas so that the received signal component at the ith receiver due to the kth transmitter can be considered at the instant n as

$$y_{ik}(n) = g_{ik} \times h_k(n) \quad , \tag{2}$$

where g_k is the path gain from the kth transmitter to the ith receiver. For narrow bandwidth waveforms, the point target model is often valid; see, for instance [29] and [30].

The received signal at the ith receiver is the superposition of all the signals originating from various transmitters plus the additive noise. Denote by $y_i(n)$ the received signal and by $w(n)$ the additive Gaussian noise at the ith receiver and then $y_i(n)$ by the following form

$$y_i(n) = \sum_{k=1}^{M} \mathbf{g}_{ik}\mathbf{h}_k(n) + w \quad , \tag{3}$$

where

$$\mathbf{g}_l^T = [g_{l1}, g_{l2}, \ldots, g_{lM}] \quad . \tag{4}$$

Denote by

$$\mathbf{y}_n = [y_1(n), \ldots, y_l(n)] \tag{5}$$

The collection of the received signals at the various receiving elements at the instant n, the received sig-nal vector can be described by the following model:

$$\mathbf{y}_n = \mathbf{g}\,\mathbf{h}_n + \mathbf{w}_n \quad , \tag{6}$$

where $M \times N$ matrix

$$\mathbf{g} = [\mathbf{g}_1, \mathbf{g}_2, \ldots, \mathbf{g}_N]^H \tag{7}$$

is the target scattering matrix similar to the channel matrix in [31] and [32]. Since entries of **g** are the path gains related to different aspects of the target, this matrix can be seen as the extended model of the target. It has been shown that **g** matrix entries obey to the Gaussian distribution law.

In order to estimate **g**, let the finite length signal $s_k(n)$ by the length $l \ge N$ be transmitted from each element. Due to the $N \times l$ transmitted matrix

$$\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_l] \quad , \tag{8}$$

the $N \times l$ received matrix **y** can be expanded as

$$\mathbf{y} = \mathbf{gS} + \mathbf{w} \quad , \tag{9}$$

where **w** is the noise matrix defined by

$$\mathbf{w} = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_l] \quad . \tag{10}$$

The ith row of **y** indicates the received signal throughout N samples by the lth element.

Alternatively, the reflected signal vector can be written in the form of diagonal matrix $diag\{\mathbf{h}\}\mathbf{g}$ or $diag\{\mathbf{g}\}\mathbf{h}$, $diag\{\mathbf{x}\}$ denotes a diagonal matrix whose diagonal elements are the elements of the vector **x**. The illumination vector is a linear function of the transmit signal and can be written as $\mathbf{g} = \mathbf{A}_M\mathbf{s}$, where $\mathbf{A}_M$ is a $l \times MN_f$ matrix whose elements are the gains from a particular transmit antenna to a scatterer location, at a specific frequency. Similarly, we define $\mathbf{A}_n$ as the $NN_f \times l$ matrix whose elements are the gains from a given scatterer location to a receive antenna, at a specific frequency.

The received signal is the reflected signal multiplied by this gain. Thus, the $NN_f \times 1$ vector **y** of received signals can be written in the following form:

$$\mathbf{y} = \mathbf{A}_N diag\{\mathbf{h}\}\mathbf{g} = \underbrace{\mathbf{A}_N diag\{\mathbf{h}\}\mathbf{A}_M}_{\mathbf{A}_s}\mathbf{s} = \mathbf{A}_s\mathbf{s} \tag{11}$$

or equivalently as

$$\mathbf{y} = \mathbf{A}_N diag\{\mathbf{g}\}\mathbf{h} = \underbrace{\mathbf{A}_N diag\{\mathbf{g}\}}_{\mathbf{A}_h}\mathbf{h} = \mathbf{A}_h\mathbf{s} \tag{12}$$

These equations simply state that the received radar signal is a bilinear function of the transmitted signal and of the scatterer reflectivity. The gain matrices $\mathbf{A}_M$ and $\mathbf{A}_N$ can be written explicitly for a given ra-dar-scatterer geometry, i.e., given the coordinates of all the transmitted and received antenna phase cent-ers and the scatterer coordinates. If the antenna ele-ments are not omnidirectional, their radiation patt-erns are also required. The detailed structure of $\mathbf{A}_M$ and $\mathbf{A}_N$ is discussed in [28] for the case where the transmitted and received arrays are collocated. The scatterer amplitudes are usually represented as a ra-ndom process. In the following, we assume that **h** is a multivariate complex Gaussian vector with zero mean and covariance $\mathbf{R}_h$, i.e., $\mathbf{h} \approx \mathcal{CN}(0, \mathbf{R}_h)$.

The covariance matrix $\mathbf{R}_h$ is assumed to have a low rank decomposition

$$\mathbf{R}_h = \mathbf{V}_h \mathbf{V}_h^H \ , \tag{13}$$

where $\mathbf{V}_h$ is a $l \times N$ matrix, N being the rank of $\mathbf{R}_h$. The covariance of the received signal is then given by

$$\mathbf{R}_x = \mathbf{A}_h \mathbf{R}_h \mathbf{A}_h^H \ . \tag{14}$$

The reflection from a target is almost always accompanied by reflections from the surrounding environment, e.g. ground, ocean, etc., referred to as the clutter. In the following discussion, we need to distinguish between the radar signal returned from the target and from clutter. We denote the target-related quantities by the superscript *t*, namely, $\mathbf{x}_t, \mathbf{h}_t$, $\mathbf{R}_{xt}, \mathbf{V}_{xt}$. The clutter-related quantities are denoted by the subscript *c*, namely, $\mathbf{x}_c, \mathbf{h}_c, \mathbf{R}_{xc}, \mathbf{V}_{xc}$. The complete signal model for the received radar signal **y** is then given by the following form

$$\mathbf{y} = \mathbf{g}_t \mathbf{S} + \mathbf{g}_c \mathbf{S} + \mathbf{w} = \mathbf{x}_t + \mathbf{x}_c + \mathbf{w} \ , \tag{15}$$

where $\mathbf{x}_t$ is the target signal, $\mathbf{x}_c$ is the clutter signal; **w** is a noise vector where $\mathbf{w} \approx \mathcal{CN}(0, \sigma_n^2 \mathbf{I})$, and **I** is the identity matrix; $\mathbf{g}_t$ and $\mathbf{g}_c$ are assumed to be the Gaussian distributed matrices with zero mean and covariance $\mathbf{R}_{xt}$ and $\mathbf{R}_{xc}$, respectively. These covariance matrices are defined by

$$\begin{cases} \mathbf{R}_{xt} = E\{\mathbf{g}_t^H \mathbf{g}_t\} , \\ \mathbf{R}_{ct} = E\{\mathbf{g}_c^H \mathbf{g}_c\} , \end{cases} \tag{16}$$

where $E\{\cdot\}$ denotes the mean.

3 Detection by GD

Given the signal model above we formulate the target detection problem as the following Gauss-Gauss binary hypothesis-testing problem:

$$\begin{cases} \mathcal{H}_0 \Rightarrow \mathbf{y} \approx \mathcal{CN}(0, \mathbf{R}_{xc} + \sigma_n^2 \mathbf{I}) ; \\ \mathcal{H}_1 \Rightarrow \mathbf{y} \approx \mathcal{CN}(0, \mathbf{R}_{xt} + \mathbf{R}_{xc} + \sigma_n^2 \mathbf{I}) , \end{cases} \tag{17}$$

where $\mathcal{CN}$ denotes the multivariate complex Gaussian distribution. The GD for the case where the statistics $(\mathbf{R}_{xt}, \mathbf{R}_{xc}, \sigma_n^2)$ are known has the following decision statistic:

$$Z = 2\mathbf{x}_t^H \mathbf{Q}\mathbf{y} - \mathbf{y}^H \mathbf{Q}\mathbf{y} + \boldsymbol{\xi}_{AF}^H \mathbf{R}_{xcn} \boldsymbol{\xi}_{AF} \ , \tag{18}$$

where the $NN_f \times 1$ vector $\boldsymbol{\xi}_{AF}$ represents the reference noise forming by the linear system at the GD front end.

For better understanding of (18), there is a need to recall the main GD functioning principles discussed in [22], [24]. There are two linear systems at the GD front end that can be presented as low-pass filters, namely, the preliminary filter (PF) with the impulse response $h_{PF}(\tau)$ and the additional filter (AF) with the impulse response $h_{AF}(\tau)$. For simplicity of analysis, we consider that these filters have the same amplitude-frequency responses or transfer functions and bandwidths by value. Moreover, a resonant frequency of the AF is detuned relative to a resonant frequency of PF on such a value that the incoming signal cannot pass through the AF. Thus, the received signal and noise can be appeared at the PF output and ***the only noise*** is appeared at the AF output.

It is well known fact that if a value of detuning between the AF and PF resonant frequencies is more than $4 \div 5\Delta f_a$, where Δf_a is the signal bandwidth, the processes forming at the AF and PF outputs can be considered as independent and uncorrelated processes [22]–[24]. In practice, the coefficient of correlation is not more than 0.05. In the case of signal absence in the input process, the statistical parameters at the AF and PF outputs will be the same, because the same noise $n(t)$ comes in at the AF and PF inputs. We may think that the AF and PF do not change the statistical parameters of input process, since they are the linear GD front-end systems. By this reason, the AF can be considered as a generator of reference sample with a priori information ***a "no" signal is obtained*** in the additional reference noise forming at the AF output.

There is a need to make some comments regarding the noise forming at the PF and AF outputs. If the mentioned above Gaussian noise comes in at the AF and PF inputs, the GD linear system front end, the noise forming at the AF and PF outputs is Gaussian, too, because AF and PF are the linear systems and, in a general case, the noise takes the following form:

$$\begin{cases} \xi_{PF}(t) = \int\limits_{-\infty}^{\infty} h_{PF}(\tau) w(t-\tau) d\tau \ ; \\ \xi_{AF}(t) = \int\limits_{-\infty}^{\infty} h_{AF}(\tau) w(t-\tau) d\tau \, . \end{cases} \quad (19)$$

If, for example, the additive white Gaussian noise (AWGN) with zero mean and two-sided power spectral density $0.5N_0$ is coming in at the AF and PF inputs (the GD linear system front end), then the noise forming at the AF and PF outputs is Gaussian with zero mean and variance given by [24]

$$\sigma_n^2 = \frac{N_0 \omega_0^2}{8\Delta_F}\,, \quad (20)$$

where, in the case if the AF (or PF) is the RLC oscillatory circuit, then the AF (or PF) bandwidth Δ_F and resonance frequency ω_0 are defined in the following manner

$$\Delta_F = \pi\beta, \omega_0 = 1/\sqrt{LC}, \quad \text{where} \quad \beta = R/(2L)\,. \quad (21)$$

Now, return to (18), where

$$\mathbf{Q} = \mathbf{R}_{xcn}^{-1} - [\mathbf{V}_{xt}\mathbf{V}_{xt}^T + \mathbf{R}_{xcn}]^{-1}\,. \quad (22)$$

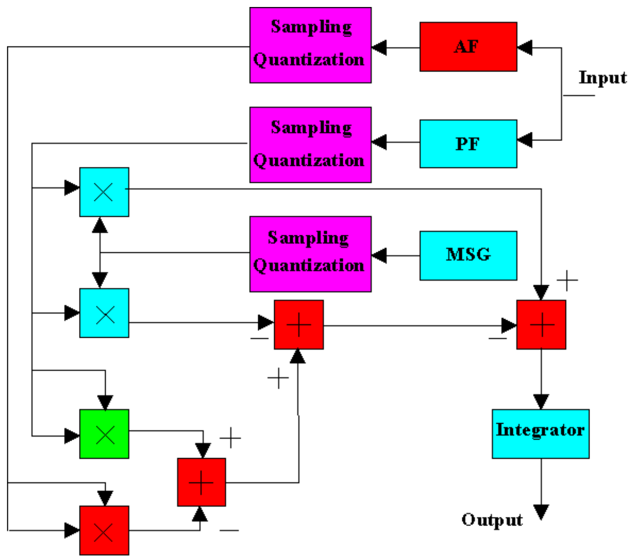

Fig.1 Block diagram of general technical interpretation of the GD.

Here

$$\mathbf{R}_{xcn} = \mathbf{R}_{xc} + \sigma_n^2 \mathbf{I} \quad (23)$$

is the clutter-plus-noise covariance matrix. This can be equivalently written in the following form:

$$\mathbf{Q} = \mathbf{R}_{xcn}^{-1}\mathbf{V}_{xt}[\mathbf{V}_{xt}^H\mathbf{R}_{xcn}^{-1}\mathbf{V}_{xt} + \mathbf{I}]^{-1}\mathbf{V}_{xt}^H\mathbf{R}_{xcn}^{-1} = \mathbf{W}\mathbf{W}^H\,, \quad (24)$$

where

$$\mathbf{W} = \mathbf{R}_{xcn}^{-1}\underbrace{\mathbf{V}_{xt}[\mathbf{V}_{xt}^H\mathbf{R}_{zcn}^{-1}\mathbf{V}_{xt} + \mathbf{I}]^{-0.5}}_{\tilde{\mathbf{V}}_{xt}}\,. \quad (25)$$

Then the detection statistic of GD given by (14) can be rewritten in the following form

$$\begin{aligned} Z &= 2\mathbf{x}_t^H\mathbf{W}\mathbf{W}^H\mathbf{y} - \mathbf{y}^H\mathbf{W}\mathbf{W}^H\mathbf{y} + \boldsymbol{\xi}_{AF}^H\mathbf{R}_{xcn}^{-1}\boldsymbol{\xi}_{AF} \\ &= 2\mathbf{x}_t^H\mathbf{W}\mathbf{W}^H\mathbf{y} - \|\mathbf{W}^H\mathbf{y}\|^2 + \boldsymbol{\xi}_{AF}^H\mathbf{R}_{xcn}^{-1}\boldsymbol{\xi}_{AF}\,. \end{aligned} \quad (26)$$

In the case of a unit rank target covariance matrix $\mathbf{V}_{xt}$ is a vector. We denote it by the lower case symbol $\mathbf{v}_{xt}$ to emphasize this fact. In this case,

$$\mathbf{W} = \mathbf{R}_{xcn}^{-1}\mathbf{v}_{xt}\,, \quad (27)$$

where we discarded a scalar multiplier term that can be absorbed into the detection threshold. We may say that in this case the GD is constructed using the principles of the well-known minimum variance distortionless response (MVDR) detector [33].

The more general case where the rank is greater than unity is the generalized MVDR (GMVDR) detector discussed in [34]. The case of the unit rank $\mathbf{R}_{xt}$ corresponds to a rigid target with fixed orientation relative to the radar. Motion relative to the radar will introduce the same random phase fluctuations to the target return signals from all of the scatterers comprising the target, so that the target scatters can be represented by $\mathbf{h}_t \exp(j\varphi)$, where $\mathbf{h}_t$ is a deterministic vector and φ is a random phase. In this case,

$$\mathbf{R}_{ht} = \mathbf{v}_t \mathbf{v}_t^H , \tag{28}$$

where $\mathbf{v}_t$ equals to $\mathbf{h}_t$ up to an arbitrary unit magnitude complex scalar. A target that is not a rigid collection of point scatterers, or is one that is rotating relative to the radar will induce different random variations along the vector $\mathbf{h}_t$ and the corresponding covariance matrix $\mathbf{R}_{ht}$ defined above will have an effective rank $n > 1$.

4 SINR Analysis

Because both the target and clutter statistics ($\mathbf{R}_{xt}$, $\mathbf{R}_{xc}$) depend on the transmit signal **s**, the matrix **W** defining the GD structure is a function of the transmit waveform **s**. We want to select this waveform so, as to maximize the performance of the GD detector. This can be achieved by maximizing the signal-to-interference-plus-noise ratio (*SINR*) at the GD output. It follows from (14) that the *SINR* is given by

$$SINR = \frac{\mathrm{tr}\{\mathbf{W}^H \mathbf{R}_{xt} \mathbf{W}\}}{\mathrm{tr}\{\boldsymbol{\xi}_{AF}^H \mathbf{R}_{xcn}^{-1} \boldsymbol{\xi}_{AF} - \boldsymbol{\xi}_{PF}^H \mathbf{R}_{xcn}^{-1} \boldsymbol{\xi}_{PF}\}} \tag{29}$$

or equivalently

$$SINR = \frac{\mathrm{tr}\{\widetilde{\mathbf{V}}_{xt}^H \mathbf{R}_{xcn}^{-1} \mathbf{R}_{xt} \mathbf{R}_{xcn}^{-1} \widetilde{\mathbf{V}}_{xt}\}}{\mathrm{tr}\{\boldsymbol{\xi}_{AF}^H \mathbf{R}_{xcn}^{-1} \boldsymbol{\xi}_{AF} - \boldsymbol{\xi}_{PF}^H \mathbf{R}_{xcn}^{-1} \boldsymbol{\xi}_{PF}\}} , \tag{30}$$

where $\mathrm{tr}\{\cdot\}$ denotes the trace operation of matrix. This *SINR* is a nonlinear function of the elements of **s** and, in general, we must resort to numerical optimization techniques to solve for the optimal transmit waveform **s**.

To gain some insight into this problem we consider next the case of a unit rank target covariance matrix. In the unit rank case, the matrix **W** given by (27) is a vector and the trace operation in (29) is not needed. We can then rewrite (29) in the following form

$$SINR = \frac{\mathbf{v}_{xt}^H \mathbf{R}_{xcn}^{-1} (\mathbf{v}_{xt} \mathbf{v}_{xt}^H) \mathbf{R}_{xcn}^{-1} \mathbf{v}_{xt}}{\boldsymbol{\xi}_{AF}^H \mathbf{R}_{xcn}^{-1} \boldsymbol{\xi}_{AF} - \boldsymbol{\xi}_{PF}^H \mathbf{R}_{xcn}^{-1} \boldsymbol{\xi}_{PF}} . \tag{31}$$

Because the numerator is the squared value of the $\mathbf{v}_{xt}^H \mathbf{R}_{xcn}^{-1} \mathbf{v}_{xt}$ we can write (18) in the following form:

$$SINR = \frac{\{\mathbf{v}_{xt}^H [\mathbf{V}_{xc} \mathbf{V}_{xc}^H + \sigma_n^2 \mathbf{I}]^{-1} \mathbf{v}_{xt}\}^2}{\boldsymbol{\xi}_{AF}^H \mathbf{R}_{xcn}^{-1} \boldsymbol{\xi}_{AF} - \boldsymbol{\xi}_{PF}^H \mathbf{R}_{xcn}^{-1} \boldsymbol{\xi}_{PF}} . \tag{32}$$

This can also be written as

$$SINR = \frac{(\mathbf{v}_{xt}^H)^2 \left\{ \mathbf{I} - \frac{1}{\sigma_n^2} \mathbf{V}_{xc} \left[\frac{1}{\sigma_n^2} \mathbf{V}_{xc}^H \mathbf{V}_{xc} + \mathbf{I} \right]^{-1} \mathbf{V}_{xc}^H \right\} (\mathbf{v}_{xt})^2}{\sigma_n^2 (\boldsymbol{\xi}_{AF}^H \boldsymbol{\xi}_{AF} - \boldsymbol{\xi}_{PF}^H \boldsymbol{\xi}_{PF})} . \tag{33}$$

If the clutter is much stronger than the noise, we can write this as

$$SINR = \frac{(\mathbf{v}_{xt}^H)^2 \mathbf{P}_{V_{xc}}^{\perp} (\mathbf{v}_{xt})^2}{4\sigma_n^4} , \tag{34}$$

where

$$\mathbf{P}_{V_{xc}}^{\perp} = \mathbf{I} - \mathbf{V}_{xc} (\mathbf{V}_{xc}^H \mathbf{V}_{xc})^{-1} \mathbf{V}_{xc}^H \tag{35}$$

is the orthogonal projection operator onto the column space of $\mathbf{V}_{xc}$.

The *SINR* expression above has the intuitive interpretation of being the energy of the target component, which is orthogonal to the clutter, i.e., what is left after completely removing the clutter component from the target, divided by the noise energy. In the following discussion, we consider only the unit-rank target covariance. We note, however, that all of our results can be extended to the general low rank case. An optimal transmit waveform **s** can be derived by maximizing the *SINR* over all possible choices of **s**. This optimization can be carried out using, for example, the numerical gradient descent algorithm. Consider the main statements of the numerical gradient descent algorithm.

5 Gradient Descent Algorithm

To solve the *SINR* maximization problem we need a more explicit expression of the *SINR* in terms of **s**. Recall that $\mathbf{x}_{st} = \mathbf{A}_{st}\mathbf{s}$, where

$$\mathbf{A}_{st} = \mathbf{A}_N diag\{\mathbf{h}_t\}\mathbf{A}_M \quad . \tag{36}$$

In the unit-rank case $\mathbf{h}_t$ is deterministic up to a random scalar phase term, so $\mathbf{h}_t$ can be replaced by $\mathbf{v}_t$, where

$$\mathbf{R}_{ht} = \mathbf{v}_t\mathbf{v}_t^H \, . \tag{37}$$

In other words, for a unit-rank target covariance we have

$$\mathbf{A}_{st} = \mathbf{A}_N diag\{\mathbf{v}_t\}\mathbf{A}_M \, . \tag{38}$$

Recall that

$$\mathbf{R}_{xc} = \sum_{i=1}^{r} \mathbf{A}_{xc}[i]\mathbf{s}\mathbf{s}^H \mathbf{A}_{sc}^H[i] \quad . \tag{39}$$

Inserting (39) into the *SINR* expression we obtain

$$SINR = \frac{\mathbf{v}_{xt}^H \mathbf{R}_{xcn}^{-1}\mathbf{v}_{xt}}{\boldsymbol{\xi}_{AF}^H\boldsymbol{\xi}_{AF} - \boldsymbol{\xi}_{PF}^H\boldsymbol{\xi}_{PF}} = \frac{\mathbf{s}^H \mathbf{A}_{st}^H \mathbf{R}_{xcn}^{-1}\mathbf{A}_{st}\mathbf{s}}{\boldsymbol{\xi}_{AF}^H\boldsymbol{\xi}_{AF} - \boldsymbol{\xi}_{PF}^H\boldsymbol{\xi}_{PF}} \; ; \tag{40}$$

$$\mathbf{R}_{xcn} = \sum_{i=1}^{r} \mathbf{A}_{sc}[i]\mathbf{s}\mathbf{s}^H \mathbf{A}_{sc}^H[i] + 4\sigma_n^4\mathbf{I} \quad . \tag{41}$$

Various numerical optimization techniques can be used to solve for the transmit waveform **s**, which maximizes the *SINR*. Here we consider the gradient descent method [35] that requires knowledge of the derivatives of *SINR* with respect to the elements of **s**. The update equation of the gradient descent method is given by

$$\mathbf{s} \leftarrow \mathbf{s} + \mu\left[\frac{\partial SINR}{\partial \mathbf{s}}\right]^H \quad , \tag{42}$$

where μ is a constant controlling the convergence rate of the algorithm. Taking the derivative of the *SINR* with respect to s_i we obtain

$$\frac{\partial SINR}{\partial s_i} = \frac{1}{\boldsymbol{\xi}_{AF}^H\boldsymbol{\xi}_{AF} - \boldsymbol{\xi}_{PF}^H\boldsymbol{\xi}_{PF}} \times\left[2\mathbf{s}^H\mathbf{A}_{st}^H\mathbf{R}_{xcn}^{-1}\mathbf{A}_{st}\mathbf{e}_i - 2\mathbf{s}^H\mathbf{A}_{st}^H\mathbf{R}_{xcn}^{-1}\frac{\partial \mathbf{R}_{xcn}}{\partial s_i}\mathbf{R}_{xcn}^{-1}\mathbf{A}_{st}\mathbf{s}\right], \tag{43}$$

where s_i is the i-th entry of **s** and $\mathbf{e}_i$ is a vector of zeros with a 1 at the i-th position. Examples of calculation of this type of derivatives are presented in [36] and [37].

Define $\mathbf{R}_{xc}$ in the following form:

$$\mathbf{R}_{xc} = \sum_{i=1}^{r} \mathbf{A}_h\mathbf{v}_c[i]\mathbf{v}_c^H[i]\mathbf{A}_h^H \quad . \tag{44}$$

Note that

$$\frac{\partial \mathbf{R}_{xcn}}{\partial s_i} = \frac{\partial \mathbf{R}_{xc}}{\partial s_i} \quad . \tag{45}$$

In this case, we obtain

$$\frac{\partial \mathbf{R}_{xcn}}{\partial s_i} = \sum_{n=1}^{r} \mathbf{A}_{sc}[n]\mathbf{e}_i\mathbf{s}^H\mathbf{A}_{sc}^H[n] \quad . \tag{46}$$

Then (43) can be represented in the following form

$$\frac{\partial SINR}{\partial s_i} = \frac{2p_i - 2q_i}{\boldsymbol{\xi}_{AF}^H\boldsymbol{\xi}_{AF} - \boldsymbol{\xi}_{PF}^H\boldsymbol{\xi}_{PF}} \quad , \tag{47}$$

where p_i are the elements of the row vector

$$\mathbf{p} = \mathbf{s}^H\mathbf{A}_{st}^H\mathbf{R}_{xcn}^{-1}\mathbf{A}_{st} \tag{48}$$

and

$$\mathbf{q}_i = \mathbf{s}^H\mathbf{A}_{st}^H\mathbf{R}_{xcn}^{-1}\left[\sum_{n=1}^{r}\mathbf{A}_{sc}[n]\mathbf{e}_i\mathbf{s}^H\mathbf{A}_{sc}^H[n]\right]\mathbf{R}_{xcn}^{-1}\mathbf{A}_{st}\mathbf{s} \quad . \tag{49}$$

Finally, collecting the derivatives into a row vector we obtain

$$\frac{\partial SINR}{\partial \mathbf{s}} = \frac{2(\mathbf{p} - \mathbf{q})}{\boldsymbol{\xi}_{AF}^H\boldsymbol{\xi}_{AF} - \boldsymbol{\xi}_{PF}^H\boldsymbol{\xi}_{PF}} \quad . \tag{50}$$

Equations (42), (48)–(50) define the gradient desc-ent algorithm for computing the *SINR*-maximizing waveform **s**. To initialize the algorithm we let **s** be one of the suboptimal waveforms described in Secti-on 6.

5.1 Iterative Optimization

A second optimization technique involves iterating between updating *SINR* in (36) assuming **s** and, consequently $\mathbf{R}_{xcn}$, are fixed, and then updating $\mathbf{R}_{xcn}$.

More specifically, this algorithm proceeds as follows:

- Maximize

$$SINR = \frac{1}{\xi_{AF}^H \xi_{AF} - \xi_{PF}^H \xi_{PF}} \times \underbrace{\mathbf{s}^H \mathbf{A}_{st}^H \mathbf{R}_{xcn}^{-1} \mathbf{A}_{st} \mathbf{s}}_{\mathbf{Q}_1} , \quad (51)$$

  assuming $\mathbf{R}_{xcn}$ is fixed. This is accomplished by letting $\mathbf{s}$ be the eigenvector corresponding to the largest eigenvalue of $\mathbf{Q}_1$;
- Use the resulting $\mathbf{s}$ to update $\mathbf{R}_{xcn}$ via (33).
- Repeat until convergence. To initialize the algorithm we let $\mathbf{s}$ be one of the suboptimal waveforms described in Section 6.

6 Suboptimal Waveforms

The optimal waveform requires knowledge of the target and clutter statistics, namely, $\mathbf{R}_{ht}$ and $\mathbf{R}_{hc}$. The waveforms considered here provide suboptimal performance, but, with one exception, require less information about the target and clutter than the optimal waveform. The transmit signal will be normalized to unit energy in all the different waveforms we consider to allow for a fair comparison of the results. In other words, the transmitted signal vector $\mathbf{s}$ must obey the constraint $\| \mathbf{s} \|^2 = 1$.

6.1 Standard Waveforms

As a reference waveform against which to compare the performance achieved using optimal and suboptimal waveforms, we selected a linear frequency-modulated (FM) waveform (chirp), which is commonly used in radar systems, fed to a single transmit antenna. Without loss of a generality, we can assume that the first antenna is active while the other antennas has zero input. Thus, we can write the standard transmit waveform $\mathbf{s}$ in the following form:

$$\mathbf{s} = \frac{1}{\sqrt{N_f}} \left[f(1), 0, \ldots, 0, f(N_f), 0, \ldots, 0 \right]^T , \quad (52)$$

where $f(\cdots)$ are the values of the discrete Fourier transform of the signal. It is possible, of course, to select other reference waveforms.

For example, we can use a linear FM waveform fed to all antennas with an appropriate set of complex weights so, as to form a transmit beam in a desired direction. However, the resulting beam will illuminate only a portion of an extended target and the results will be highly dependent on the orientation of the beam relative to the target scatterers. We prefer instead to use for reference a system employing a single transmit antenna, so that the entire area of interest is uniformly illuminated.

6.2 Target-Based Waveforms

Consider the case, where we know the target statistics but not the clutter statistics. This case corresponds, for instance, to the situation where the radar is searching for specific targets with known signatures. The target signature $\mathbf{v}_t$ represents the azimuth-frequency target scattering function. By designing $\mathbf{s}$, we can control the azimuth-frequency radar illumination of the target.

In the following, we consider two ways to design the illuminating function. In the first, we focus all of the radar energy to illuminate the azimuth-frequency element of the target, i.e., the component of $\mathbf{v}_t$, which has the largest magnitude. The second approach is to match the illuminating function to the target scattering function. In other words, put more of the radar energy, where the scattering is strong, and less, where it is weak. The second approach will produce less target energy at the receiver than the first approach. In the case, where the clutter energy is uniformly distributed in azimuth-frequency, the first approach is expected to provide the better performance. However, given a non-uniform clutter distribution the second approach might be preferred.

Consider, for example, the case, where the azimuth frequency cell, where the target scattering has its largest magnitude, happens to also have a strong clutter component. In this case, the better performance will be obtained by illuminating a part of the target scattering function, where the clutter is weak. Since we assume here that the clutter distribution is unknown, the second approach will be more robust than the first. Both of these approaches are made more precise next.

6.2.1 Maximizing Target Energy

Consider designing of the transmitted waveform $\mathbf{s}$ so that the received target energy is maximized. In other words, we want to maximize $\mathbf{W}^H \mathbf{R}_{xt} \mathbf{W}$. In the case, where

$$\mathbf{R}_{ht} = \mathbf{v}_t \mathbf{v}_t^H \quad (53)$$

is the unit rank, we have

$$\mathbf{W}^H\mathbf{R}_{xt}\mathbf{W} = \mathbf{W}^H(\mathbf{A}_h\mathbf{v}_t\mathbf{v}_t^H\mathbf{A}_h^H)\mathbf{W} = \|\mathbf{W}^H\mathbf{A}_h\mathbf{v}_t\|^2 \quad (54)$$

or

$$\mathbf{W}^H\mathbf{R}_{xt}\mathbf{W} = \mathbf{W}^H(\mathbf{A}_{st}\mathbf{s}\mathbf{s}^H\mathbf{A}_{st}^H)\mathbf{W} = \|\mathbf{W}^H\mathbf{A}_{st}\mathbf{s}\|^2 \ , \quad (55)$$

where

$$\mathbf{A}_{st} = \mathbf{A}_N diag\{\mathbf{v}_t\}\mathbf{A}_M \ . \quad (56)$$

Clearly $\mathbf{W}^H\mathbf{R}_{xt}\mathbf{W}$ will be maximized if **W** and **s** are chosen to be left and right eigenvectors of $\mathbf{A}_{st}$ corresponding to its largest eigenvalue. Without loss of generality, we assume that **W** is normalized to have unit norm, so as to keep the noise variance at the GD output constant. We can interpret this result as follows. The vector $\mathbf{v}_t$ represents the azimuth-frequency distribution of the target scattering function. The maximizing solution focuses the transmit energy on the azimuth-frequency portion of the target, which provides the strongest radar return.

6.2.2 Matching the Target Distribution

Consider the illumination vector $\mathbf{g} = \mathbf{A}_M\mathbf{s}$, which was introduced earlier. We now design **s**, so as to have the illumination function match the target scattering function, that is to have $\mathbf{g} = \mathbf{v}_t$ .In other words, let $\mathbf{A}_M\mathbf{s} = \mathbf{v}_t$ .The target matching transmit waveform **s** is the least-squares solution of this equation,

$$\mathbf{s} = (\mathbf{A}_M^H\mathbf{A}_M)^{-1}\mathbf{A}_M^H\mathbf{v}_t \ . \quad (57)$$

6.3 Minimizing Clutter Energy

Now, consider the case where we know the clutter statistics but not the target statistics. This will be the situation if we estimate the clutter covariance from measurement assumed to be target free, but do not have a priori knowledge about the target characteristics. Consider designing **s** so that the received clutter energy is minimized, i.e., we want to minimize $\mathbf{W}^H\mathbf{R}_{xc}\mathbf{W}$. Recall that

$$\mathbf{R}_{xc} = \mathbf{A}_h\mathbf{R}_{hc}\mathbf{A}_h^H \ , \quad (58)$$

where

$$\mathbf{R}_{hc} = \mathbf{V}_c\mathbf{V}_c^H \ . \quad (59)$$

Let

$$\mathbf{V}_c = \{\mathbf{v}_c[1], \ldots, \mathbf{v}_c[r]\} \ , \quad (60)$$

where r is the rank of the clutter covariance matrix, and $\mathbf{v}_c[n]$ is the column of $\mathbf{V}_c$. Note that

$$\mathbf{A}_h\mathbf{v}_c[i] = \mathbf{A}_{sc}[i]\mathbf{s} \ , \quad (61)$$

where

$$\mathbf{A}_{sc}[i] = \mathbf{A}_N diag\{\mathbf{v}_c[i]\}\mathbf{A}_M \ . \quad (62)$$

Referring to (44) we obtain

$$\mathbf{R}_{xc} = \sum_{i=1}^{r}\mathbf{A}_{sc}[i]\mathbf{s}\mathbf{s}^H\mathbf{A}_{sc}^H[i] \ . \quad (63)$$

We can write

$$\mathbf{W}^H\mathbf{R}_{xc}\mathbf{W} = \mathbf{W}^H\underbrace{\Big(\sum_{i=1}^{r}\mathbf{A}_{sc}[i]\mathbf{s}\mathbf{s}^H\mathbf{A}_{sc}^H[i]\Big)}_{\mathbf{Q}_{sc}}\mathbf{W} \quad (64)$$

or

$$\mathbf{W}^H\mathbf{R}_{xc}\mathbf{W} = \mathbf{s}^H\underbrace{\Big(\sum_{i=1}^{r}\mathbf{A}_{sc}[i]\mathbf{W}\mathbf{W}^H\mathbf{A}_{sc}^H[i]\Big)}_{\mathbf{Q}_{wc}}\mathbf{s} \ . \quad (65)$$

We want to minimize $\mathbf{W}^H\mathbf{R}_{xc}\mathbf{W}$ jointly over **s** and **W** subject to the norm constraints

$$\|\mathbf{s}\|^2 = 1 \quad \text{and} \quad \|\mathbf{W}\|^2 = 1 \ . \quad (66)$$

Note that if **W** is known then **s**, which minimizes the received clutter power, is the eigenvector of $\mathbf{Q}_{wc}$ corresponding to its smallest eigenvalue. If **s** is known then **W**, which minimizes the received clutter power, is the eigenvector of $\mathbf{Q}_{sc}$ corresponding to its smallest eigenvalue. This suggests an iterative solution procedure where we solve successively for **W** and **s**. More specifically, the following procedure is used:

- Use an initial waveform **s**, for example, the linear FM signal.
- Compute $\mathbf{Q}_{sc}$ corresponding to **s**.
- Compute $\mathbf{Q}_{sc}$ corresponding to **W**.
- Let **W** be eigenvector of $\mathbf{Q}_{sc}$ corresponding to its smallest eigenvalue.
- Let **s** be eigenvector of $\mathbf{Q}_{wc}$ corresponding to its smallest eigenvalue.

- Repeat previous steps until convergence.

6.3.1 Maximize Signal-to-Interference Ratio

Finally, we consider an approach where the target-to-clutter ratio or signal-to-interference ratio (*SIR*) at the GD output is maximized rather than the *SINR*. As in the case of the optimal waveform design, both the clutter and the target statistics are known. The *SIR* is defined as

$$SIR = \frac{\mathbf{W}^H \mathbf{R}_{xt} \mathbf{W}}{\mathbf{W}^H \mathbf{R}_{xc} \mathbf{W}} \quad . \tag{67}$$

Recall that the clutter energy can be alternatively written as $\mathbf{W}^H \mathbf{Q}_{xc} \mathbf{W}$ or $\mathbf{s}^H \mathbf{Q}_{xc} \mathbf{s}$. Similarly, the target energy can be written as $\mathbf{W}^H \mathbf{Q}_{st} \mathbf{W}$ or $\mathbf{s}^H \mathbf{Q}_{wt} \mathbf{s}$,where

$$\mathbf{Q}_{st} = \mathbf{A}_{st} \mathbf{s}\mathbf{s}^H \mathbf{A}_{st}^H \tag{68}$$

and

$$\mathbf{Q}_{wt} = \mathbf{A}_{st} \mathbf{W}\mathbf{W}^H \mathbf{A}_{st}^H \, . \tag{69}$$

Thus, the *SIR* can be written as

$$SIR = \frac{\mathbf{W}^H \mathbf{Q}_{st} \mathbf{W}}{\mathbf{W}^H \mathbf{Q}_{sc} \mathbf{W}} \tag{70}$$

or

$$SIR = \frac{\mathbf{s}^H \mathbf{Q}_{wt} \mathbf{s}}{\mathbf{s}^H \mathbf{Q}_{wc} \mathbf{s}} \quad . \tag{71}$$

We want to maximize *SIR* jointly over all norm-constrained choices of **W** and **s**. Note that if **s** is assumed known the value of **W**, which maximizes the *SIR*, is generalized eigenvector of $\{\mathbf{Q}_{st}, \mathbf{Q}_{sc}\}$ corresponding to the largest generalized eigenvalue. Similarly, if **W** is known, the value of **s**, which maximizes *SIR*, is the generalized eigenvector of $\{\mathbf{Q}_{wt}, \mathbf{Q}_{wc}\}$ corresponding to the largest generalized eigenvalue. This suggests the following iterative solution procedure:

- Use an initial waveform **s**, for example, the linear FM signal.
- Compute $\mathbf{Q}_{st}$ and $\mathbf{Q}_{sc}$ corresponding to **s**.
- Let **W** be the generalized eigenvector of $\{\mathbf{Q}_{st}, \mathbf{Q}_{sc}\}$ corresponding to its smallest eigenvalue.
- Compute $\mathbf{Q}_{wt}$ and $\mathbf{Q}_{wc}$ corresponding to **W**.
- Let **s** be the generalized eigenvector of $\{\mathbf{Q}_{wt}, \mathbf{Q}_{wc}\}$ corresponding to its smallest eigenvalue.
- Repeat previous steps until convergence.

Note that the only difference between the *SIR* and the *SINR* is that the denominator of the *SINR* expression has an additional noise term $4\sigma_n^4 \mathbf{W}^H \mathbf{W}$. As the clutter-to-noise ratio increases, the *SIR* approaches the *SINR*. Therefore, we expect that the performance of the *SIR* maximizing the waveform will be close to that of the optimal waveform for large values of the clutter-to-noise ratio.

7 Simulation

In this section, we present some numerical examples illustrating the performance of the radar, when using the waveforms described earlier. It should be emphasized that these examples are presented only to understand the problem. To evaluate the actual performance tradeoffs of different waveform design methods requires studying them in the context of a specific radar system and a well-defined surveillance scenario, and requires addressing other issues, which are beyond the scope of the present paper.

7.1 Simulation Conditions

The clutter and target are modeled by a collection of scatterers placed on a grid in the azimuth/range plane. The grid consists of $N_a \times N_r$ points, where N_a is the number of azimuth sample points and N_r is the number of range sample points. Different clutter and target models correspond to different distributions of the complex scatterer amplitudes over this grid. For the examples presented here both target and clutter were represented by independent random scatterers whose amplitudes are complex Gaussian with zero mean and variance equal to the signal-to-noise ratio (*SNR*) and the clutter-to-noise ratio (*CNR*), respectively.

In other words, we did not assume any particular structure for the target and clutter. We note however that it is straightforward to incorporate any desired clutter and target distribution into this type of simulation. The clutter scatterers were placed at all $N_a \times N_r$ grid points, while the target scatters were placed on a rectangular portion of the grid. The surveillance scenario is assumed stationary, i.e., the target and clutter scatters are not moving and neither is the radar. Therefore, both the target and clutter covariance matrices are assumed to have unit rank so that

$$\mathbf{R}_{ht} = \mathbf{v}_t \mathbf{v}_t^H \tag{72}$$

and

$$\mathbf{R}_{hc} = \mathbf{v}_c \mathbf{v}_c^H \, . \tag{73}$$

Given the clutter and target statistics we can now proceed to compute the optimal and suboptimal waveforms $\mathbf{s}$ as described earlier. The optimal waveform was computed using the gradient descent algorithm discussed in [35]. For each waveform $\mathbf{s}$, we then compute the *SINR* at the GD output. The procedure above provides the GD *SINR* for a single randomly chosen scenario with a particular clutter and target. To get a more representative assessment of performance we repeat this procedure for many randomly selected scenarios and collect the probability density function (pdf) and cumulative density function (cdf) of the *SINR* at the GD output. Comparison of cdfs corresponding to the different waveform design methods provides valuable insight into their relative performance. A performance comparison of the GD and GMVDR detector [34] employed by radar system is presented.

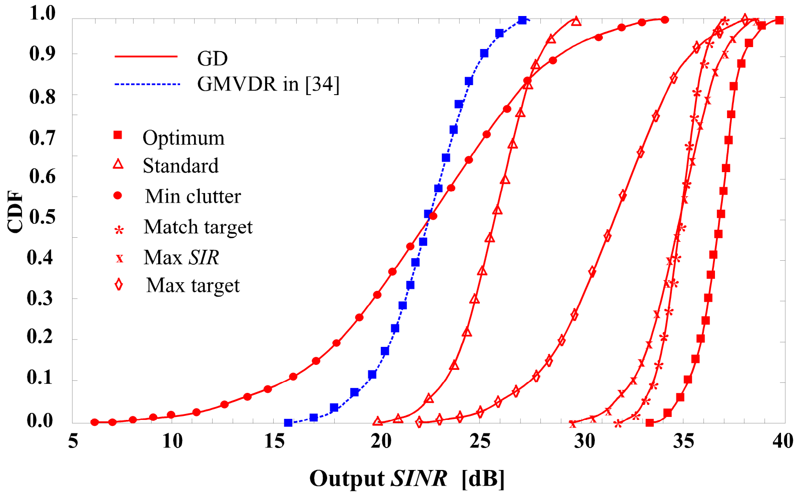

Fig.2 CDF of *SINR* at the GD output for different transmit waveforms. Comparison with GMVDR detector. Case 1.

7.2 Discussion of Results

Next, we present results for three cases differing by the size of the target relative to footprint of the radar. We present the cdf of the *SINR* at the outputs of GD and GMVDR detector discussed in [34] for the following transmit waveforms: standard, target maximizing, matched to target, clutter minimizing, *SIR* maximizing, and the optimum computed using the gradient descent method [35]. The radar is assumed to employ a 10 element uniformly spaced linear array.

7.2.1 Target is Small Compared with Radar Footprint

The results for this case are shown in Fig. 2. Investigation of Fig.2 leads us to the following observations. The optimum waveform offers the best performance and provides a 10 dB advantage over the standard waveform. We carry out a comparison at the level of 0.9 for *SINR* cdf. This gain may be interpreted as a combination of an 8 dB transmit array gain, and a 2 dB temporal gain. The first is the gain of the transmit array due to the fact, that the transmit power is concentrated almost entirely on the part of the azimuth-frequency plane occupied by the target, whereas in the standard case the same power is spread evenly over the entire plane. The second is the gain due to the fact, that the transmit energy is concentrated at those frequencies where the target has the strongest scattering. In other words, we have gain due to both spatial and temporal effects. Note, however, that conventional beamforming would capture the 8 dB array gain. The cdfs of the *SINR* for the matched-to-target and maximum *SIR* waveform are almost identical, and are approximately 2 dB for the matched-to-target and 1 dB for the maximum *SIR* from the optimum. Both of these capture most of the performance gain of the optimal waveform in this case. The target maximizing waveform is next in order of performance. Note that its *SINR* has a larger variance than the *SINR*s of the previously mentioned waveforms. This is due to the variability for clutter at the azimuth-frequency cells where the target energy is largest. The clutter minimizing waveform performs poorly because it often focuses the energy where the target energy is low or nonexistent. In other words, illuminating area of low clutter may cause us to miss the target. Superiority of the GD *SINR* cdf over GMVDR one in the case of optimum waveform is for about 12 dB.

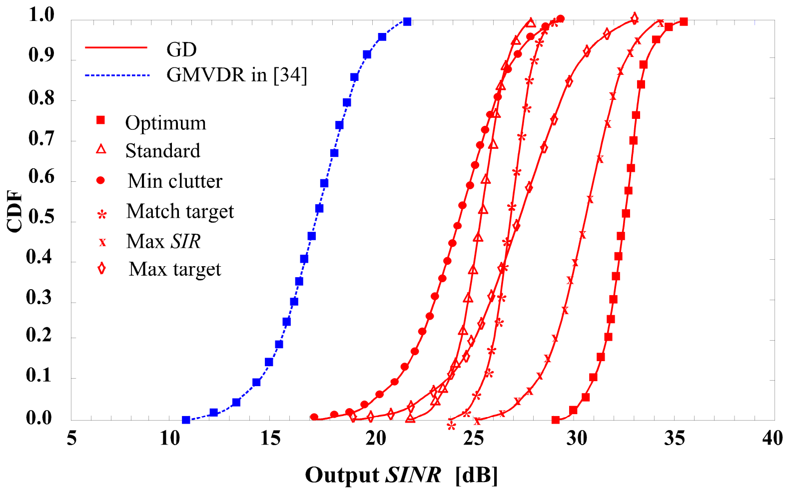

Fig.3 CDF of *SINR* at the GD output for different transmit waveforms. Comparison with GMVDR detector. Case 2.

7.2.2 Target and Radar Footprint Have the Same Size

The results for this case are depicted in Fig.3. Examination of Fig. 3 shows that the optimum waveform offers a 7 dB advantage over the standard waveform. Note that the target-matched waveform offers 1 dB improvement only. These waveforms spread the transmit power over the entire azimuth-frequency plane so they do not benefit from the transmit array gain. The target-to-clutter maximizing waveform gives close to optimal performance. As before, the clutter-based waveforms perform poorly. Superiority of the GD *SINR* cdf over GMVDR one in the case of optimum waveform is for about 14 dB.

7.2.3 Target Size is Half of the Radar Footprint

The results for this case are depicted in Fig.4. As expected the results fall in between those of the two cases discussed above. Superiority of the GD *SINR* cdf over GMVDR one in the case of optimum waveform is for about 13 dB.

8 Conclusions

Techniques for designing transmit waveforms for MIMO radar systems employing the GD were presented. We have shown that by controlling the space-time (or azimuth-frequency) distribution of the transmitted signal it is possible to get significant improvements in detection performance employing the GD. To achieve this advantage it is necessary to have knowledge of the clutter and/or target statistics.

In this paper, we assumed that this statistical information is available. Statistics of specific targets of interest may be assumed known through measurements of their radar signatures. By tuning the transmit waveform to a given target type, using, for example, the matched target waveform described earlier, it is possible to enhance significantly the detectability of targets of that type. Knowledge of clutter statistics can be obtained by collecting data over multiple pulse periods. The use of GD by radar systems allows us to get a great advantage in detection performance. A more complete discussion of issues related to estimating target and clutter statistics and the impact of estimation accuracy on detection performance are beyond the scope of this paper and are the subject of ongoing research.

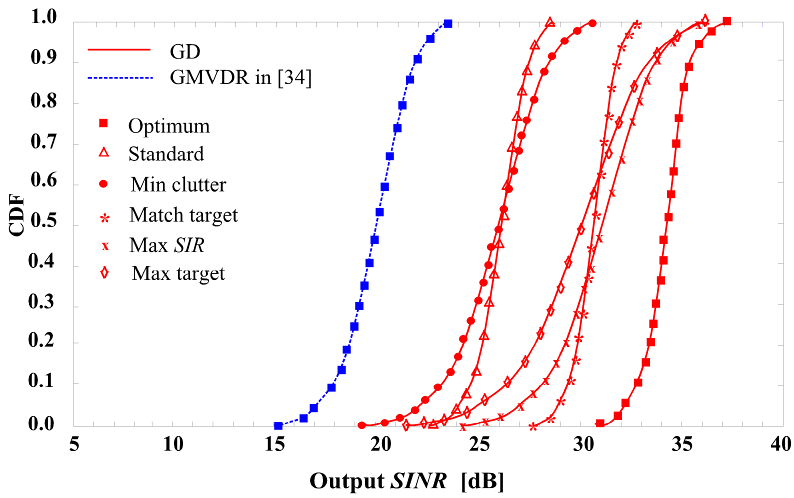

Fig.4 CDF of *SINR* at the GD output for different transmit waveforms. Comparison with GMVDR detector. Case 3.

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