An Energy-segmented Moth-flame Optimization Algorithm for Function Optimization and Performance Measures Analysis

YUANFEI WEI[1], PENGCHUAN WANG[2,3], QIFANG LUO[2,3*], YONGQUAN ZHOU[2,3]

[1]Xiangsihu College of Gunagxi University for Nationalities, Nanning, Guangxi 532100, CHINA
[2]College of Information Science and Engineering, Guangxi University for Nationalities, Nanning, Guangxi 530006, CHINA
[3]Guangxi Key Laboratories of Hybrid Computation and IC Design Analysis, Nanning 530006, CHINA

Abstract: The moth-flame optimization algorithm (MFO) is a novel metaheuristic algorithm for simulating the lateral positioning and navigation mechanism of moths in nature, and it has been successfully applied to various optimization problems. This paper segments the flame energy of MFO by introducing the energy factor from the Harris hawks optimization algorithm, and different updating methods are adopted for moths with different flame-detection abilities to enhance the exploration ability of MFO. A new energy-segmented moth-flame optimization algorithm (ESMFO) is proposed and is applied on 21 benchmark functions and an engineering design problem. The experimental results show that the ESMFO yields very promising results due to its enhanced exploration, exploitation, and convergence capabilities, as well as its effective avoidance of local optima, and achieves better performance than other the state-of-the-art metaheuristic algorithms in terms of the performance measures.

KeyWords: moth-flame optimization algorithm, energy-segmented moth-flame optimization, benchmark function, metaheuristic optimization

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1. Introduction

Over the last three decades, some classical methods for solving various optimization problems have been successfully established. As the number of optimization problem parameters increases, the complexity of these problems increases. Therefore, when dealing with high-dimensional problems, classical methods cannot obtain accurate global minima or maxima, and so they fall into local minima. Thus, finding the exact solutions of such problems becomes a challenge for classical methods [1]. To solve this problem of classical optimization algorithms, a natural heuristic algorithm is defined under the same background. In fact, for millions of years, nature has been a great source of inspiration for the solving of real-world problems by human beings. There are many real-world processes that describe nature-inspired computations, such as those related to decision-making, immune systems, collective behaviors, and learning. Based on these phenomena, various natural heuristic optimization algorithms have been designed and are now widely applied in most research fields.

According to the inspiration sources of these algorithms, natural heuristic algorithms can be divided into four categories: evolution-based, group-based, physics-based and human-based approaches [2][3]. The evolutionary methods were developed based on the law of natural evolution. Among these methods, the most the genetic algorithm which simulates Darwin's evolutionary process, is undoubtedly the most popular [4]. In this category, other popular methods include evolutionary strategies [5] and genetic programming [6]. On the other hand, group-based approaches are designed to simulate the social and collective behaviors of animal groups (birds, insects, fish, etc.). The particle swarm optimization [7][8] algorithm, which was inspired by the social behaviors of flocking birds, is the most representative and successful example in this field. Other related methods include the ant colony optimization [9][10], artificial bee colony [11][12][46], and whale optimization algorithms [13][14], etc. In addition, there are some physics-based algorithms that have been developed on the basis of simulating the laws of physics observed in our universe. The most popular methods in this category are simulated

annealing [15], the gravity search algorithm [16][17], and the material search state [18]. Finally, we describe human-based algorithms. These methods, inspired by nature, are unique because they derive inspiration from phenomena that are usually associated with human behaviors, lifestyles or perception. The most famous methods in the literature include harmony search [19][20], the pyrotechnics algorithm [21], the heap-based optimizer [47], the slime mold algorithm[48], the gradient-based optimizer [49], and so on.

The moth-flame optimization (MFO) algorithm [22-24] is an evolutionary algorithm that was recently proposed. The algorithm is inspired by the navigation behavior or lateral orientation of moths found in nature. According to their lateral orientation, moths fly straight toward the moon by forming a fixed angle with the moon. This phenomenon only applies to light sources far away from moths. Therefore, any artificial light in their path causes them to follow the light, and they persist around it as they continue to move in a deadly spiral path. This phenomenon was used to develop the MFO algorithm, and since its inception, it has been applied to solve various real-world tasks.

For any generalized algorithm, exploration and development are its most important features. Here, exploration refers to exploring the search space or the use of global search, while development refers to local search. Although MFO is a good algorithm, because there is no perfect theorem [25], no algorithm is the most suitable one for all optimization problems. Therefore, in order to improve the performance of MFO, some improvement schemes are proposed to improve the development and exploration capabilities of the MFO algorithm and maintain a proper balance between the two. Here, the energy classification system of the Harris hawks optimization (HHO) algorithm [26] is introduced to segment the flame energy in the basic MFO algorithm and balance the exploration and exploitation abilities of the algorithm. The new algorithm is called energy-segmented moth-flame optimization (ESMFO), which focuses on improving the search ability and convergence speed of the MFO algorithm. In this paper, we apply the ESMFO algorithm to the tasks of function optimization and performance measures analysis.

In this paper, by introducing the energy factor from the Harris hawks optimization algorithm, the flame energy in MFO is segmented, and different updating methods are adopted for moths of different flame-detection abilities to enhance the exploration ability of MFO. The ESMFO is applied to test 21 benchmark functions. The experimental results show that among the compared algorithms, the ESMFO algorithm has the best comprehensive performance and achieves better performance than other the state-of-the-art metaheuristic algorithms in terms of the chosen performance measures.

The rest of the paper is organized as follows. Section II provides a description of related works. Section III describes the proposed energy-segmented moth-flame optimization (ESMFO) algorithm. The results are discussed in Section IV. Section V introduces the conclusions and future work ideas.

2. Related Work

2.1. Original Moth-flame Optimization Algorithm (MFO)

Moths are winged insects belonging to the butterfly family. At night, moths fly by moonlight. For the purpose of flying, they use a special navigation mechanism known as lateral positioning. According to this method, moths maintain a fixed angle with the moon when flying. However, moths have been observed to fly around light in a spiral manner rather than horizontally. This is because lateral positioning is effective only when the light source is far away from the moth. Because the alternate light source is far from the moon, the straight path becomes a spiral path.

In the basic moth optimization algorithm, the position of a moth in space is the variable of the problem (assuming that the moth is the candidate solution). By changing the moth's position vector, the algorithm can solve problems in low-dimensional and multidimensional space. In addition, it is worth noting that the moth and flame are both candidates in the moth optimization algorithm. The moth is the main moving body in the solution space, and the flame is the optimal value obtained by the moth during the current iteration of the algorithm. The flame can be understood as the location marker at the end of the moth's final search path. When the algorithm is locally exploited, moths search for the optimal position in the field of flame. Under this mechanism, the optimization accuracies of moths are improved.

The framework of the moth optimization algorithm is as follows:

$$MFO = (I, P, T) \tag{1}$$

The function I generates a moth population M randomly and generates the corresponding fitness matrix OM .

$$I : \phi \rightarrow \{M, OM\} \tag{2}$$

The function P is the principal function of the moth search process in the solution space. After updating, the matrix M is returned:

$$P : M \rightarrow M \tag{3}$$

The function T belongs to the judgment statement. If it satisfies the termination condition, “true” is returned; if not, “false” is returned:

$$T : M \rightarrow \{true, false\} \tag{4}$$

As mentioned above, the algorithm is inspired by lateral positioning. We use the following equation to update the moth's position:

$$M_i = S(M_i, F_j) \tag{5}$$

Here, M_i denotes moth i, F_j denotes flame j, and S is a spiral function.

In the moth optimization algorithm, the main updating mechanism for a moth is simulated by a logarithmic helix

function, and the following conditions must be satisfied:

- The starting point of the helix is at the position of the moth.
- The end of the helix is at the position of the flame.
- The floating range of the helix does not exceed the search space.

Based on the above conditions, the logarithmic helix function of the moth optimization algorithm is defined as follows:

$$S(M_i, F_j) = D_i \cdot e^{bt} \cdot \cos(2\pi t) + F_j \quad (6)$$

Here, the distance between the *i*th moth and the *j*th flame is denoted by D_i. b is a constant that defines the logarithmic helix and t is a random number between $[-1,1]$. D_i is calculated by the following formula:

$$D_i = |F_j - M_i| \quad (7)$$

In the whole search space, the position of a moth is updated relative to *n* different positions, resulting in a decrease in the search accuracy and an increased likelihood of falling into a local optimum. Therefore, an adaptive mechanism based on the number of flames is proposed. The formula is as follows:

$$flame\_no = round\left(N - l * \frac{N-1}{T}\right) \quad (8)$$

Here, l is the current iteration number, N is the maximum number of flames and T is the maximum number of iterations.

2.2. Harris Hawks Optimization (HHO)

The main inspiration for HHO comes from the cooperative behavior and pursuit strategy of the Harris eagle, and this strategy is called a "raid" in nature [27-30]. This desert predator demonstrates the innovative evolutionary ability of teaming to track, surround, rush at and eventually attack potential prey. Harris hawks' main strategy for catching prey is called assault, also known as the “seven killing strategy.” In this clever strategy, several eagles converge on their prey from different directions in an attempt to perform a surprise attack. Harris hawks can reveal many kinds of chase patterns according to the dynamic characteristics of the scene and the chosen escape mode of their prey. In this section, inspired by exploring the different attacking and raiding strategies of Harris hawks, we simulate the exploration and exploitation stages of the proposed HHO. HHO is a population-based, gradient-free optimization technique; therefore, it can be applied to any optimization problem as long as there is an appropriate formula. HHO algorithms are mainly divided into three stages: the exploratory stage, the transitional stage between the exploratory mining and exploitation stages, and the exploitation stage. Since the energy factor of our improved algorithm comes from the latter two stages of the Harris hawks optimization algorithm, we briefly introduce the latter two stages of the Harris hawks optimization algorithm.

(1) *Transition from exploration to exploitation*

The HHO algorithm can transfer from the exploration phase to the exploitation phase and then change between different exploitative behaviors based on the escaping energy of the prey. The energy of a prey decreases considerably during the escaping behavior. To model this fact, the energy of a prey is modeled as:

$$E = 2E_0\left(1 - \frac{t}{T}\right) \quad (9)$$

where E indicates the escaping energy of the prey, T is the maximum number of iterations, and E_0 is the initial state of the energy of the prey. In HHO, E_0 randomly changes within the interval (-1,1) during each iteration. When the value of E_0 decreases from 0 to -1, the prey is physically flagging, but when the value of E_0 increases from 0 to 1, the prey is strengthening. This dynamic escaping energy exhibits a decreasing trend as the number of iterations increases. In short, exploration occurs when $|E| \geq 1$, while exploitation occurs in later steps when $|E| < 1$. For defining the threshold of E_0, we can refer to the original paper [26] for a more detailed introduction than the one given here.

(2) *Exploitation phase*

In this phase, the Harris hawks perform a surprise attack by attacking the prey detected in the previous phase. However, prey often attempt to escape from dangerous situations. According to the escape behaviors of the prey and the chasing strategies of the Harris hawks, four possible strategies were proposed in the HHO.

The authors proposed a probability parameter ***r*** for the successful escape of prey from pursuit. When $r<0.5$, the escape is successful, and when $r \geq 0.5$, the prey does not successfully escape. To model this strategy and enable the HHO algorithm to switch between the soft and hard sieging processes, the parameter ***E*** is utilized. In this regard, when $|E| \geq 0.5$, the soft attack occurs, and when $|E| < 0.5$, the hard attack occurs. Since this article quotes the third form of sieging described by the original author at this stage (hard sieging with progressive, rapid dives), we only introduce this kind of sieging strategy. For more detailed information, please read the source article about HHO.

When $|E| < 0.5$ and $r < 0.5$, the prey does not have enough energy to escape and a hard siege is conducted before the surprise pounce to catch and kill the prey. Therefore, the following rule is obeyed by a hard siege:

$$X(t+1) = \begin{cases} Y & if\ F(Y) \prec F(X(t)) \\ Z & if\ F(Z) \prec F(X(t)) \end{cases} \quad (10)$$

where Y and Z are obtained using the new rules in Eqs. (12) and (13).

$$Y = X_{rabbit}(t) - E|JX_{rabbit}(t) - X_m(t)| \quad (11)$$

where $X_m(t)$ is the average position of the hawks. Then, the possible result of such a movement is compared to that of

the previous dive to detect whether it be a good dive. If the result is not reasonable, they also start to perform irregular, abrupt, and rapid dives when approaching the prey. We suppose that they dive according to LF-based patterns using the following rule:

$$Z = Y + S \times LF(D) \tag{12}$$

where D is the dimension of the problem, S is a random vector of size $1 \times D$ and LF is the Levy flight function, which is calculated using Eq. (13) [48]:

$$LF(x) = 0.01 \times \frac{u \times \sigma}{|v|^{\frac{1}{\beta}}}, \sigma = \left(\frac{\Gamma(1+\beta) \times \sin\left(\frac{\pi\beta}{2}\right)}{\Gamma\left(\frac{1+\beta}{2}\right) \times \beta \times 2^{\left(\frac{\beta-1}{2}\right)}} \right)^{\frac{1}{\beta}} \tag{13}$$

where u and v are random values bounded by (0,1) and β is a constant set to 1.5 by default.

3. Energy-segmented Moth-flame Optimization Algorithm

As we all know, the authors who developed the MFO algorithm only considered the update process for a moth in the ideal state and did not consider the influence of the distance between the moth and the light source on the moth when the it circles the artificial light source. Therefore, in order to better conform to the biological characteristics of nature, the algorithm proposed in this paper accounts for the influence of the brightness of the light source on moth flight. According to the previous section, we compare moths to Harris hawks, and then we compare flames to prey. Considering the moth's irregular flight and the effect of light intensity on the moth, we borrow the adaptive parameter for the energy factor from the HHO algorithm. This parameter is called the light source intensity E.

The light source intensity E is used to distinguish between the two stages (i.e., exploration and exploitation) of the MFO algorithm; when the moth is far away from the flame, the moth feels a low amount of flame energy, and the ESMFO algorithm performs the exploration phase; otherwise, the moth and flame are close, the flame energy felt by the moth is high, and the algorithm performs the exploitation phase. E exhibits a rising trend as the number of iterations increases. When the flame energy $|E|<0.5$, the moth searches different regions to detect the position of the flame, so the ESMFO algorithm performs the exploration phase. When $|E| \geq 0.5$, the algorithm attempts to exploit the neighborhood of the solutions during the exploitation phase. In short, exploration occurs when $|E|<0.5$, while exploitation occurs in later steps when $|E| \geq 0.5$. The two phases of the ESMFO algorithm are described in the next subsections.

3.1. Exploration Phase

In this subsection, the exploration mechanism of ESMFO is proposed. The main inspiration of the MFO algorithm is the navigation mechanism of moths in nature called transverse orientation. When the moth is far away from the light source, the energy value E felt by the moth is less than 0.5, and the proposed algorithm still uses the spiral update formula from the MFO algorithm. This behavior is modeled by the following rule:

$$S(M_i, F_i) = D_i \cdot e^{bt} \cdot \cos(2\pi t) + F_j \tag{14}$$

3.2. Exploitation Phase

In this phase, the moth feels increasingly intense energy as the distance between the moth and the flame is reduced; when the energy value is greater than 0.5, the influence of the flame on the moth is intensified. The moth cannot continue to maintain the navigation mechanism (lateral positioning), so it instead performs a useless and fatal irregular flight strategy. The method proposed in this paper uses a new update formula instead of the update formula contained in the MFO algorithm. The current positions are updated using Eq. (15):

$$M(t+1) = \begin{cases} Y & if \ F(Y) \prec F(S(t)) \\ Z & if \ F(Z) \prec F(S(t)) \end{cases} \tag{15}$$

where Y and Z are obtained using Eqs. (16)-(17).

$$Y = M(t) - E|jF(t) - M(t)| \tag{16}$$

$$Z = M(t) - E|jF(t) - M(t)| \cdot Levy(\beta) \tag{17}$$

where ***Levy*** is the Levy flight function, which is calculated using Eq. (13).

3.3. Pseudocode of the ESMFO Algorithm

The pseudocode of the proposed ESMFO algorithm is given in Algorithm 1.

Algorithm 1 The pseudo code of the ESMFO algorithm

Update *flame_ no* using Eq. (8)
OM = FitnessFunction(*M*);
If iteration ==1
  F=sort(*M*);
    OF=sort(*OM*);
else
  F = sort(M_{t-1}, M_t);
    OF = sort(M_{t-1}, M_t);
end
for $i = 1:n$
  for $j = 1:d$
    Update initial energy E_0
    Update r and t
    If $(|E| \geq 0.5)$ ***then***

Calculate D using Eq. (7) with respect to the corresponding moth;

else $(|E| < 0.5)$ ***then***

Update $M(i, j)$ using Eqs. (16) and (17) with respect to the corresponding moth;

end

end

end

3.4. Computational Complexity of the ESMFO Algorithm

The computational complexity of an algorithm is a key metric for evaluating its run time, and this metric can be defined based on the structure and implementation of the algorithm. The computational complexity of the ESMFO algorithm depends on the number of moths, number of variables, maximum number of iterations, and sorting mechanism for the flames in each iteration. Since the Quicksort algorithm is utilized, the computational complexity of the sorting process is of $O(nlogn)$ and $O(n^2)$ for the best and worst cases, respectively. Therefore, the overall computational complexity is defined as follows:

$$O(ESMFO) = O\,(t\,(O\,(Quicksort)+O\,(position\ update))) \tag{18}$$

$$O(ESMFO) = O\,(t\,(O\,(n^2 + n\times d))) = O\,(tn^2 + tnd) \tag{19}$$

where n is the number of moths, t is the maximum number of iterations, and d is the number of variables.

4. Results and Discussion

In this section, we analyze the effectiveness of the proposed algorithm using benchmark functions. Additionally, the results of ESMFO are compared with those of the basic MFO algorithm as well as with the results of other metaheuristic algorithms. The results are obtained for different dimension sizes to evaluate the effect of the algorithm.

4.1. Parameter Settings

For performance evaluation purposes, the results are obtained using a computer with a 64-bit Windows 10 operating system, an Intel Core i5-7200U, CPU@ 2.50 GHZ, 8 GB RAM, and MATLAB version R2017a. All the simulation experiments are performed on MATLAB. All the algorithms are run 30 times, each with 1000 iterations. The results of ESMFO are compared with those of the moth-flame optimization algorithm (MFO) [22-24], Levy flight moth-flame optimization algorithm (LMFO) [32], Harris hawks optimization algorithm (HHO) [26], whale optimization algorithm (WOA) [10][31][33], bat algorithm (BA) [34-37], and cuckoo search (CS) algorithm [38-41]. The various parameters of these algorithms are given in Table 1.

TABLE 1
PARAMETER SETTINGS

Algorithm	Parameter	Value
BA	A Loudness; r Pulse rate	0.5; 0.5
CS	Discovery rate of unknown solutions pa	0.25
WOA	$\vec{\alpha}$	Linearly decreased from 2 to 0
MFO	Spiral factor b	1
LMFO	Spiral factor b	1
ESMFO	Spiral factor b	1

4.2. Test Functions

To check its performance, the algorithm proposed in this paper is been verified on twenty one benchmark functions [42-44]. These benchmark functions include unimodal functions, multimodal functions and multimodal functions with fixed dimensions. Unimodal functions have only one global mining operation. On the other hand, multimodal functions have a large number of local mining operations. Here, it should be noted that unimodal functions help to check the exploitative ability of an algorithm, whereas multimodal functions help keep a check on the explorative capabilities of any algorithm. The benchmark functions are discussed in detail in Tables 2, 3 and 4.

TABLE 2
UNIMODAL, HIGH-DIMENSIONAL BENCHMARK TEST FUNCTION

Benchmark function	Dim	Range	$f_{\min}$
$f_1(x) = \sum_{i=1}^{n} x_i^2$	30	$x_i \in [-100,100]$	0
$f_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	30	$x_i \in [-10,10]$	0

$f_3(x)=\sum_{i=1}^{n}(\sum_{j=1}^{i}x_j)^2$	30	$x_i\in[-100,100]$	0
$f_4(x)=\max_i\{\lvert x_i\rvert,1\le i\le D\}$	30	$x_i\in[-100,100]$	0
$f_5(x)=\sum_{i=1}^{D-1}[100(x_{i+1}-x_i^2)^2+(x_i-1)^2]$	30	$x_i\in[-30,30]$	0
$f_6(x)=\sum_{i=1}^{n}x_i^4+random(0,1)$	30	$x_i\in[-1.28,1.28]$	0

TABLE 3
MULTIMODAL, HIGH-DIMENSIONAL BENCHMARK TEST FUNCTION

Benchmark function	Dim	Range	$f_{\min}$
$f_7(x)=\sum_{i=1}^{n}[x_i^2-10\cos(2\pi x_i)+10]$	30	$x_i\in[-5.12,5.12]$	0
$f_8(x)=-20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}-\exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos 2\pi x_i\right)\right)+20+e$	30	$x_i\in[-32,32]$	0
$f_9(x)=\frac{1}{4000}\sum_{i=1}^{n}(x_i^2)-\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})+1$	30	$x_i\in[-600,600]$	0
$f_{10}(x)=0.1\sin^2(3\pi y_1)+\sum_{i=1}^{n-1}(y_i-1)^2[1+\sin^2(3\pi y_{i+1})+(y_n-1)^2(1+\sin^2(2\pi y_n))]+\sum_{i=1}^{n}\mu(x_i,5,100,4)$	30	$x_i\in[-50,50]$	0
$f_{11}(x)=\sum_{i=1}^{n}(\sum_{j=1}^{i}\lvert x_j\sin(x_i)+0.1x_j\rvert)$	30	$x_i\in[-10,10]$	0
$f_{12}(x)=\sum_{i=1}^{n}x_i^2+(\sum_{i=1}^{n}0.5ix_i)^2+(\sum_{i=1}^{n}0.5ix_i)^4$	30	$x_i\in[-5,10]$	0

TABLE 4
MULTIMODAL, FIXED-DIMENSIONAL BENCHMARK TEST FUNCTION

Benchmark function	Dim	Range	$f_{\min}$
$f_{13}(x)=\left(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6}\right)^{-1}$	2	$x_i\in[-65,65]$	1
$f_{14}(x)=\sum_{i=1}^{11}\left[a_i-\frac{x_1(b_i^2+b_ix_2)}{b_i^2+b_ix_3+x_4}\right]^2$	4	$x_i\in[-5,5]$	0.0003075
$f_{15}(x)=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	2	$x_i\in[-5,5]$	-1.0316285

$f_{16}(x) = (x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 + 6)^2 + 10(1 - \frac{1}{8\pi})\cos x_1 + 10$	2	$x_i \in [-5,5]$	0.398
$f_{17}(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	2	$x_i \in [-5,5]$	3
$f_{18}(x) = -\sum_{i=1}^{4} c_i \exp\left[\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2\right]$	3	$x_i \in [0,1]$	-3.86278
$f_{19}(x) = -\frac{1 + \cos(12\sqrt{x_1^2 + x_2^2})}{0.5(x_1^2 + x_2^2) + 2}$	2	$x_i \in [-5.12,5.12]$	-1
$f_{20}(x) = 0.5 + \frac{\sin^2(\sqrt{x_1^2 + x_2^1}) - 0.5}{(1 + 0.001(x_1^2 + x_2^2))^2}$	2	$x_i \in [-100,100]$	-1
$f_{21}(x) = -\cos(x_1)\cos(x_2)\exp(-(x_1 - \pi)^2 - (x_2 - \pi)^2)$	2	$x_i \in [-100,100]$	-1

4.3. Test Function Results and Analysis

In this section, we perform simulation experiments on the algorithm, analyze the effectiveness of its HHO-based modification, and verify its optimization performance. Tables 5 and 6 show the comparison of the ESMFO algorithms with other algorithms based on fixed-dimensional and high-dimensional functions. Table 5 shows that the ESMFO algorithm performs better than other algorithms on nine benchmark functions for fixed-dimensional functions, ranking first in the optimization rankings of all seven algorithms. This demonstrates the efficiency of the optimization performance achieved by the ESMFO algorithm.

Fig. 1 to Fig. 9 show the best fitness curves for each function, and Fig. 10 to Fig. 18 show the variance diagrams for each function. From the curves and variance diagrams, we can see that the ESMFO algorithm presented in this paper has the highest optimization accuracy and fastest convergence speed on the fixed-dimensional test function. At the same time, we can see that the algorithm shows strong stability. Therefore, the ESMFO algorithm achieves excellent performance on fixed-dimensional benchmark functions.

TABLE 5
COMPARISON OF THE MULTIPEAK FIXED-DIMENSIONAL TEST FUNCTION RESULTS

	Algorithm	MFO	LMFO	HHMFO	HHO	WOA	BA	CS	RANK
f_{13}-2dim	Best	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	1.99E+00	9.98E-01	1
	Worst	4.95E+00	1.26E+01	1.26E+01	5.93E+00	1.08E+01	1.27E+01	9.98E-01	
	Mean	1.69E+00	4.98E+00	5.42E+00	1.20E+00	1.82E+00	1.16E+01	9.98E-01	
	Std	9.78E-01	4.06E+00	4.75E+00	9.12E-01	1.87E+00	2.90E+00	0.00E+00	
f_{14}-4dim	Best	6.27E-04	3.10E-04	**3.08E-04**	**3.08E-04**	3.16E-04	3.08E-04	3.07E-04	1
	Worst	2.04E-02	1.24E-03	1.22E-03	1.49E-03	2.24E-03	5.69E-03	3.08E-04	
	Mean	0.001556	3.85E-04	3.95E-04	3.68E-04	8.29E-04	1.20E-03	3.08E-04	
	Std	0.0035656	1.69E-04	2.32E-04	2.13E-04	4.59E-04	1.53E-03	4.84E-08	
f_{15}-2dim	Best	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	1
	Worst	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	
	Mean	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	
	Std	6.78E-16	1.14E-05	3.69E-06	1.61E-11	3.21E-11	6.54E-08	6.78E-16	

f_{16}-2dim	Best	3.98E-01	**0.39791**	3.98E-01	3.98E-01	-1.00E+00	0.39789	0.39789	2
	Worst	3.98E-01	3.99E-01	3.98E-01	3.98E-01	-9.36E-01	0.39789	0.39789	
	Mean	3.98E-01	3.98E-01	3.98E-01	3.98E-01	-9.85E-01	0.39789	0.39789	
	Std	0.00E+00	3.48E-04	7.68E-05	6.04E-07	2.74E-02	3.96E-08	0.00E+00	
f_{17}-2dim	Best	**3**	**3**	**3**	**3**	**3**	**3**	**3**	1
	Worst	3	3.0003	3	3	3.0001	84	3	
	Mean	3	3.0001	3	3	3	7.5	3	
	Std	1.29E-15	7.50E-05	1.28E-04	1.38E-07	2.54E-05	1.60E+01	1.87E-15	
f_{18}-3dim	Best	-3.86E+00	-3.86E+00	**-3.86E+00**	-3.86E+00	-3.86E+00	-3.86E+00	-3.00E-01	1
	Worst	-3.86E+00	-3.85E+00	-3.85E+00	-3.86E+00	-3.85E+00	-1.00E+00	-3.00E-01	
	Mean	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.62E+00	-3.00E-01	
	Std	2.71E-15	2.50E-03	2.31E-03	1.53E-03	2.47E-03	7.38E-01	2.26E-16	
f_{19}-2dim	Best	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**	1
	Worst	-9.36E-01	-1	-1	-1	-0.93625	-0.78575	-1	
	Mean	-9.76E-01	-1	-1	-1	-0.98937	-0.9376	-1	
	Std	-3.12E-02	0	0	0	0.024166	0.034634	3.75E-12	
f_{20}-2dim	Best	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**	1
	Worst	-9.62E-01	-1	-1	-1	-0.99028	-0.99028	-0.9993	
	Mean	-9.89E-01	-1	-1	-1	-0.99611	-0.99741	-0.99997	
	Std	7.28E-03	0	0	0	0.0048412	0.00437	0.00012846	
f_{21}-2dim	Best	**-1**	-0.9999	**-1**	**-1**	**-1**	**-1**	**-1**	1
	Worst	-1	-0.9988	-1	-1	-1	-8.11E-05	-1	
	Mean	-1	-0.9997	-1	-1	-1	-0.53337	-1	
	Std	0	2.91E-04	7.07E-05	9.94E-07	2.17E-07	5.07E-01	0	

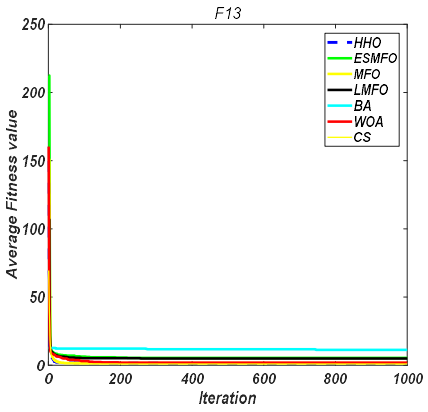


FIGURE 1 Evolution curves of the fitness values for f_{13}

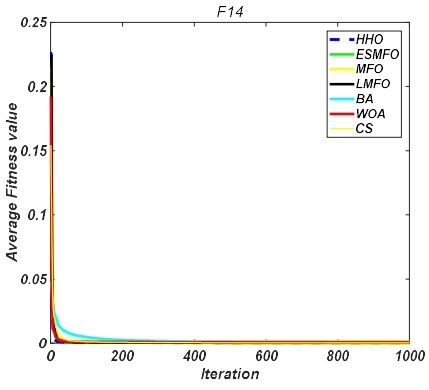


FIGURE 2 Evolution curves of the fitness values for f_{14}

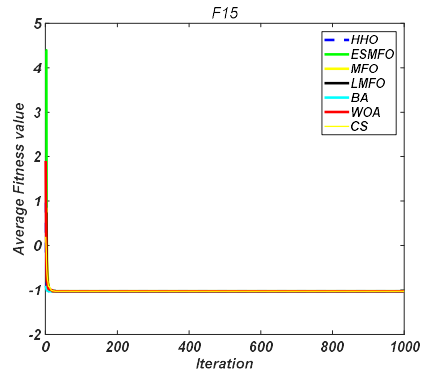


FIGURE 3 Evolution curves of the fitness values for f_{15}

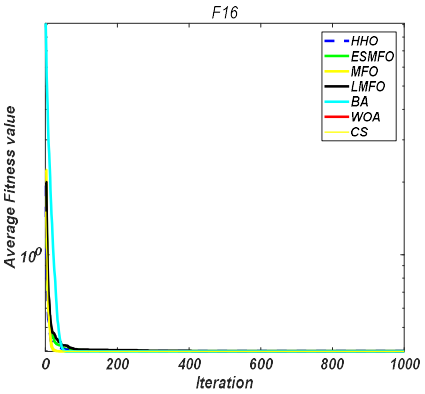


FIGURE 4 Evolution curves of the fitness values for f_{16}

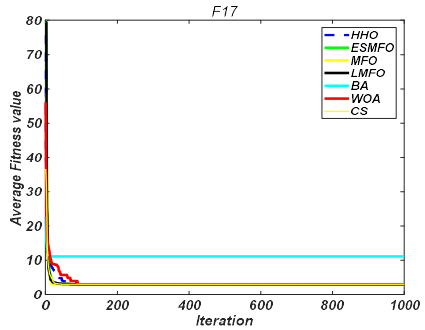


FIGURE 5 Evolution curves of the fitness values for f_{17}

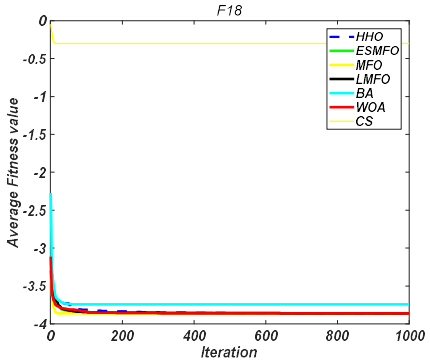


FIGURE 6 Evolution curves of the fitness values for f_{18}

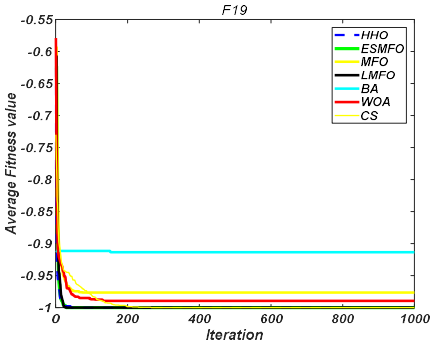


FIGURE 7 Evolution curves of the fitness values for f_{19}

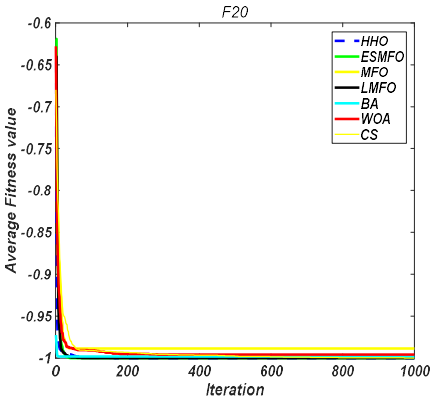


FIGURE 8 Evolution curves of the fitness values for f_{20}

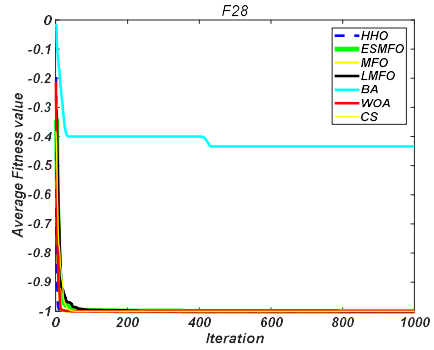


Figure 9 Evolution curves of the fitness values for f_{21}

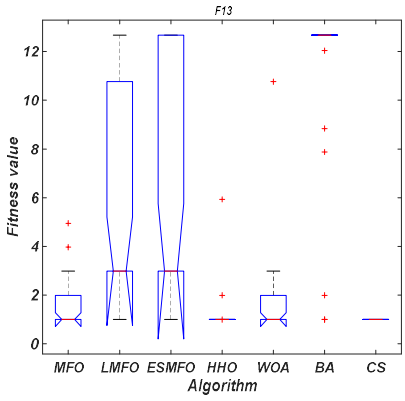


Figure 10 Variance diagrams for f_{13}

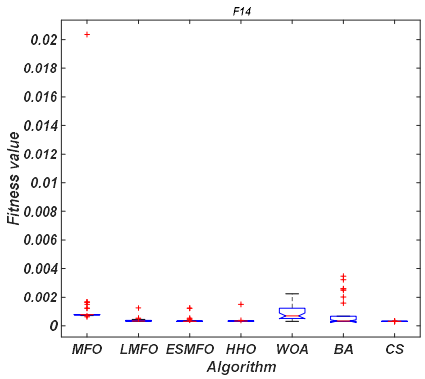


FIGURE 11 Variance diagrams for f_{14}

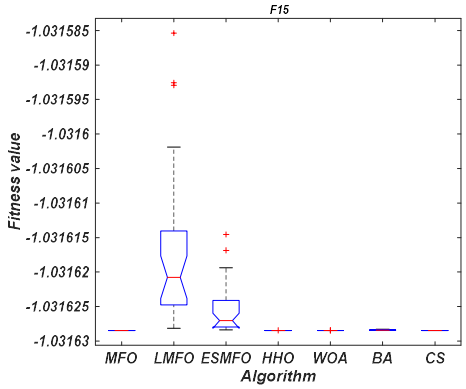


FIGURE 12 Variance diagrams for f_{15}

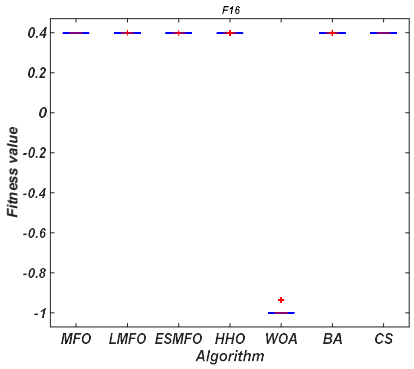


FIGURE 13 Variance diagrams for f_{16}

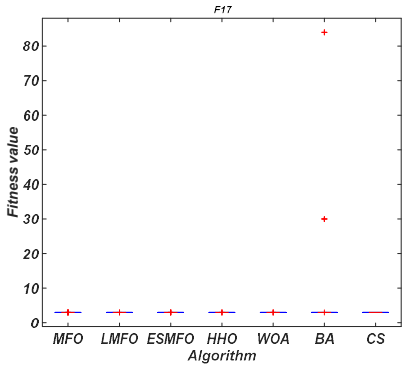


FIGURE 14 Variance diagrams for f_{17}

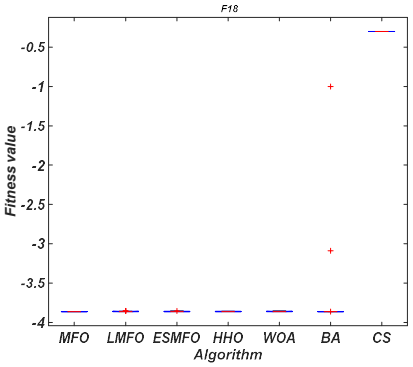


FIGURE 15 Variance diagrams for f_{18}

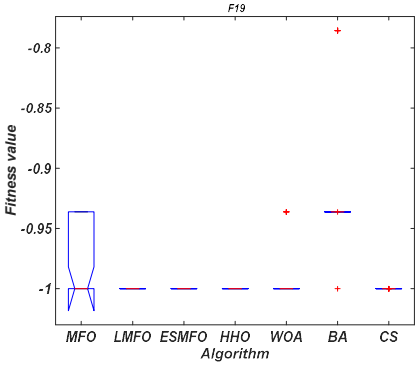


FIGURE 16 Variance diagrams for f_{19}

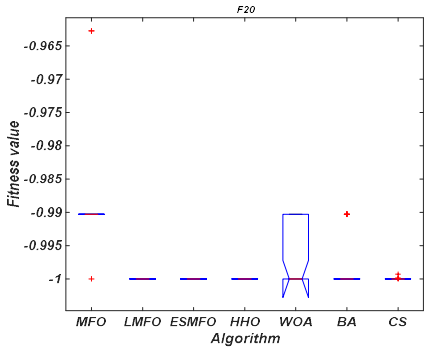


FIGURE 17 Variance diagrams for f_{20}

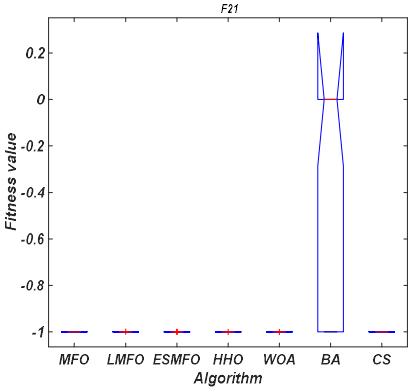


FIGURE 18 Variance diagrams for f_{21}

From Table 6, we can see that the ESMFO algorithm has strong competitiveness in the remaining 11 high-dimensional standard test functions (except for the midstream of the best fitness values obtained on the f_{10} function), and its optimization accuracy ranks first. Fig. 19 to Fig. 42 represent the curves and variance graphs for 12 high-dimensional functions. Similarly, we can see that except for the instability of ESMFO for individual functions and the fact that it occasionally falls into local optima, the functions have strong stability in search accuracy and convergence speed.

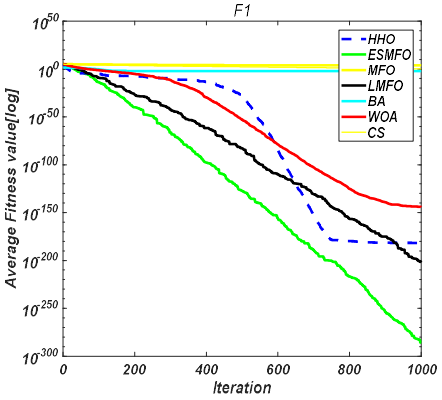


FIGURE 19 Evolution curves of the fitness values for f_1

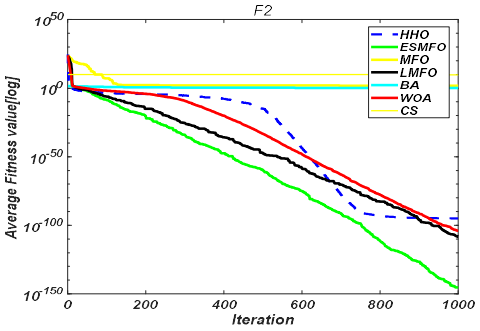


FIGURE 20 Evolution curves of the fitness values for f_2

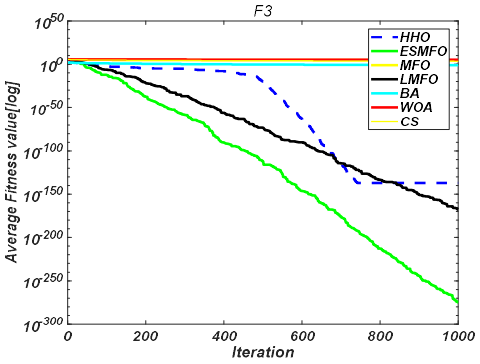


FIGURE 21 Evolution curves of the fitness values for f_3

TABLE 6
RESULTS OF ESMFO COMPARED WITH THOSE OF THE OTHER ALGORITHMS FOR THE CASE WITH DIMENSION SIZE=50

	Algorithm	MFO	LMFO	ESMFO	HHO	WOA	BA	CS	RANK
f_1-50dim	Best	1.42E+00	4.17E-275	**0**	2.58E-216	1.23E-177	3.93E-03	6.82E-01	1
	Worst	2.00E+04	2.69E-201	4.72E-285	6.65E-181	2.08E-143	5.94E-03	2.74E+00	
	Mean	5.37E+03	8.95E-203	1.66E-286	2.22E-182	6.95E-145	4.82E-03	1.71E+00	
	Std	5.69E+03	0	0	0	3.80E-144	4.88E-04	6.27E-01	
f_2-50dim	Best	3.01E+01	2.01E-127	**1.73E-174**	2.67E-106	3.01E-112	4.22E-01	2.78E+01	1
	Worst	1.30E+02	5.89E-108	1.11E-144	1.75E-94	1.78E-103	2.83E+00	1.00E+10	
	Mean	7.32E+01	4.36E-109	4.19E-146	1.23E-95	9.72E-105	1.34E+00	6.67E+09	
	Std	3.26E+01	1.37E-108	2.03E-145	3.90E-95	3.34E-104	6.25E-01	4.79E+09	
f_3-50dim	Best	1.99E+04	8.63E-222	**0**	1.34E-186	7.20E+04	7.16E-02	2.33E+03	1
	Worst	7.38E+04	5.82E-167	4.21E-274	3.00E-136	1.81E+05	1.71E-01	4.92E+03	
	Mean	4.31E+04	1.94E-168	1.40E-275	1.00E-137	1.25E+05	1.20E-01	3.56E+03	
	Std	1.52E+04	0	0	5.47E-137	2.70E+04	2.62E-02	7.95E+02	
f_4-50dim	Best	7.37E+01	4.10E-118	**2.25E-165**	1.17E-105	1.68E+01	2.05E-01	1.02E+01	1
	Worst	9.19E+01	4.41E-93	2.84E-138	1.57E-91	9.28E+01	6.56E-01	1.69E+01	
	Mean	8.40E+01	1.48E-94	9.74E-140	5.98E-93	6.54E+01	4.76E-01	1.39E+01	
	Std	4.71E+00	8.05E-94	5.19E-139	2.86E-92	2.48E+01	1.08E-01	2.18E+00	
f_5-50dim	Best	7.59E+02	4.66E+01	4.66E+01	**2.99E-05**	4.68E+01	4.54E+01	1.42E+02	2
	Worst	8.00E+07	4.87E+01	4.87E+01	3.86E-02	4.85E+01	1.10E+02	1.25E+03	
	Mean	7.88E+06	4.75E+01	4.77E+01	6.07E-03	4.76E+01	5.21E+01	5.39E+02	

	Std	2.40E+07	5.23E-01	6.71E-01	7.91E-03	4.77E-01	1.50E+01	2.52E+02	
f_6-50dim	Best	3.05E-01	2.28E-06	**2.03E-06**	7.96E-06	5.04E-05	8.79E-02	8.99E-02	1
	Worst	7.57E+01	1.55E-04	1.69E-04	5.29E-04	8.05E-03	2.23E-01	3.31E-01	
	Mean	1.71E+01	6.15E-05	6.52E-05	8.22E-05	1.86E-03	1.40E-01	1.86E-01	
	Std	1.84E+01	5.02E-05	4.94E-05	1.05E-04	2.10E-03	3.78E-02	5.98E-02	
F_7-50dim	Best	2.13E+02	**0**	**0**	**0**	**0**	2.98E+01	1.20E+02	1
	Worst	4.08E+02	0	0	0	0	7.15E+01	2.28E+02	
	Mean	3.08E+02	0	0	0	0	4.91E+01	1.68E+02	
	Std	4.56E+01	0	0	0	0	9.47E+00	2.16E+01	
f_8-50dim	Best	1.74E+01	**8.88E-16**	**8.88E-16**	**8.88E-16**	**8.88E-16**	2.48E+00	3.79E+00	1
	Worst	2.00E+01	8.88E-16	8.88E-16	8.88E-16	7.99E-15	3.19E+00	1.14E+01	
	Mean	1.93E+01	8.88E-16	8.88E-16	8.88E-16	3.61E-15	2.91E+00	6.24E+00	
	Std	8.14E-01	0	0	0	2.41E-15	2.11E-01	2.15E+00	
f_9-50dim	Best	9.89E-01	**0**	**0**	**0**	**0**	1.42E-04	5.74E-01	1
	Worst	2.71E+02	0	0	0	0	2.53E-04	1.04E+00	
	Mean	5.91E+01	0	0	0	0	1.89E-04	8.52E-01	
	Std	7.23E+01	0	0	0	0	2.89E-05	1.24E-01	
f_{11}-50dim	Best	3.92E+00	4.60E+00	**4.05E-08**	1.19E-01	2.28E-02	1.36E+01	1.74E+01	1
	Worst	4.71E+00	4.96E+00	2.72E-04	1.07E+00	7.01E+00	4.76E+01	4.10E+08	
	Mean	4.37E+00	4.80E+00	3.67E-05	4.74E-01	4.33E+00	2.89E+01	2.79E+07	
	Std	2.03E-01	1.00E-01	6.06E-05	2.68E-01	1.79E+00	1.11E+01	1.04E+08	
f_{12}-50dim	Best	2.06E-02	4.62E-132	**2.04E-181**	7.08E-106	2.79E-116	3.44E-02	1.13E+01	1
	Worst	3.27E+01	2.83E-108	4.51E-142	3.92E-05	2.14E-101	1.57E-01	2.20E+01	
	Mean	10.3078	1.37E-109	1.50E-143	1.31E-06	9.25E-103	5.55E-02	1.69E+01	
	Std	8.6941	5.41E-109	8.23E-143	7.16E-06	3.97E-102	2.56E-02	2.91E+00	
f_{13}-50dim	Best	5.22E+02	3.73E-191	**1.15E-304**	1.11E-137	1.36E+03	2.48E-02	2.39E+02	1
	Worst	1.77E+03	1.36E-154	1.22E-253	6.99E-47	2.00E+03	5.28E-02	7.03E+02	
	Mean	1.08E+03	4.61E-156	4.08E-255	2.33E-48	1.66E+03	3.80E-02	4.54E+02	
	Std	3.50E+02	2.48E-155	0.00E+00	1.28E-47	1.93E+02	7.90E-03	1.09E+02	

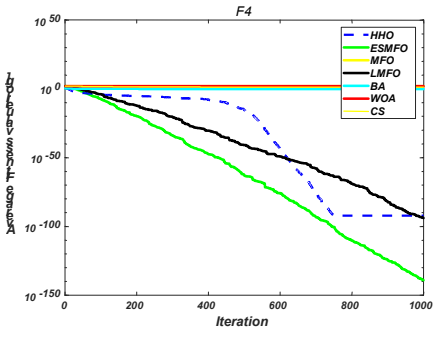


FIGURE 22 Evolution curves of the fitness values for f_4

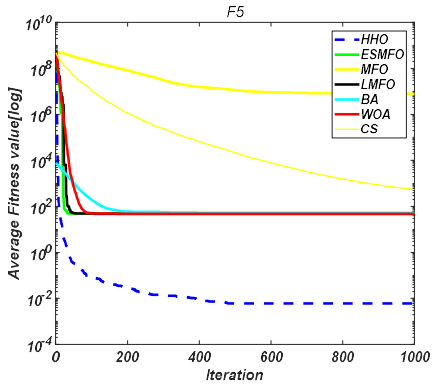


FIGURE 23 Evolution curves of the fitness values for f_5

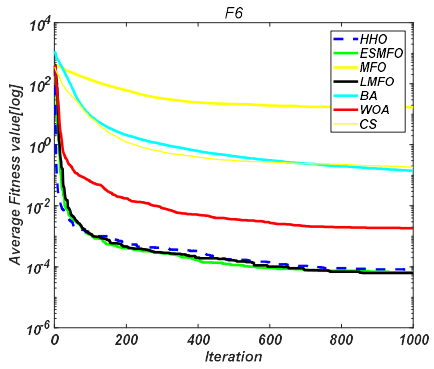


FIGURE 24 Evolution curves of the fitness values for f_6

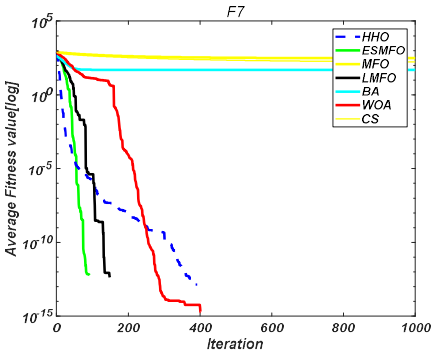


FIGURE 25 Evolution curves of the fitness values for f_7

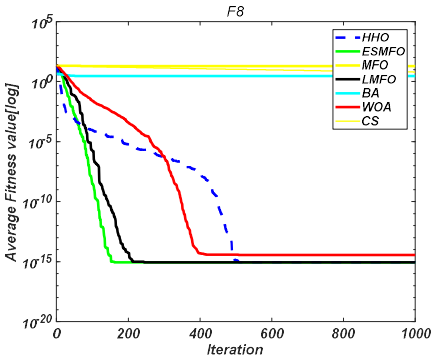


FIGURE 26 Evolution curves of the fitness values for f_8

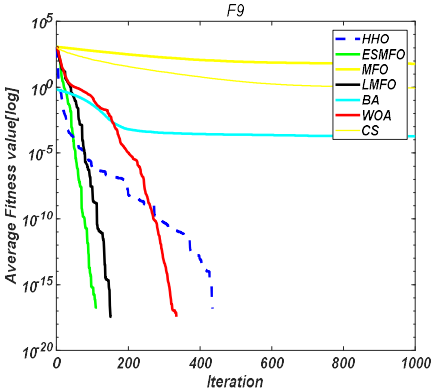


FIGURE 27 Evolution curves of the fitness values for f_9

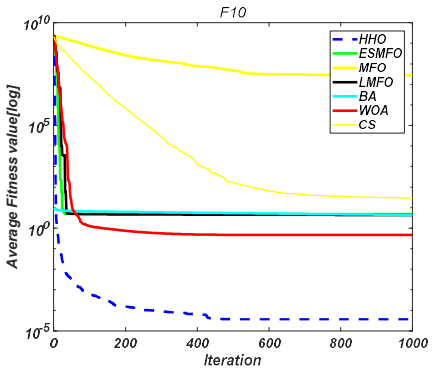


FIGURE 28 Evolution curves of the fitness values for f_{10}

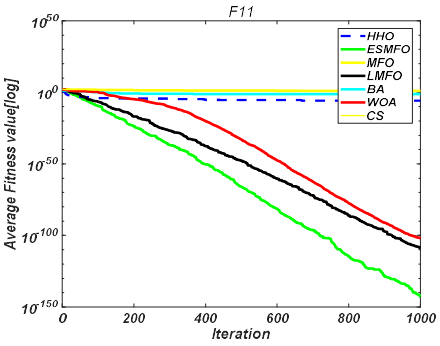


FIGURE 29 Evolution curves of the fitness values for f_{11}

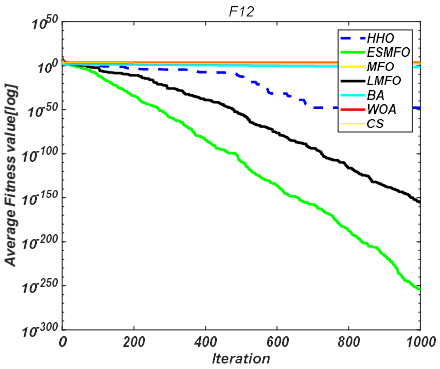


FIGURE 30 Evolution curves of the fitness values for f_{12}

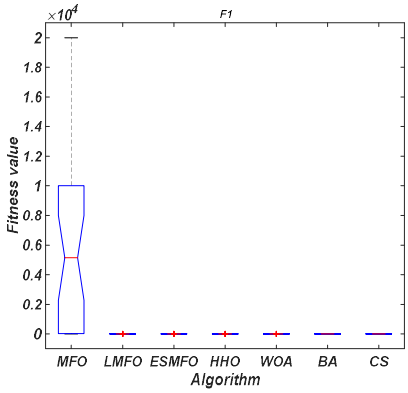


FIGURE 31 Variance diagrams for f_1

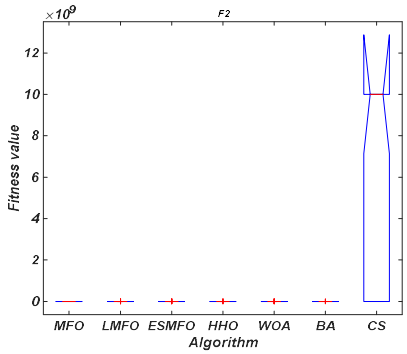


FIGURE 32 Variance diagrams for f_2

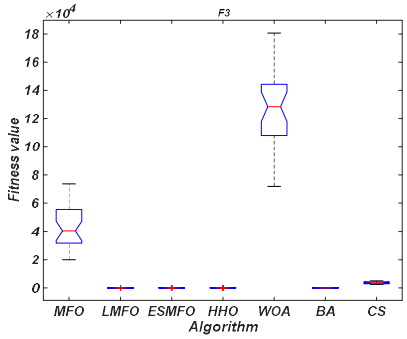


FIGURE 33 Variance diagrams for f_3

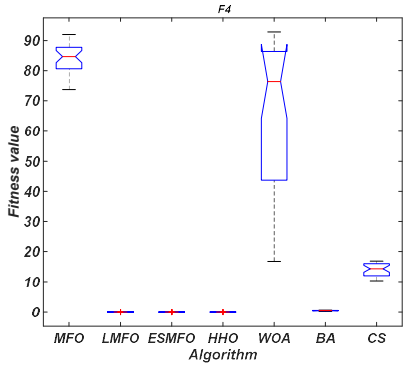


FIGURE 34 Variance diagrams for f_4

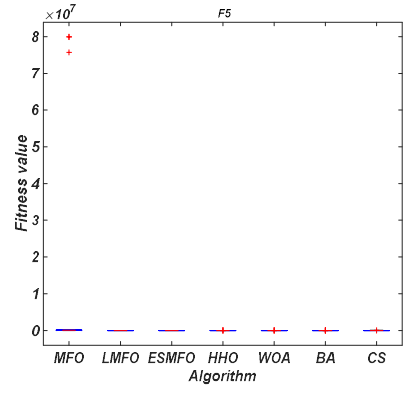


FIGURE 35 Variance diagrams for f_5

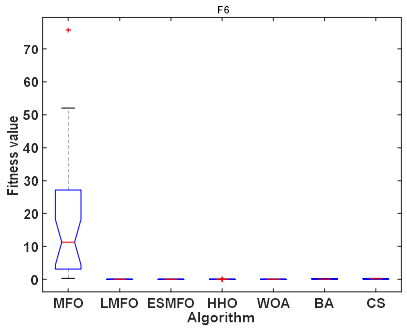


FIGURE 36 Variance diagrams for f_6

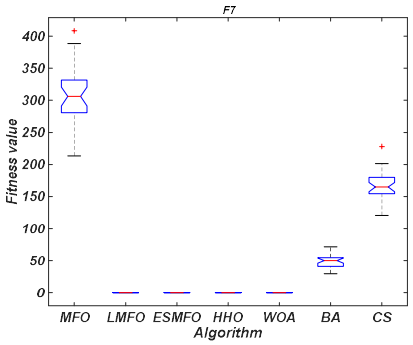


FIGURE 37 Variance diagrams for f_7

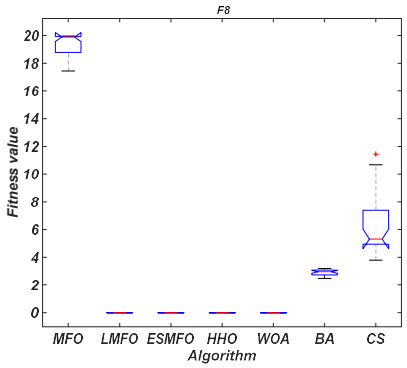


FIGURE 38 Variance diagrams for f_8

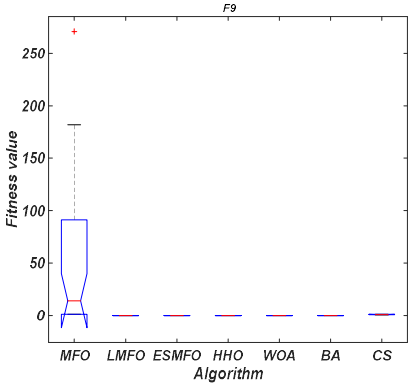


FIGURE 39 Variance diagrams for f_9

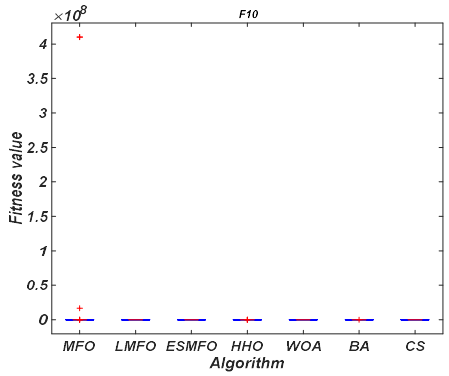


FIGURE 40 Variance diagrams for f_{10}

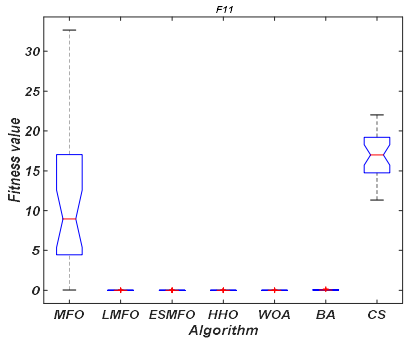


FIGURE 41 Variance diagrams for f_{11}

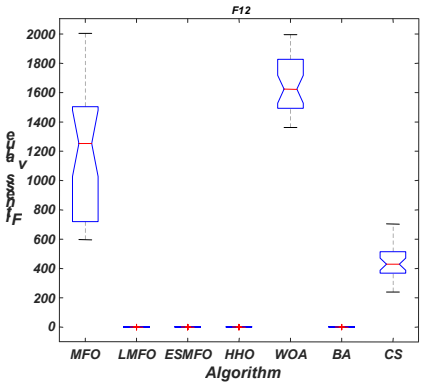


FIGURE 42 Variance diagrams for f_{12}

It can be seen from the above figures that the energy factor in the HHO algorithm provides MFO with a variety of optimization solutions, which improve the exploration task and enable the algorithm to achieve the global minima of more functions than those of the HHO algorithm. The results show that almost all the functions can provide good optimal values and standard deviation values for fixed-dimensional and high-dimensional benchmark functions. The next section details the impact of the number of dimensions on ESMFO performance.

4.4. Effect of the Number of Dimensions (Scalability Study)

The performance of the method is evaluated with different numbers of dimensions. Four different dimensions (50, 100, 200, and 500) are considered to investigate their effects on MFO and ESMFO. They are compared with meta-heuristic algorithms such as the BA, HHO, WOA, and CS algorithms. The results of MFO and ESMFO for different dimension sizes are evaluated only on unimodal and multimodal, functions which are given in Tables 2 and 3. The performances of the 12 functions are evaluated; however, the dimensions of the fixed-dimensional function cannot be changed. In this experiment, the population size is taken as 30.

The results obtained with 50 dimensions are discussed in the previous section. It is apparent from Table 6 that ESMFO performs best in 12 functions, and with 50 dimensions, HHO performs well on function f10. In the case where dimension=100, ESMFO reaches a global minimum for each functions except f5 and f10, as shown in Table 7. For functions f5 and f10, HHO provides the best results. Table 8 shows the simulation results when the number of dimensions is 500, and the results show that ESMFO performs well on all functions. The simulation results with dimension=1000 are shown in Table 9. The results show that HHO achieves good results on f5, and ESMFO has excellent performance for all other functions. For the f5 and f10 functions, all algorithms provide competitive results in terms of the best results, worst results, average results and standard deviations. However, HHO is considered to be the best due to the fact that it obtains the best optimal values.

TABLE 7
RESULTS OF ESMFO COMPARED THOSE OF THE OTHER ALGORITHMS FOR THE CASE WITH DIMENSION SIZE=100

Function	Algorithm	MFO	LMFO	ESMFO	HHO	WOA	BA	CS	RANK
F_1-100dim	Best	8.35E+03	1.75E-240	**0**	6.86E-205	2.62E-165	2.12E-02	1.37E+02	1
	Worst	5.45E+04	5.66E-203	2.87E-274	3.61E-178	6.94E-146	2.60E-02	3.86E+02	
	Mean	3.19E+06	1.89E-204	9.56E-276	1.20E-179	2.34E-147	2.36E-02	2.69E+02	
	Std	1.31E+04	0	0	0	1.27E-146	1.19E-03	7.30E+01	
F_2-100dim	Best	9.05E+01	8.42E-127	**2.63E-173**	7.93E-110	4.19E-112	2.66E+00	1.00E+10	1
	Worst	2.65E+02	1.13E-104	4.34E-143	5.07E-95	3.10E-99	7.03E+00	1.00E+10	

	Mean	1.77E+02	3.90E-106	1.49E-144	2.62E-96	1.04E-100	4.25E+00	1.00E+10	
	Std	4.45E+01	2.05E-105	7.92E-144	1.03E-95	5.66E-100	1.20E+00	0.00E+00	
F_3-100dim	Best	1.00E+05	9.36E-224	**6.36E-320**	6.60E-168	5.47E+05	3.02E+00	2.38E+04	1
	Worst	3.37E+05	1.26E-177	1.65E-272	6.07E-108	1.23E+06	5.35E+00	4.81E+04	
	Mean	2.04E+05	4.21E-179	5.60E-274	2.02E-109	8.66E+05	4.43E+00	3.18E+04	
	Std	6.02E+04	0	0	1.11E-108	1.63E+05	6.59E-01	6.09E+03	
F_4-100dim	Best	8.80E+01	6.83E-112	**1.08E-163**	2.72E-101	2.75E-01	7.14E-01	1.65E+01	1
	Worst	9.68E+01	8.26E-93	2.21E-136	1.93E-90	9.68E+01	1.08E+00	3.08E+01	
	Mean	9.30E+01	3.93E-94	8.42E-138	6.67E-92	6.22E+01	9.00E-01	2.31E+01	
	Std	2.08E+00	1.62E-93	4.04E-137	3.53E-91	3.46E+01	9.20E-02	3.24E+00	
F_5-100dim	Best	9.54E+06	9.71E+01	9.71E+01	**1.78E-06**	9.72E+01	9.53E+01	2.60E+04	2
	Worst	1.85E+08	9.87E+01	9.86E+01	3.58E-02	9.84E+01	2.03E+02	1.19E+05	
	Mean	7.36E+07	9.80E+01	9.81E+01	8.62E-03	9.79E+01	1.06E+02	4.72E+04	
	Std	4.70E+07	5.00E-01	5.04E-01	1.06E-02	3.92E-01	2.30E+01	2.14E+04	
F_6-100dim	Best	1.66E+01	1.61E-05	**1.29E-06**	2.05E-06	1.28E-05	2.28E-01	7.59E-01	1
	Worst	5.20E+02	2.66E-04	1.95E_04	4.47E-04	1.93E-02	5.94E-01	1.92E+00	
	Mean	1.45E+02	9.34E-05	5.75E-05	8.27E-05	2.33E-03	3.54E-01	1.28E+00	
	Std	1.13E+02	6.02E-05	4.64E-05	9.86E-05	4.01E-03	7.84E-02	2.90E-01	
F_7-100dim	Best	5.86E+02	**0**	**0**	**0**	**0**	7.42E+01	3.62E+02	1
	Worst	8.87E+02	0	0	0	0	1.55E+02	5.08E+02	
	Mean	7.47E+02	0	0	0	0	1.03E+02	4.35E+02	
	Std	7.03E+01	0	0	0	0	1.64E+01	3.99E+01	
F_8-100dim	Best	1.92E+01	**8.88E-16**	**8.88E-16**	**8.88E-16**	**8.88E-16**	2.62E+00	6.98E+00	1
	Worst	2.00E+01	8.88E-16	8.88E-16	8.88E-16	7.99E-15	3.42E+00	1.95E+01	
	Mean	1.98E+01	8.88E-16	8.88E-16	8.88E-16	3.49E-15	3.03E+00	9.87E+00	
	Std	2.32E-01	0	0	0	2.46E-15	2.11E-01	2.18E+00	
F_9-100dim	Best	1.21E+02	**0**	**0**	**0**	**0**	4.89E-04	2.45E+00	1
	Worst	5.76E+02	0	0	0	1.32E-01	8.07E-04	5.13E+00	
	Mean	3.20E+02	0	0	0	4.39E-03	6.07E-04	3.45E+00	
	Std	1.20E+02	0	0	0	2.40E-02	6.81E-05	6.65E-01	
F10-100dim	Best	2.56E+07	9.25E+00	9.61E+00	**5.97E-08**	5.94E-01	1.04E+01	1.72E+02	4
	Worst	8.18E+08	9.73E+00	9.95E+00	5.24E-04	3.65E+00	1.59E+01	2.79E+04	
	Mean	2.71E+08	9.55E+00	9.85E+00	4.35E-05	1.61E+00	1.32E+01	3.58E+03	
	Std	2.36E+08	1.29E-01	8.95E-02	9.79E-05	6.54E-01	1.36E+00	5.50E+03	
F_{11}-100dim	Best	1.78E+01	2.14E-132	**1.76E-172**	4.20E-113	1.92E-112	1.04E-01	3.43E+01	1
	Worst	6.88E+01	8.51E-106	2.68E-147	6.76E-94	3.34E-103	1.82E+00	5.74E+01	
	Mean	3.92E+01	2.88E-107	8.93E-149	2.29E-95	2.43E-104	3.44E-01	4.56E+01	
	Std	1.13E+01	1.55E-106	4.89E-148	1.23E-94	7.37E-104	3.57E-01	5.73E+00	
F_{12}-100dim	Best	2.26E+03	1.11E-174	**1.72e-315**	2.39E-88	2.73E+03	1.15E+01	1.31E+03	1
	Worst	5.28E+03	5.51E-135	3.02E-248	1.73E-17	3.93E+03	2.69E+01	2.32E+03	
	Mean	3.53E+03	1.84E-136	1.01E-249	5.77E-19	3.35E+03	1.74E+01	1.75E+03	

	Std	8.41E+02	1.01E-135	0	3.16E-18	2.67E+02	3.57E+00	2.78E+02	

TABLE 8
RESULTS OF ESMFO COMPARED THOSE OF THE OTHER ALGORITHMS FOR THE CASE WITH DIMENSION SIZE=500

Function	Algorithm	MFO	LMFO	ESMFO	HHO	WOA	BA	CS	RANK
F_1-500dim	Best	8.74E+05	5.57E-247	**0**	3.54E-209	2.07E-164	1.03E+00	3.24E+04	1
	Worst	1.01E+06	1.94E-191	3.20E-280	4.50E-183	1.08E-146	1.22E+00	5.01E+04	
	Mean	9.52E+05	7.16E-193	1.22E-281	1.99E-184	5.31E-148	1.13E+00	3.91E+04	
	Std	3.32E+04	0	0	0	2.05E-147	5.72E-02	3.94E+03	
F_2-500dim	Best	2.00E+03	1.31E-123	**2.16E-177**	4.80E-108	5.62E-109	5.18E+01	1.00E+10	1
	Worst	2.54E+03	1.50E-96	2.95E-139	1.06E-94	6.36E-99	7.12E+01	1.00E+10	
	Mean	2.24E+03	5.02E-98	9.96E-141	5.29E-96	2.26E-100	6.30E+01	1.00E+10	
	Std	9.92E+01	2.74E-97	5.38E-140	1.98E-95	1.16E-99	4.36E+00	0.00E+00	
F_3-500dim	Best	2.67E+06	4.83E-206	**0**	2.93E-146	1.43E+07	3.03E+02	9.11E+05	1
	Worst	5.33E+06	1.79E-167	3.34E-251	4.62E-81	7.09E+07	3.92E+02	1.47E+06	
	Mean	3.81E+06	7.61E-169	1.11E-252	1.57E-82	3.05E+07	3.40E+02	1.20E+06	
	Std	7.02E+05	0	0	8.44E-82	1.17E+07	2.18E+01	1.60E+05	
F_4-500dim	Best	9.83E+01	6.25E-112	**4.74E-167**	1.99E-103	4.08E+00	1.61E+00	3.30E+01	1
	Worst	9.95E+01	9.53E-85	2.36E-138	1.95E-90	9.91E+01	1.99E+00	4.11E+01	
	Mean	9.90E+01	3.18E-86	9.89E-140	6.64E-92	7.52E+01	1.79E+00	3.67E+01	
	Std	3.23E-01	1.74E-85	4.32E-139	3.55E-91	2.65E+01	8.21E-02	2.07E+00	
F_5-500dim	Best	3.65E+09	4.98E+02	4.98E+02	**2.31E-04**	4.95E+02	6.38E+02	7.57E+06	3
	Worst	4.57E+09	4.99E+02	4.98E+02	5.76E-01	4.97E+02	1.02E+03	1.73E+07	
	Mean	4.08E+09	4.99E+02	4.98E+02	8.75E-02	4.96E+02	8.05E+02	1.16E+07	
	Std	2.41E+08	1.21E-01	1.23E-01	1.25E-01	4.71E-01	8.53E+01	2.27E+06	
F_6-500dim	Best	2.59E+04	1.68E-06	**8.74E-07**	2.34E-06	7.89E-05	1.11E+01	6.75E+01	1
	Worst	3.33E+04	2.51E-04	2.26E-04	6.26E-04	1.06E-02	1.94E+01	1.56E+02	
	Mean	3.03E+04	7.39E-05	6.18E-05	1.03E-04	2.02E-03	1.40E+01	1.01E+02	
	Std	2.08E+03	6.35E-05	6.01E-05	1.45E-04	2.23E-03	1.60E+00	2.11E+01	
F_7-500dim	Best	6.08E+03	**0**	**0**	**0**	**0**	6.52E+02	3.37E+03	1
	Worst	6.64E+03	0	0	0	0	7.77E+02	3.87E+03	
	Mean	6.45E+03	0	0	0	0	7.16E+02	3.67E+03	
	Std	1.39E+02	0	0	0	0	3.49E+01	1.08E+02	
F_8-500dim	Best	2.00E+01	**8.88E-16**	**8.88E-16**	**8.88E-16**	**8.88E-16**	3.25E+00	1.13E+01	1
	Worst	2.03E+01	8.88E-16	8.88E-16	8.88E-16	7.99E-15	3.56E+00	1.48E+01	
	Mean	2.01E+01	8.88E-16	8.88E-16	8.88E-16	3.49E-15	3.39E+00	1.23E+01	
	Std	1.32E-01	0	0	0	2.46E-15	7.91E-02	1.07E+00	
F_9-500dim	Best	8.00E+03	**0**	**0**	**0**	**0**	1.34E-02	2.99E+02	1
	Worst	9.46E+03	0	0	0	0	1.76E-02	4.13E+02	
	Mean	8.62E+03	0	0	0	0	1.58E-02	3.65E+02	
	Std	3.37E+02	0	0	0	0	1.17E-03	3.04E+01	
F10-500dim	Best	1.56E+10	4.95E+01	4.97E+01	**4.28E-06**	5.25E+00	8.40E+01	1.00E+10	4
	Worst	2.06E+10	4.98E+01	4.99E+01	4.08E-04	1.99E+01	1.00E+02	1.00E+10	

	Mean	1.77E+10	4.97E+01	4.99E+01	1.21E-04	1.11E+01	9.41E+01	1.00E+10	
	Std	1.07E+09	6.62E-02	4.99E-02	1.06E-04	3.61E+00	3.50E+00	0.00E+00	
F_{11}-500dim	Best	7.43E+02	1.07E-124	**1.13E-183**	2.91E-108	2.44E-112	9.02E+00	2.64E+02	1
	Worst	8.80E+02	3.68E-104	9.23E-143	5.51E-05	1.80E-99	1.85E+01	3.18E+02	
	Mean	795.3246	1.64E-105	4.38E-144	1.84E-06	6.04E-101	1.33E+01	2.90E+02	
	Std	30.1765	6.85E-105	1.73E-143	1.01E-05	3.29E-100	2.93E+00	1.41E+01	
F_{12}-500dim	Best	1.50E+04	8.99E-138	**4.62E-261**	2.90E-23	1.53E+04	4.28E+02	9.32E+03	1
	Worst	2.91E+04	5.53E-95	1.54E-205	1.80E+04	1.82E+04	5.67E+02	1.64E+04	
	Mean	2.26E+04	1.89E-96	5.13E-207	5.62E+03	1.65E+04	4.76E+02	1.21E+04	
	Std	4.09E+03	1.01E-95	0.00E+00	7.16E+03	6.84E+02	3.51E+01	2.23E+03	

TABLE 9
RESULTS OF ESMFO COMPARED THOSE OF THE OTHER ALGORITHMS FOR THE CASE WITH DIMENSION SIZE=1000

Function	Algorithm	MFO	LMFO	ESMFO	HHO	WOA	BA	CS	RANK
F1-1000dim	Best	2.35E+06	1.72E-229	**0**	3.34E-206	3.88E-158	8.48E+00	1.12E+05	1
	Worst	2.56E+06	5.94E-184	2.56E-270	1.44E-179	9.93E-145	1.12E+01	1.39E+05	
	Mean	2.47E+06	2.28E-185	8.53E-272	6.64E-181	7.06E-146	9.90E+00	1.26E+05	
	Std	5.14E+04	0.00E+00	0.00E+00	0.00E+00	2.21E-145	6.65E-01	6.88E+03	
F2-1000dim	Best	Inf	4.69E-122	**1.39E-163**	2.08E-103	3.77E-115	1.86E+02	1.00E+10	1
	Worst	Inf	2.10E-99	3.23E-138	1.73E-91	1.17E-98	2.27E+02	1.00E+10	
	Mean	Inf	7.02E-101	1.08E-139	6.16E-93	5.72E-100	2.04E+02	1.00E+10	
	Std	NaN	3.84E-100	5.90E-139	3.16E-92	2.31E-99	9.77E+00	0.00E+00	
F3-1000dim	Best	9.56E+06	1.51E-201	**0**	2.89E-164	5.92E+07	1.25E+03	3.11E+06	1
	Worst	1.98E+07	2.93E-160	5.76E-254	2.31E-62	2.08E+08	1.74E+03	6.73E+06	
	Mean	1.38E+07	9.77E-162	1.92E-255	7.71E-64	1.18E+08	1.44E+03	5.26E+06	
	Std	2.57E+06	5.34E-161	0	4.22E-63	4.26E+07	1.08E+02	8.43E+05	
F4-1000dim	Best	9.92E+01	9.03E-109	**2.04E-165**	2.71E-108	3.54E+01	1.96E+00	3.53E+01	1
	Worst	9.97E+01	2.27E-80	6.32E-137	1.15E-89	9.95E+01	2.23E+00	4.60E+01	
	Mean	9.95E+01	7.57E-82	3.83E-138	3.96E-91	7.92E+01	2.09E+00	4.00E+01	
	Std	1.49E-01	4.15E-81	1.39E-137	2.09E-90	1.82E+01	6.61E-02	2.68E+00	
F5-1000dim	Best	1.03E+10	9.98E+02	9.98E+02	**1.33E-04**	9.91E+02	3.81E+03	2.83E+07	3
	Worst	1.16E+10	9.99E+02	9.98E+02	6.08E-01	9.95E+02	4.79E+03	5.74E+07	
	Mean	1.11E+10	9.98E+02	9.98E+02	1.20E-01	9.93E+02	4.25E+03	4.10E+07	
	Std	3.78E+08	7.42E-02	1.31E-01	1.57E-01	9.57E-01	2.90E+02	6.41E+06	
F6-1000dim	Best	1.59E+05	1.31E-06	**2.10E-06**	3.11E-06	9.85E-05	3.35E+02	5.00E+02	1
	Worst	1.87E+05	3.07E-04	2.39E-04	3.11E-04	1.10E-02	6.27E+02	8.97E+02	
	Mean	1.73E+05	6.83E-05	5.36E-05	6.67E-05	2.45E-03	4.90E+02	6.51E+02	
	Std	7.03E+03	7.45E-05	4.95E-05	7.04E-05	2.54E-03	6.35E+01	9.13E+01	
F7-1000dim	Best	1.43E+04	**0**	**0**	**0**	**0**	1.93E+03	8.08E+03	1
	Worst	1.52E+04	0	0	0	0	2.18E+03	8.70E+03	
	Mean	1.48E+04	0	0	0	0	2.10E+03	8.31E+03	

	Std	246.3925	0	0	0	0	6.37E+01	1.33E+02	
F8-1000dim	Best	2.00E+01	**8.88E-16**	**8.88E-16**	**8.88E-16**	**8.88E-16**	3.44E+00	1.11E+01	1
	Worst	2.05E+01	8.88E-16	8.88E-16	8.88E-16	7.99E-15	3.73E+00	1.57E+01	
	Mean	2.02E+01	8.88E-16	8.88E-16	8.88E-16	4.09E-15	3.59E+00	1.28E+01	
	Std	2.27E-01	0	0	0	2.85E-15	7.05E-02	1.04E+00	
F9-1000dim	Best	2.15E+04	**0**	**0**	**0**	**0**	7.29E-02	9.36E+02	1
	Worst	2.34E+04	0	0	0	1.11E-16	9.97E-02	1.38E+03	
	Mean	2.23E+04	0	0	0	3.70E-18	8.08E-02	1.10E+03	
	Std	4.72E+02	0	0	0	2.03E-17	5.27E-03	1.10E+02	
F10-1000dim	Best	4.63E+10	9.95E+01	9.97E+01	**1.54E-08**	1.14E+01	1.97E+02	1.00E+10	3
	Worst	5.23E+10	9.98E+01	9.99E+01	5.21E-04	4.40E+01	2.16E+02	1.00E+10	
	Mean	4.99E+10	9.97E+01	9.98E+01	9.18E-05	2.31E+01	2.08E+02	1.00E+10	
	Std	1.43E+09	7.54E-02	5.67E-02	1.13E-04	6.98E+00	5.00E+00	0.00E+00	
F11-1000dim	Best	1.97E+03	5.96E-126	**4.34E-171**	2.15E-109	4.21E-114	4.62E+01	5.85E+02	1
	Worst	2.21E+03	3.01E-100	1.18E-141	4.69E-04	1.05E-97	7.18E+01	6.97E+02	
	Mean	2.07E+03	1.60E-101	5.10E-143	1.56E-05	3.53E-99	6.08E+01	6.52E+02	
	Std	5.09E+01	6.30E-101	2.23E-142	8.57E-05	1.92E-98	6.57E+00	2.74E+01	
F12-1000dim	Best	3.42E+04	2.45E-126	**4.94E-258**	6.58E-04	3.13E+04	8.86E+02	1.98E+04	1
	Worst	6.07E+04	2.05E-89	1.58E-193	3.54E+04	3.46E+04	1.07E+03	3.37E+04	
	Mean	4.75E+04	9.58E-91	5.79E-195	2.11E+04	3.32E+04	9.87E+02	2.77E+04	
	Std	7.74E+03	3.91E-90	0	1.39E+04	9.94E+02	4.93E+01	4.72E+03	

The effects of using 50, 100, 200, and 500 dimensions are evaluated. We randomly select several functions from the 12 total test functions to compare the optimal values obtained by the algorithms in three-dimensional stereo histograms and record the optimal fitness values of each algorithm in different dimensions, as shown in Figs. 43-48. From the graph, we can see that ESMFO has obvious advantages over the MFO, WOA, BA and CS algorithms in several test functions. As the number of dimensions increases, the convergence accuracy of each of the four algorithms decreases successively. Compared with the accuracy of the LMFO and HHO algorithms, this difference is not obvious, but we can see from the local comparison graphs for the f2 function (Figs. 45- 46), the performance of ESMFO is better than those of the LMFO and HHO algorithms. The results show that ESMFO is superior to the other algorithms in all dimensions. ESMFO provides good results for all functions except f5 and f10. The results show that the performance of the ESMFO algorithm decreases as the number of dimensions increases, but it is better than those of the other algorithms.

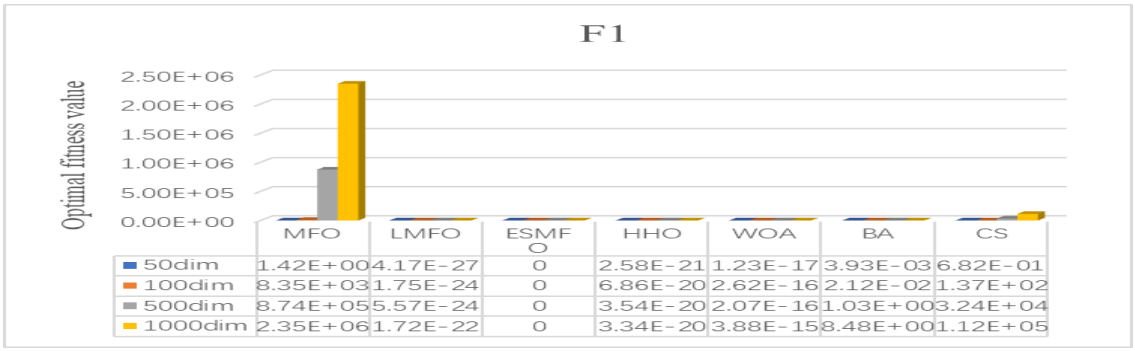
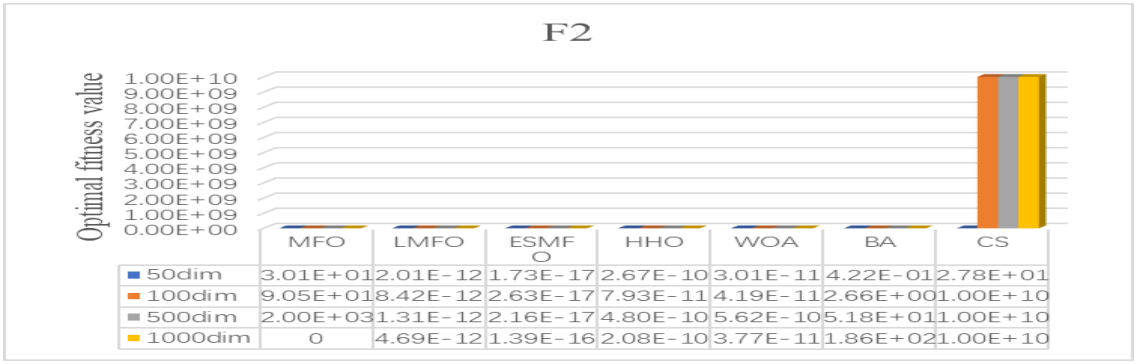


FIGURE 43 Comparison of the optimal fitness values obtained by the algorithms based on the number of dimensions of f_1



	MFO	LMFO	ESMFO	HHO	WOA	BA	CS
50dim	3.01E+01	2.01E-12	1.73E-17	2.67E-10	3.01E-11	4.22E-01	2.78E+01
100dim	9.05E+01	8.42E-12	2.63E-17	7.93E-11	4.19E-11	2.66E+00	1.00E+10
500dim	2.00E+03	1.31E-12	2.16E-17	4.80E-10	5.62E-10	5.18E+01	1.00E+10
1000dim	0	4.69E-12	1.39E-16	2.08E-10	3.77E-11	1.86E+02	1.00E+10

FIGURE 44 Comparison of the optimal fitness values obtained by the algorithms based on the number of dimensions of f_2

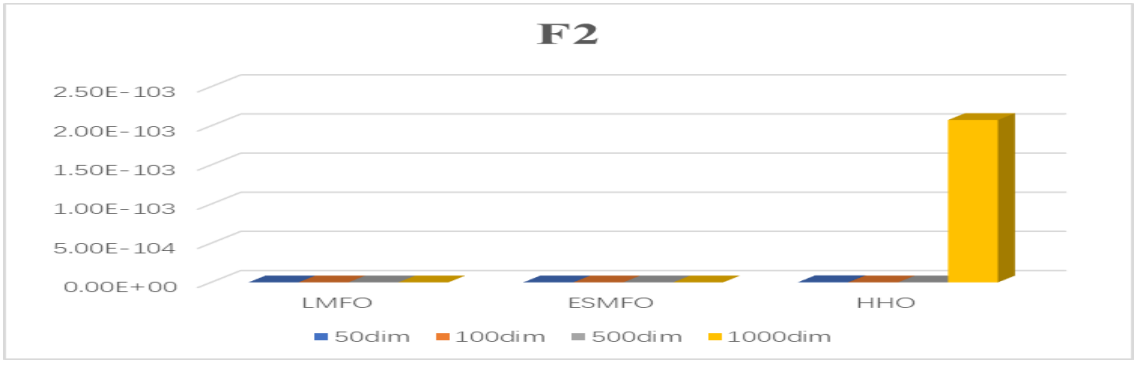


FIGURE 45 Local comparison of the optimal fitness values obtained by the algorithms based on the number of dimensions of f_2

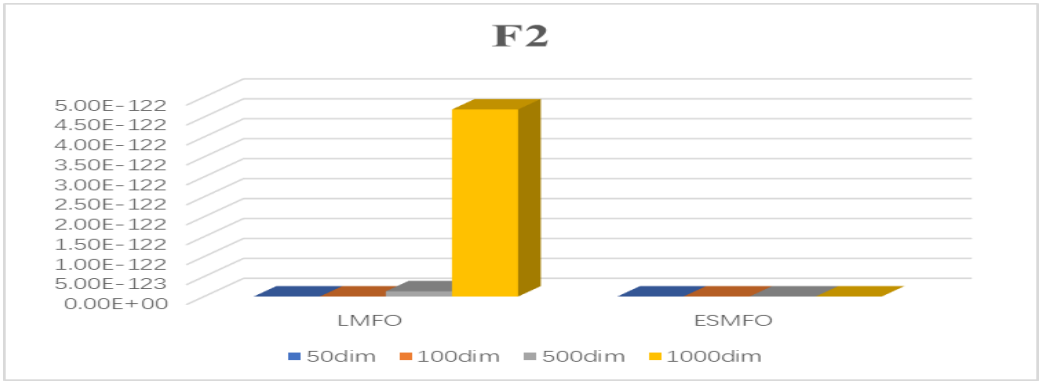
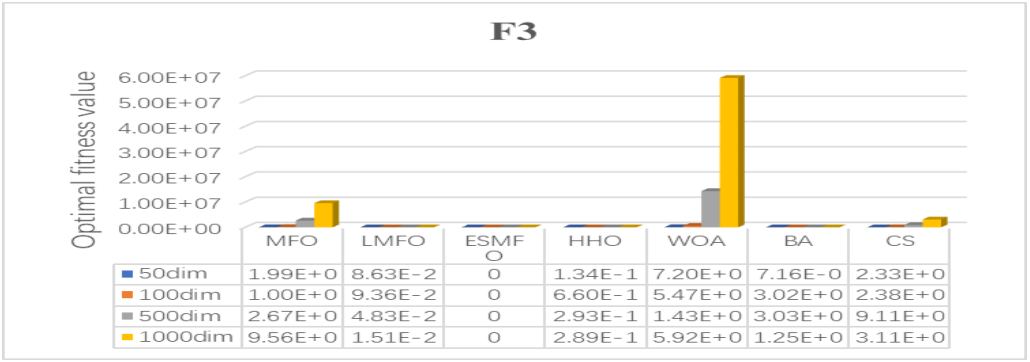


FIGURE 46 Local comparison of the optimal fitness values obtained by the algorithms based on the number of dimensions of f_2



	MFO	LMFO	ESMFO	HHO	WOA	BA	CS
50dim	1.99E+0	8.63E-2	0	1.34E-1	7.20E+0	7.16E-0	2.33E+0
100dim	1.00E+0	9.36E-2	0	6.60E-1	5.47E+0	3.02E+0	2.38E+0
500dim	2.67E+0	4.83E-2	0	2.93E-1	1.43E+0	3.03E+0	9.11E+0
1000dim	9.56E+0	1.51E-2	0	2.89E-1	5.92E+0	1.25E+0	3.11E+0

FIGURE 47 Comparison of the optimal fitness values obtained by the algorithms based on the number of dimensions of f_2

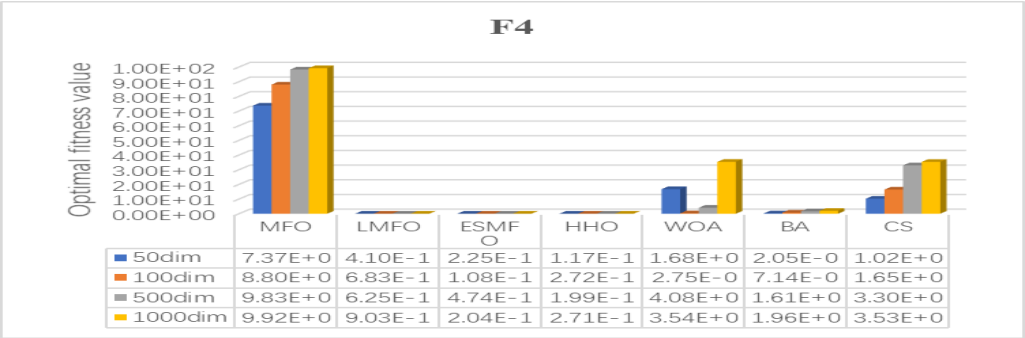


FIGURE 48 Comparison of the optimal fitness values obtained by the algorithms based on the number of dimensions of f_4

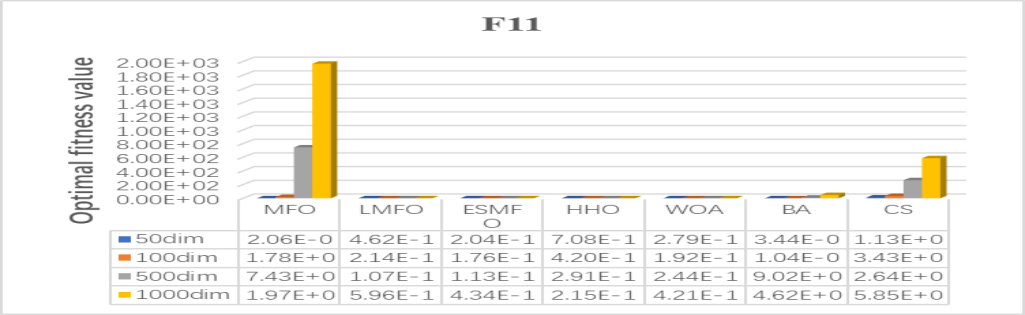


FIGURE 49 Comparison of the optimal fitness values obtained by the algorithms based on the number of dimensions of f_{11}

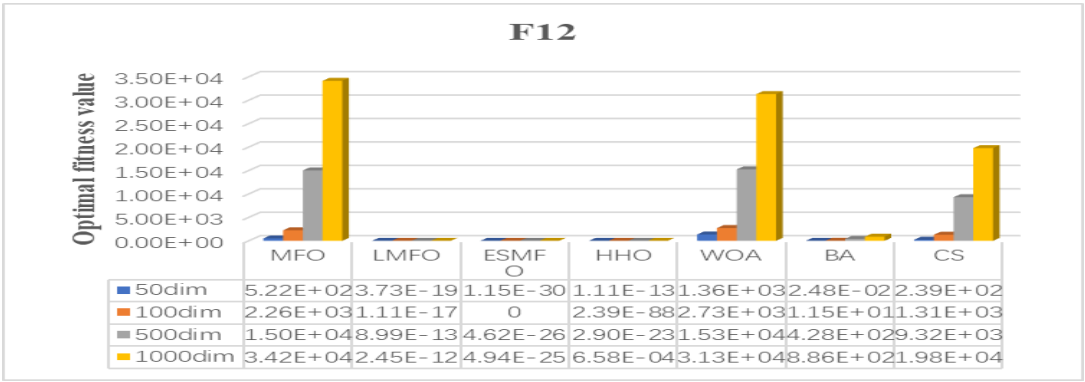


FIGURE 50 Comparison of the optimal fitness values obtained by the algorithms based on the number of dimensions of f_{12}

4.5. Effect of the Parameter B on the Performances of Mfo and Esmfo

In the basic MFO algorithm, the parameter b defines the shape of the spiral. The spiral chosen for this paper is a logarithmic spiral whose equation is as follows:

$$r = e^{b\theta} \tag{18}$$

where r and θ are the polar coordinates of the curve that define the distance from the origin and the angle from the x-axis, respectively, and b is any real positive constant. The rate of change of the radius with respect to θ is given as:

$$\frac{dr}{d\theta} = b \cdot e^{b\theta} = b \cdot r \tag{19}$$

From these equations, it is observed that b is the parameter that defines how tightly and in which direction a spiral is spun. There are two extreme cases for the values of b: for b = 0, the spiral is converted into a circle, and for b = ∞, the spiral becomes a straight line. The value chosen in the original article about the MFO algorithm for this parameter is 1, and no parametric study for its use has been given. Therefore, in order to evaluate the effect of b on the performance of the algorithm, four different values of this parameter are compared.

Experimental analysis: The results are obtained for different values of b, such as 0.5, 1 and 2. The population size and number of iterations are set as 30 and 1000, respectively, during the experimental analysis. This analysis is conducted on both the MFO and ESMFO algorithms. First, for MFO with b = 0:5, the algorithm achieves the best results for six functions in terms of the best results, worst results, average results and standard deviations. When the

experiment is performed with b = 1, five functions yield good results, whereas at b = 2 only one functions yields good results. Eight functions yield highly competitive results for all values of b. Furthermore, for the ESMFO algorithm, it is concluded from Table 10 that twelve functions offer highly competitive results for all values of b. The five functions f2, f4, f5, f10 and f12 are successful in obtaining global optima for b = 1; on the other hand, for b = 0:5, only 3 out of 21 functions, i.e., f6, f11 and f14, yield good results. For b = 2, only one function is successful in obtaining optimal results (Table 10).

Inference: In the basic MFO algorithm, the parameter b basically defines the shape of the spiral, and its value is set to 1. The author does not provide any explanation for the chosen value. Therefore, an experimental analysis is conducted to analyze the effect of b on the algorithm. Different values of b are chosen: 0.5, 1 and 2. It is clear from the above analysis that b = 1 is the ideal value for ESMFO.

TABLE 10
EXPERIMENTAL ANALYSIS WITH DIFFERENT VALUES OF b

Function	Performance	b=0.5		b=1		b=2	
		MFO	ESMFO	MFO	ESMFO	MFO	ESMFO
f1	Best	2.65E-06	**0**	**2.76E-06**	**0**	1.12E-03	**0**
	Worst	2.00E+04	8.50E-281	1.33E-02	1.49E-289	1.00E+04	5.14E-289
	Average	1.00 E+03	2.87E-282	7.49 E-04	4.98E-291	2.00 E+03	1.89E-290
	SD	4.26E+03	0	2.41E-03	0	4.07E+03	0
f2	Best	**3.26 E-05**	5.40E-178	9.67 E-04	**7.59E-179**	1.00E+01	6.89E-175
	Worst	5.00E+01	1.71E-143	6.00E+01	4.67E-146	1.00E+02	4.44E-145
	Average	2.37E+01	5.71E-145	3.10E+01	1.56E-147	4.87E+01	1.48E-145
	SD	1.43E+01	3.13E-144	1.71E+01	8.52E-147	2.61E+01	8.11E-145
f3	Best	**4.82E+02**	**0**	8.48E+02	3.93E-178	8.93E+02	**0**
	Worst	3.35E+04	2.99E-288	5.80E+04	1.89E-148	5.83E+04	1.26E-269
	Average	1.48E+04	9.97E-290	2.04E+04	7.26E-150	2.65E+04	4.22E-271
	SD	107E+04	0	1.46E+04	3.49E-149	1.51E+04	0
f4	Best	5.83E+01	1.93E-175	**4.26E+01**	**3.34E-176**	2.01E+01	6.53E-165
	Worst	8.62E+01	7.63E-142	8.17E+01	4.45E-175	5.88E+01	3.58E-135
	Average	7.19E+01	2.54E-143	6.66E+01	1.48E-131	4.08E+01	1.94E-136
	SD	8.01E+00	1.39E-142	1.08E+01	8.12E-131	9.28E+00	6.53E-136
f5	Best	1.62E+01	2.65E+01	**1.36E+01**	**2.64E+01**	7.09E+01	2.66E+01
	Worst	9.01E+04	2.88E+01	9.00E+04	2.80E+01	8.00E+07	2.87E+01
	Average	6.35E+04	2.75E+01	6.45E+04	2.72E+01	8.02E+06	2.73E+01
	SD	2.28E+04	6.81E-01	2.27E+04	4.40E-01	2.24E+07	5.12E-01
f6	Best	**2.88E-02**	**2.06E-07**	3.17E-02	2.01E-06	5.91E-02	1.73E-06
	Worst	5.11E+01	2.83E-04	1.08E+01	1.44E-04	4.84E+01	2.03E-04
	Average	4.42E+00	5.88E-05	1.49E+00	5.32E-05	6.31E+00	4.79E-05
	SD	1.09E+00	6.62E-05	2.99E+00	3.81E-05	1.02E+01	5.23E-05
f7	Best	1.12E+02	**0**	8.46E+01	**0**	**2.98E+01**	**0**
	Worst	2.77E+02	0	2.24E+02	0	2.18E+02	0

	Average	1.69E+02	0	1.65E+02	0	9.34E+01	0
	SD	3.86E+01	0	3.75E+01	0	4.21E+01	0
f8	Best	1.65E+00	**8.88E-16**	**0.81E-02**	**8.88E-16**	3.29E-02	**8.88E-16**
	Worst	1.99E+01	8.88E-16	1.99E+01	8.88E-16	1.99E+01	8.88E-16
	Average	1.24E+01	8.88E-16	1.43E+01	8.88E-16	1.86E+01	8.88E-16
	SD	7.63E+00	0	7.98E+00	0	5.04E+00	0
f9	Best	**7.91E-06**	**0**	2.55E-05	**0**	2.98E-03	**0**
	Worst	1.80E+02	0	1.81E+02	0	1.81E+02	0
	Average	2.41E+01	0	3.32E+01	0	2.11E+01	0
	SD	1.69E+01	0	6.05E+01	0	5.83E+01	0
f10	Best	**1.00E-04**	1.43E+00	9.42E-04	**9.80E-01**	2.28E-04	1.08E-00
	Worst	8.65E+00	2.97 E+00	8.97E-01	2.97E+00	1.68E+00	2.80E+00
	Average	8.52E-01	2.65 E+00	1.04E-01	2.48E+00	1.07E-01	2.44E+00
	SD	1.73E+00	3.49 E-01	2.01E-01	5.45E-01	3.01E-01	4.03E-01
f11	Best	**4.71E-06**	**1.07E-180**	4.19E-05	2.11E-179	3.11E-04	3.66E-176
	Worst	2.42E+01	5.28E-147	1.93E+01	4.02E-140	1.98E+01	1.69E-140
	Average	3.97E+00	1.76E-148	4.83E+00	1.34E-141	8.51E+00	5.66E-142
	SD	5.14E+00	9.63E-148	5.26E+00	7.33E-141	6.58E+00	3.09E-141
f12	Best	1.86E+01	7.02E-322	**1.14E+01**	**0**	2.17E+01	8.65E_319
	Worst	6.21E+02	1.36E-245	8.16E+02	2.35E-264	9.12E+02	4.81E-259
	Average	2.40E+02	8.37E-247	2.81E+02	7.83E-266	3.35E+02	1.60E-260
	SD	1.79E+02	0	2.67E+02	0	3.15E+02	0
f13	Best	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**
	Worst	1.08E+01	1.27E+01	1.17E+01	1.27E+01	5.93E+00	1.27E+01
	Average	2.87E+00	3.22E+00	2.48E+00	4.98E+00	1.66E+00	4.59E+00
	SD	2.49E+00	3.34E+00	2.37E+00	4.53E+00	1.33E+00	4.02E+00
f14	Best	**3.56E-04**	3.08E-04	6.18E-04	3.09E-04	6.56E-04	**3.07E-04**
	Worst	2.03E-02	6.79E-04	2.04E-02	5.83E-04	2.04E-02	1.23E-03
	Average	1.81E-03	3.44E-04	2.03E-03	3.48E-04	2.37E-03	4.06E-04
	SD	3.77E-03	8.00E-05	3.73E-03	7.61E-05	4.91E-03	2.39E-04
f15	Best	**-1.306**	**-1.306**	**-1.306**	**-1.306**	**-1.306**	**-1.306**
	Worst	-1.306	-1.306	-1.306	-1.306	-1.306	-1.306
	Average	-1.306	-1.306	-1.306	-1.306	-1.306	-1.306
	SD	6.78E-16	2.53E-06	6.78E-16	1.90E-06	6.78E-16	2.25E-06
f16	Best	**0.39789**	**0.39789**	**0.39789**	**0.39789**	**0.39789**	**0.3979**
	Worst	0.39789	0.39823	0.39789	0.39813	0.39789	0.39862
	Average	0.39789	0.39797	0.39789	0.39795	0.39789	0.39798
	SD	0	1.07E-04	0	6.68E-05	0	1.40E-04
f17	Best	**3**	**3**	**3**	**3**	**3**	**3**

	Worst	3	3.0005	3	3.0004	3	3.0002
	Average	3	3.0001	3	3.0001	3	3.0001
	SD	1.86E-15	1.58E-04	1.59E-15	8.62E-05	2.24E-15	7.41E-05
f18	Best	**-0.30048**	**-0.30048**	**-0.30048**	**-0.30048**	**-0.30048**	**-0.30048**
	Worst	-0.30048	-0.30048	-0.30048	-0.30048	-0.30048	-0.30048
	Average	-0.30048	-0.30048	-0.30048	-0.30048	-0.30048	-0.30048
	SD	2.26E-16	2.26E-16	2.26E-16	2.26E-16	2.26E-16	2.26E-16
f19	Best	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**
	Worst	-9.36E-01	-1	-9.36E-01	-1	-9.36E-01	-1
	Average	-9.72E-01	-1	-9.62E-01	-1	-9.72E-01	-1
	SD	3.21E-02	0	3.18E-02	0	3.21E-02	0
f20	Best	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**
	Worst	-9.90E-01	-1	-9.90E-01	-1	-9.90E-01	-1
	Average	-9.91E-01	-1	-9.92E-01	-1	-9.91E-01	-1
	SD	2.47E-03	0	3.95E-03	0	2.47E-03	0
f21	Best	**-1**	**-1**	**-1**	**-1**	**-1**	**-1**
	Worst	-1	-0.99974	-1	-1	-1	-0.99957
	Average	-1	-0.99994	-1	-1	-1	-0.99993
	SD	0	5.72E-05	0	8.98E-05	0	8.62E-05

4.6. Statistical Testing

The statistical testing used includes the Wilcoxon rank-sum test [38][39], which tests the performances of two different algorithms. Here, the performance of ESMFO is checked with respect to those of the BA, CS, WOA, HHO, MFO and LMFO algorithms. Basically, this test is used to find the difference in performance between two algorithms, and at the end of this test we obtain a *p*-value as the output. This *p*-value signifies the significance of the algorithm being tested. If the *p*-value is lower than 0.05, then the corresponding algorithm is said to be statically significant. The data shown by the red horizontal line represent a large similarity between ESMFO and another algorithm. It can be seen from the table that ESMFO has similarity with two or more algorithms for functions f5, f6, and f18. The data obtained from Table 5 and Table 6 can be seen in these three functions. The algorithm reaches the convergence state and the optimization obtains the optimal value. It is proven that the comparison of the data is statistically significant. The results shown in Table 10 demonstrate that ESMFO performs better than the competing algorithms on twenty functions. An NA value for any algorithm shows the algorithm's superiority over other algorithms, and the *p*-values of these algorithms are given with respect to the superior algorithm. The statistical results prove that ESMFO is a better algorithm than the others (Table 11).

TABLE 11
P-VALUES OF THE WILCOXON RANK-SUM TEST OVER ALL RUNS ($P \geqslant 0.05$ ARE UNDERLINED)

Function	MFO	LMFO	HHO	WOA	BA	CS
1	2.92E-11	2.92E-11	2.92E-11	2.92E-11	2.92E-11	2.92E-11
2	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	1.27E-11
3	3.00E-11	3.00E-11	3.00E-11	3.00E-11	3.00E-11	3.00E-11
4	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
5	3.02E-11	**0.8883**	3.02E-11	**0.7506**	0.0042	3.02E-11
6	3.02E-11	**0.8534**	**0.9117**	1.69E-09	3.02E-11	3.02E-11
7	1.21E-12	NA	NA	NA	1.21E-12	1.21E-12

8	1.21E-12	NA	NA	2.51E-07	1.21E-12	1.21E-12
9	1.21E-12	NA	NA	NA	1.21E-12	1.21E-12
10	3.02E-11	8.99E-11	3.02E-11	3.02E-11	0.9823	3.02E-11
11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
12	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
13	5.39E-07	**0.836**	9.76E-10	1.61E-06	5.09E-06	1.21E-12
14	3.50E-09	0.0012	0.0933	2.60E-08	**1**	3.02E-11
15	1.21E-12	1.11E-04	2.98E-11	3.02E-11	1.33E-10	1.21E-12
16	1.21E-12	2.57E-07	4.50E-11	7.77E-12	3.02E-11	1.21E-12
17	2.00E-11	**0.2519**	3.02E-11	9.51E-06	0.0012	1.57E-11
18	1.21E-12	**0.2009**	**0.0824**	**0.3953**	8.48E-09	1.21E-12
19	3.06E-04	NA	NA	0.0216	1.21E-12	5.37E-06
20	2.57E-13	NA	NA	1.44E-04	1.21E-12	1.21E-12
21	1.21E-12	5.86E-06	1.46E-10	4.08E-11	**0.379**	1.21E-12

4.7. Pressure Vessel Design Problem

This problem was first proposed by Kannan and Kramer (1994) to minimize the total cost of the materials and the processes of forming and welding cylindrical containers. The schematic view of the pressure vessel design problem is shown in **Fig. 51**. There are four design variables:

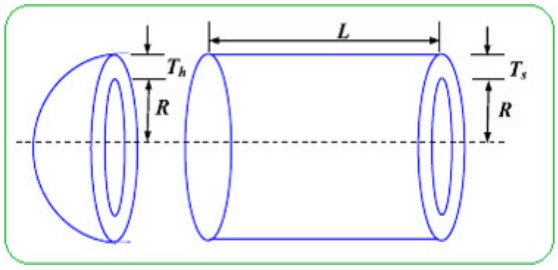


Fig. 51 Pressure vessel design problem

T_s (x_1 , thickness of the shell); T_h (x_2 , thickness of the head); R (x_3 , inner radius); and L (x_4 , length of the cylindrical section without considering the head). R and L are continuous variables, whereas T_s and T_h are integer values that are multiples of 0.0625 in. The mathematical formulation of this problem is described below:

Consider $\vec{x} = [x_1, x_2, x_3, x_4] = [T_s, T_h, R, L]$;

Minimize

$$f(\vec{x}) = 0.6224 x_1 x_3 x_4 + 1.7781 x_2 x_3^2 + 3.1661 x_1^2 x_4 + 19.84 x_1^2 x_3 \quad (20)$$

Subject to:

$$g_1(\vec{x}) = -x_1 + 0.0193 x_3 \le 0,$$

$$g_2(\vec{x}) = -x_3 + 0.00954 x_3 \le 0,$$

$$g_3(\vec{x}) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0,$$

$$g_4(\vec{x}) = x_4 - 240 \le 0,$$

where

$1 \times 0.0625 \le x_1$, $x_2 \le 99 \times 0.0625$, $10.0 \le x_3$, and $x_4 \le 200.0$.

The structure of the pressure vessel is optimized with IMFO and the results are compared to those of the MFO, GSA, PSO, GA, ES, DE, IMFO and ACO algorithms. The results in Table 12 show the superiority of ESMFO with respect to the other algorithms.

TABLE 12
COMPARISON RESULTS FOR THE PRESSURE VESSEL DESIGN PROBLEM

Algorithm	Optimal values for the variables				Optimum cost
	*T*s	*T*h	R	L	
ESMFO	0.78246	0.37243	40.21238	177.897865	5867.4375
IMFO[50]	0.77455	0.38320	40.31962	200.00000	5870.12398
MFO [22]	0.8125	0.4375	42.098445	176.636596	6059.7143
GSA [16]	1.1250	0.6250	55.988659	84.4542025	8538.8359
PSO [7]	0.8125	0.4375	42.091266	176.746500	6061.0777
DE [6]	0.8125	0.4375	42.098411	176.637690	6059.7340
ACO [9]	0.8125	0.4375	42.103624	176.572656	6059.0888
ES [51]	0.8125	0.4375	42.098087	176.640518	6059.7456
GA [52]	0.8125	0.4375	42.097398	176.654050	6059.9463

The proposed approach is verified on 21 benchmark functions and compared with the LMFO, HHO, MFO, BA, WOA and CS algorithms. It is clear from the results (comparison between the improved ESMFO algorithm and the other algorithms based on solution quality) that this approach is successful in obtaining optimal results and avoiding local minima. Additionally, there is wide discussion about parameter b, whose explanation is not given in the original paper; this paper clearly describes the importance of b. In this paper, the effect of problems with different numbers of dimensions (i.e., 50, 100, 500 and 1000 dimensions) on MFO, ESMFO and other algorithms is also analyzed. In addition, the pressure vessel design

problem is examined. It is found that the proposed ESMFO algorithm yields highly competitive results for most of the test functions considered.

5. Conclusions and Future Work

In this paper, by introducing the energy factor from the Harris hawks algorithm, the flame energy of the MFO algorithm is segmented, and the energy-segmented moth-flame optimization algorithm (ESMFO) is proposed. The algorithm is applied to 21 benchmark functions, along with several mainstream heuristic algorithms. The simulation experiment is carried out and the influence of changes in the number of dimensions on the algorithm is considered. The results are statistically tested (Wilkerson value test), and the statistical conclusions that the ESMFO algorithm exhibits high convergence speed and high convergence accuracy are obtained. For future work, there are many ways by which ESMFO can be efficiently improved. These include applying ESMFO in different applications, such as PV parameter estimation, neural network applications, image processing applications, text and data mining applications, big data applications, signal denoising, recourse management applications, network applications, industry and engineering applications, other benchmark test functions, smart home applications, feature selection, image segmentation, task scheduling, and more. The algorithm can also be extended to real-world applications that depend on binary, discrete, and multiobjective optimization. Moreover, the performance of ESMFO can be improved by combining it with Levy fight, disruption, mutation, other stochastic components, local search methods, global search methods, and other evolutionary operators. Additionally, binary and multiobjective versions can be developed to solve practical optimization problems.

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Compliance With Ethical Standards

Conflict of interest The authors declare that they have no conflicts of interest.

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