# Model Order Reduction of Transmission Line Model

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*Abstract:* Transmission Line model are an important role in the electrical power supply. Modeling of such system remains a challenge for simulations are necessary for designing and controlling modern power systems.In order to analyze the numerical approach for a benchmark collection Comprehensive of some needful real-world examples, which can be utilized to evaluate and compare mathematical approaches for model reduction. The approach is based on retaining the dominant modes of the system and truncation comparatively the less significant once.as the reduced order model has been derived from retaining the dominate modes of the large-scale stable system, the reduction preserves the stability. The strong demerit of the many MOR methods is that, the steady state values of the reduced order model does not match with the higher order systems. This drawback has been try to eliminated through the Different MOR method using sssMOR tools. This makes it possible for a new assessment of the error system Offered that the Observability Gramian of the original system has as soon as been thought about, an $H_\infty$ and $H_2$ error bound can be calculated with minimal numerical effort for any minimized model attributable to The reduced order model (ROM) of a large-scale dynamical system is essential to effortlessness the study of the system utilizing approximation Algorithms. The response evaluation is considered in terms of response constraints and graphical assessments. the application of Approximation methods is offered for arising ROM of the large-scale LTI systems which consist of benchmark problems. The time response of approximated system, assessed by the proposed method, is also shown which is excellent matching of the response of original system when compared to the response of other existing approaches .

Key-Words: Benchmarks, Order Reduction, Error Estimation, Transmission Line Model, Balanced Truncation.
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## 1. Introduction

Transmission line models for simulations are compulsory for designing and controlling modern power systems. present day exiting electricity structures should be reform to the superior energy grid.To make the strength grid become ‘superior’, specifically in phrases of stability and versatility, one have to need to make a controller which successful to control the above issue after integration of HDG-hybrid distribution generation system into the utility grid However[1][2][3], the orders of these models are very big as well as their simulations are acknowledged to be computationally intensive Simulation of transmission line transients caused by faults and switching operations are needed for computation of surge responses, Corona effects[4]. Model-order Reduction techniques have been developed to reduce the computational complexity of simulations and controller synthesis for realtime applications[5][6].MOR of big-scale systems has been an essential topic area in control engineering for several years.The most important problem in any appearance at concerning the dynamic behaviors of the Higher-dimensional system are all around and exist in multiple fields such as complicated transportation device, ecological systems, electrical power device aeronautics, hydraulic. And adequately of extra and so on. Or a hybrid of these, a system is stated to be large if it could be decoupled into a number of the coordinated device or modest -scale system for both computational or sensible reasons[1][7] Otherwise, a system is an enormous scale while its size are so huge, such that the traditional strategies of modeling, analysis, manage system design and computation fail to present appropriate solutions with affordable computational efforts. The evaluation of such physical device begins through building up of the model which can be taken into consideration as an enthusiastic representation of such structures, the mission of control engineer instigates with the formulation of a model and evaluation, In this first segment we present a high stage negotiation on computational technological know-how, and the essential for dense models of occurrence observed in perspective and industry[8]. We argue that rather much more sophisticated problems can be addressed by way of using most recent computing technology and innovative algorithms, however, that there is a want for model order reduction so that you can deal with even greater complex concerns (Schilders,2008)[9].

Approximation Techniques play an essential foreword nowadays Large-Scale layout. Direct numerical simulation of high order dynamical system is commonly used to validate the design and examine the behavior of the region of the system earlier than consultation and implementation. One of the principal studies regions in the large-scale real-global system[8],[9]. The Dynamical system simulation is to apply mathematical modeling to forecast the manners of a real-world system. A system is typically represented by way of a fixed of partial differential equations (PDEs)- Distributed parameter structures or ordinary differential equations (ODEs)- Lumped Parameter Systems. Therefore, the dynamical system simulation without a doubt includes solving large-scale ODEs, which now and again takes numerous days or maybe weeks. Consequently, quickly and unique system reproduction algorithms are proper to quicken the reproduction cycle[12].

The MOR is numerical method for order Decrease of large -scale system to enhance the simulation with a suitably easy system, which captures the main characteristics of the original character one, which reduces the complication of the original large-scale system and produces a ROM (reduced-order model) to characterize the original one[13].in this present basically three MOR (model order reduction) methods, however, there may be no technique that gives the quality consequences for all of the structures[14]. So, each system makes use of the quality approach in keeping with its application. So, there is nevertheless want for novel model order reduction approach [13]. This paper gives innovative MOR procedures based on fully MATLAB environment for the linear systems and non-linear. MOR is one area wherein "approximation-based techniques" have tested effective applications. We propose MATLAB Software (sssMOR toolbox)[15] [16]-based totally algorithms for model order reduction that dynamically studies the outstanding explanation. We put in strength our method and compare its techniques. Our method to progress run-time without any offensive the ordinary accuracy.

## 2. Problem Statement

In this section, the problem statement for the MOR of LTI large-scale dynamical system with its detailed methodological procedure has been presented.

### 2.1. Brief descriptions

This short, discussion is focused on computing the decreased -order approximation estimation for linear dynamical systems are mainly regarded as more than their input-output plot $\delta$ : u →y, mapping inputs u to outputs y by way of a state-space realization provided as LTI system

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

with an associated transfer function matrix of the system

Original system $G(s) = C(sI_n - A)^{-1}B + D$

where $x \in \mathbb{R}^n$ is the system state, $y \in \mathbb{R}^p$ is the system output, u(t) is the manage input, A ∈ $\mathbb{R}^{n\times n}$, B ∈ $\mathbb{R}^{n\times m}$, C ∈ $\mathbb{R}^{p\times n}$ and $D \in \mathbb{R}^{p\times m}$. The systems are all stable and minimal phase and the number of state variables n is hence the order of the system[17].

We mention to $\delta$ as a SISO-single-input/single-output system when *m*= *p* = 1 (scalar-valued input, scalar-valued output) and as a multivariable system otherwise[18]. Systems of the form (1) with enormously the large state-space dimension n follow in several orders; see and for an assembly of such examples. In spite of the high order dimension, as a rule, the state space directions, x(t), slash near subspaces with expressively lower[19] the size and develop in behaviors that do not completely acquire the state space[8]. We need the reduced input-output plot, $\delta_r$ : $\mathbf{u} \rightarrow y_r$, to be close to $\delta$ in a suitable sense[20]. Being a smaller version of the original dynamical model, the input-output plot $\delta_r$ is described by the reduced system in state-space form as in model reduction, one attempts to the invention of a reduced-order model of the system.

The essential knowledge behind model order reduction is to originate a low-order approximation $G_r(s)$ for a given high-order system $G(s)$ The Reduced order model (ROM) can be represented as

$$\dot{x}_r(t) = A_r x_r(t) + B_r u_r(t)$$
$$y_r(t) = C_r x_r(t) + D_r u_r(t)$$

of the order $n_r \ll n$ such that the transfer function of the original System

Reduced system $G_r = C_r(sI_{n_r} - A_r)^{-1}B_r + D_r$

approaches original system G(s) in a specific sense, and model reduction approaches diverge characteristically in the error estimated that is being minimized. Throughout this brief, subscript "r " is used to denote parameters related to the ROMs[21].

- A is the matrix of the large system so the eigenvalues (or at least the nearby ones to the $j\omega$

axis), which are also the poles of large-scale system G(s)

- control theory to describe controllability Gramian $G_c$ and observability Gramian $G_o$ of the higher-order system, the explanations of the Lyapunov equations is defined as

$$AG_c + G_c A^T + BB^T = 0,$$
$$A^T G_o + G_o A + CC^T = 0$$

- singular values of the system are a Hankel plot so-called HSV, which are also the square-roots of the eigenvalues of controllability Gramian, observability Gramian ( $G_c G_o$ )
- the transfer function as a function of frequency called the frequency response for the largest singular value.

$$\sigma(\omega) = \|G(j\omega)\|2$$

These characteristics can be associated with these of the decreased order model $G_r(s)$, When they are presented, in this short, a benchmark example is regarded as a test system. Such models are characterized by linear with the dynamical system, we give all of the above properties for the benchmark examples. For each example, we provide the matrix model $\{A, B, C, and, D\}$ [22][23]. In this section, we have introduced briefly these basic parameters of the benchmarks discussed. Large Scale Dynamical System Examples

In this part, we apply the procedures mentioned above to four different dynamical systems: Including control systems, biological structures, analog circuits, and multi-agent systems Since most of the benchmarks are high-dimensional, their dynamic equations cannot be offered in detail on this paper. We refer readers to [16] for in addition info and derivations, in addition to our provided supplementary material Benchmarks example for the order- reduction concept method in which N is the measurement of the system order; inputs as define m and outputs p respectively [18][11].

Notwithstanding these advances, reachability examination of huge scale frameworks with hundreds to thousands of measurements is as yet infeasible even for LTI- linear time-invariant systems, i.e., with no discrete exchanging conduct. It is critical to growing new strategies and tools that can be utilized to confirm the security of such high-dimensional frameworks, which more often than not exist in an expansive scope of fields and applications[24]

## 3. Application of Reduction Methods

**Balanced truncation Algorithm**[25][26]

1.Calculate $X = RR^T$ and $Y = LL^T$

2.Calculate the SVD

$$L^T R = [U_1, U_2]\begin{bmatrix}\Sigma_1 & \\ & \Sigma_2\end{bmatrix}[V_1, V_2]^T,$$

With $\Sigma_1 = diag(\xi_1, ...., \xi_l)$,

$\Sigma_2 = diag(\xi_{l+1}, ..., \xi_n)$.

3.Calculate the Reduced order model

$$(A_r, B_r, C_r, D_r) = (W^T AT, W^T B, CT, D)$$

With

$$W = LU_1 \Sigma_1^{-1/2} \in \mathbb{R}^{n\times l}, T = RV_1 \Sigma_1^{-1/2} \in \mathbb{R}^{n\times l}.$$

**Properties**

- $(A_r, B_r, C_r, D_r)$ is asymptotically stable
- Error bound:
  $\|G_r - G\|_{H_\infty} \le 2(\xi_{l+1} + ... + \xi_n)$
- Need to solve large -scale Lyapunov equations

### 3.3. Balanced Truncation (BT) algorithm

In this section, describe the methodology of Balanced Truncation, firstly presented by B. C. Moore [25], is based on control theory, somewhere one basically examines in what way a system can be explored and how its response can be found. In such manner, the essential thought of the BT[27] is to initially group, which states x is difficult to influence and which states x are hard to gather from staring at the yield y, at that point to redevelop the framework with the end goal that the 2 units of states agree and inevitably truncate the framework to such an extent that the diminished system does now not the interface significance to these complex cases. balanced truncation approach basically applies the truncation procedure to the BRT-balanced realization techniques (A, B, C, and D) of an original system G for model order reduction. The main aim is balanced truncation is to change the original stable LTI system to a representation of the same size, but with the assets that states of changed system that controllable are also observable and vice versa.

## 4. Numerical Experiments

In this paper, we present a globalized way for the approximation of Benchmark Problems. We apply the algorithms mentioned above to a Transmission Line dynamical Model from the collection of benchmark example [28].in this contribution, implementiion of balanced truncation methods mentioned above have been developed MATLAB based, a new software used the sssMOR toolbox (Castagnotto *et al,*.2017). a package that is used for a high-scale dynamical system.

### 5.1 Transmission Line Example Model

A transmission line Example Model[28], has 256th state, 2 input, and 2 outputs, hence MIMO system. A Transmission Line system model is a circuit model modeling the impedance of interconnecting strictures accounting for both the charge accumulation on the surface of conductors and current traveling along the conductor[29][30][31][32]. singular values (state contributions) of the system are shown in Fig.1. we approximate the system in 6 states with reduced model's effort quite well. The resultant reduced states Transmission Line system model is shown in Fig.1 in bode diagram plot and Fig.2.Singular values of 6 states of the Line system model. as seen from this below figure, approximated in this contribution, implementing four methods mentioned above have been developed MATLAB based, an innovative software used the sssMOR toolbox (Castagnotto *et al,*.2017). a package which is used for a high-scale dynamical system.

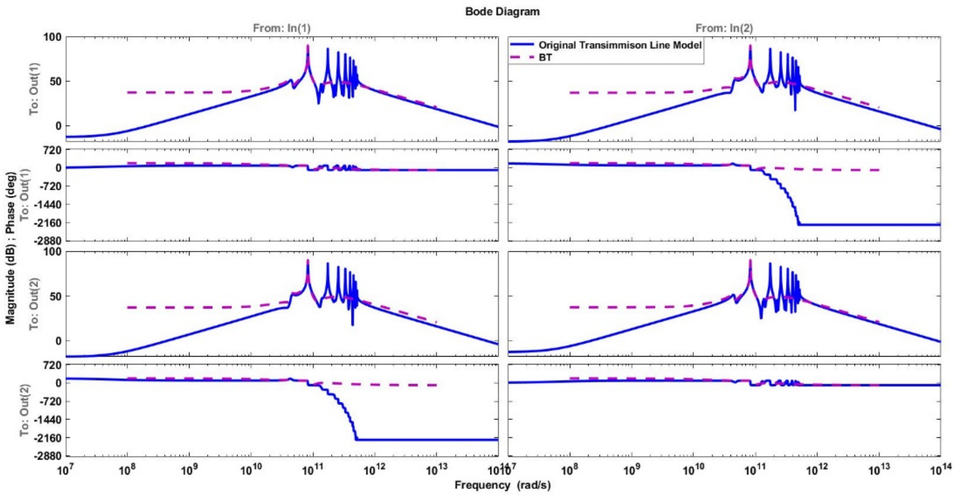


Fig.1. Bode Diagram of Transmission Line model and Reduced states.

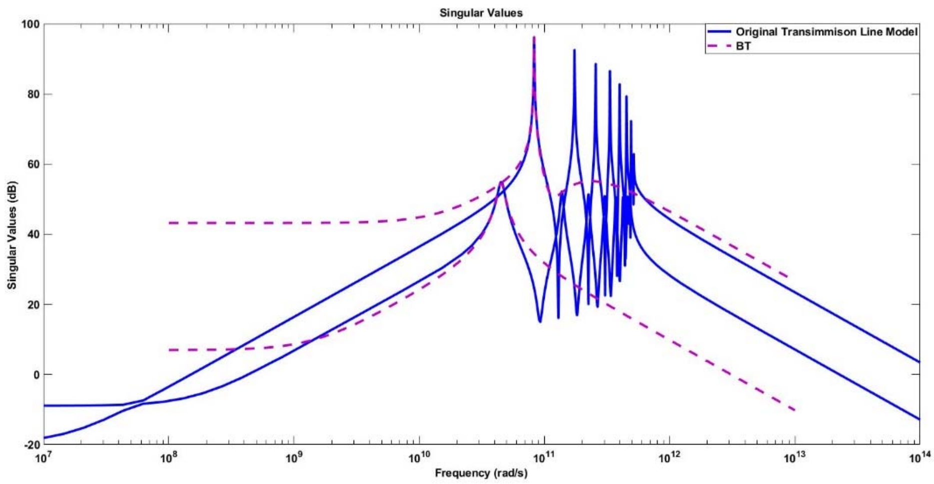


Fig.2.Singular Values of Transmission Line model and Reduced states.

In this discussion, the HSV-Hankel singular values of this model do not deterioration rapidly and hence the model is moderately hard to reduce the Transmission Line model. Both Fig.1(frequency response) and Fig.2(Sigular Values-Sigma)). show the response of the original system to reduced order model with different algorithms. In this contribution, we have been used balanced truncation Furthermore, even however the BT approaches lead to good local behavior, they are practical to produce large $H_\infty$ and $H_2$ errors related to algorithms[33]. In this section, we present a reasonable study of using approximation algorithms for model order reduction, an error estimation of a globalized method for approximation of Benchmark Problems. We apply the algorithms mentioned above.Benchmark collection to transmission line large scale dynamical systems [28][39].In this contribution, implementing all of the methods mentioned above have been developed MATLAB based, a new software used the sssMOR toolbox (Castagnotto et al,.2017). a package which is used for large scale dynamical system. In this analysis, we have been used balanced truncation method for model order reduction. We find Error estimation using Approximation of Original system. This enables a new evaluation of the error system offered that the Observability Gramian of the original system has once been regarded as, an $H_\infty$ and $H_2$ error bound Can be calculated with negligible numerical challenge for any reduced model because of The decreased order demonstrating of a large-scale dynamical system is compulsory to effortlessness the analysis of the system the usage of approximation techniques. Clearly it is observed to close the comparison of the original system with the reduced one with the lesser error.

## 5. CONCLUSIONS

In this paper, a novel approach based reduced order modelling of higher-order dynamical systems using Balanced Truncation method using the sssMOR toolbox. This algorithms have been applied to Transmission line MIMO dynamical systems. This Approach has been used to effectively demonstrate a examples of continuous-LTI system. Furthermore, the ROM estimated by the proposed method provides a close approximation to the higher-order system. Further, the accuracy, superior performance and the validation of the presented methods are proven by comparison infinity norm with present literature. Applicability of large-scale systems allows the approach more desirable The outcomes illustration that balanced the truncation method are best over the whole frequency range.

Table. 1. A summary of Error estimation Analysis for Large Scale system and Reduced system of Transmission Line Model

| Reduced Method | Response of system | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | N | Error | | Selected Model | | | Difference System | | | |
| | | Error bond | Relative error bound | $H_\infty$ Norm | $H_2$ Norm | Decay Time(s) | $H_\infty$ Norm | | $H_2$ Norm | |
| | | | | | | | Absolute | Relative | Absolute | Relative |
| Original System | 256 | - | - | 6.514E04 | Inf | 3.67375E-10 | 4.183E-11 | 6.422E-16 | inf | NaN |
| Balancing truncation(BT) | 6 | 1.049E05 | 3.221E00 | 6.514E04 | 1.418E09 | 1.89616E-04 | 2.136E04 | 0.328 | inf | inf |

**Nomenclatures**

| | |
|---|---|
| A, C, B, D | Original system Constant Matrix |
| Ar, Br, Cr,Dr, | Reduced Matrix |
| G(s) | Original System |
| □ | Real Number |
| ∑ | General dynamical system |

**Abbreviation**

| | |
|---|---|
| TBR | Balancing & Truncation |
| M T | Model Techniques |
| PDE | Partial differential equation |
| SISO | Single -input/Single-output |
| sssMOR | Sparse State-Space Through Model Order Reduction |
| MIMO | Multi-input/Multi-output |
| MOR | Model Order Reduction |
| HSV | Hankel singular values |
| ROM | Reduced Order Model |

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