Novel Chaotic Strange Phenomena in Piecewise Linear Negative Resistance/Conductance L-C Oscillators: Feasibility Study and Virtual simulation

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Abstract: In this paper, the chaotic phenomena are outlined in a class of simple NRO (negative resistance oscillator) or NCO (negative conductance oscillator) equivalently. Compared to most chaotic electronic schemes encountered in nonlinear dynamic systems theory and practice, the proposed class of chaotic circuits consists of simple piecewise linear NRO/NCO, involving a minimum number of building components and dynamic states. The feasibility of relevant chaotic phenomena is proven from the existence of their Van Der Pol dynamic models according to the least square estimation principle. Then, despite the simplicity of the 2nd order NRO/NCO, a relevant parameterization strategy associated with virtual simulation techniques, are used as key exploration means for creating relevant chaotic phenomena. Therefore, new strange chaotic attractors with 2D/3D shapes are created and presented. Moreover, a comparative study with a sample of existing chaotic NRO/NCO schemes, show that the proposed class of chaotic NRO/NCO is optimal since it provides minimum building constituents and lower dynamic order, while offering a new palette of chaotic attractors with great topological strangeness.

Key-Words: Negative resistance/conductance, nonlinear dynamic systems, oscillators, chaotic phenomena, strange chaotic attractors, topological strangeness, virtual simulation.

1 Introduction
The chaotic phenomena were surprisingly discovered a long time ago in different contexts by a few pioneering researchers, including Henri Poincaré in 1892 (when solving the interaction problem among planets [1-2]), Lyapunov for his PhD thesis presented in 1892 on the study of initial conditions sensibility of chaotic systems [3], Van Der Pol (when simulating a class of nonlinear electric circuits [4-5]), and Edwar Lorenz in 1963 [6]. However, following the state-of-the art of further researches on chaos theory and relevant applications [7-8], it is known nowadays that the chaos is a strange behaviour of natural and artificial nonlinear dynamic systems, e.g., planet interactions, weather dynamics, electrical machines, animal world, electronic oscillators, communication systems and more. It is important to recall that,
independently of their building technology, chaotic
dynamic systems are characterized on by two
following key properties: a) sensibility to
neighbouring initial conditions, e.g., small changes
of initial conditions in state or input variable(s),
might cause local instability and global bounded
trajectories after long runs; b) sensibility to
parameter(s), e.g., small changes one or a few key
parameters, might also cause local instability and
global bounded trajectories. As a consequence,
a strange visual attraction object can arise from
instability and bounded trajectory phenomena. Since
the discover of chaos, abundant research works
have been published in that emerging research topic
[1-18]. However, in most of these research works,
the relevant contributions rely on the creation of
new complex chaotic systems from existing chaotic
schemes. Even though a significant effort for the
reduction of the complexity of electronic chaotic
circuits have been done in a few recent research
works [19-22], the final chaotic electronic circuit
involved still remain notoriously intricate in terms
of the total number of building constituents. As a
novelty, the major emphases of this paper is on a
better trade off strategy between the structural
simplicity and the topological strangeness of the
proposed class of simplest chaotic oscillators. It
consists of a basic piecewise linear NRO (negative
resistance oscillators) or NCO (negative
conductance oscillators equivalently), involving
the minimum number of building components and
high topological strangeness of chaotic attractors.

In the following sections of the paper are
organized as follows: A brief recall on building
components of NRO/NCO is provided in Section
2. Then, the feasibility of chaotic phenomena in
piecewise linear NRO/NCO is outlined in Section
3 from the existence of its equivalent Van Der Pol
versions according to least square estimation
principle. Then, in section 4, virtual simulations
under Multisim and Simulink environment are
conducted under key parameterization strategies,
in order to create and show a rich palette of strange
chaotic attractors. Furthermore, a comparative
study is conducted in section 5 between the
proposed class of piecewise linear NRO/NCO and
a sample of existing chaotic NRO schemes, in order
to outline the novelty of this research work. Finally,
the paper is concluded in section 6.

2 Piecewise NRC/NCC and related
NRO/NCO works
The piecewise NRC (negative resistance circuit) or
its dual piecewise NCC (negative conductance
circuit), is widely used as a building element in a
wide variety of electronic instrumentation systems,
including autonomous oscillator, switching or sine
wave modulators for communication systems,
interfacing driver for power electronics converters,
signal processing device for analog-to-digital and
digital-to-analog converters. As shown in Fig. 1, the
basic NRC/NCC circuits can be analogically
implemented using a single operational amplifier.

Fig. 1 Simple piecewise NRC and NCC
The input-output characteristics $ue \sim ie$ and $ie \sim ue$ of NRC and NCC respectively, are both piecewise linear functions given by:

\begin{align}
ue &= N(ie) \quad (a) \\
\text{ie} &= f(ue) \quad (b)
\end{align} \tag{1}

In (1) the slopes parameters $p$ and $p1$ for $N(ie)$ in Fig. 1(a), and $q$ and $q1$ for $f(ue)$ in Fig. 1(b), are given by:

\begin{align}
p &= -\frac{R1}{R2} R3, \quad p1 = R3 \quad (a) \\
q &= -\frac{R2}{R1 R3}, \quad q1 = \frac{1}{R3} \quad (b)
\end{align} \tag{2}

The piecewise linear NRC/NCC are used as building components of a class of NRO/NCO as shown in Fig.2. The serial L-C oscillator presented in Fig. 2(a) is built using a NRC, and its dual L-C parallel configuration shown in Fig. 2(b), relies on a NCC core. In each case, the NRO/NCO can be under autonomous or controlled operating conditions from an external voltage source $Vs(t)$.

![Serial L-C Circuit](image)

**Fig. 2** A Class of NRO and NCO circuits

For the sake of simplicity, let us consider the new notations,

\begin{align}
y &= ue \quad \text{(in Fig. 1(a) and Fig. 2(a))} \\
x &= ie
\end{align} \tag{3}

and

\begin{align}
y = ie \\
x = ue \quad \text{(in Fig. 1(b) and Fig. 2(b))}
\end{align} \tag{4}

Then, a generic form of (1) could be written as follows:

\begin{align}
y &= N(x) \quad \text{(in Fig. 1(a), Fig. 2(a))} \quad (a) \\
y &= f(x) \quad \text{(in Fig. 1(b), Fig. 2(b))} \quad (b)
\end{align} \tag{5}

Because of the duality principle between NRO and NCO, the emphasis in this paper without loss of generality, is on the relevant subclass of serial piecewise NRO circuits, which are governed by the following 2nd order nonlinear dynamic model:

\begin{align}
d^2 x(t) = -\frac{R}{L} \frac{dx(t)}{dt} - \frac{1}{L} x(t) + \frac{1}{L} \frac{dV_s(t)}{dt} \\
-\frac{R}{L} \frac{dN(x(t))}{dt} + \frac{1}{L} \frac{dV_s(t)}{dt} \\
-\frac{1}{L} \frac{\partial N(x(t))}{\partial x(t)} d(x(t)) + \frac{1}{L} \frac{dV_s(t)}{dt}
\end{align} \tag{6}

where $N(x)$ is a piecewise linear function outlined earlier in Fig. 1(a).

### 3 Chaotic Phenomena in Piecewise Linear NRO/NCO

It is important to understand that a class of Van Der Pol oscillators could be derived from piecewise NRO modelled by (5) using LSE (Least Square Estimation) principles. Indeed, let us consider a polynomial estimation model of the resulting piecewise NRC/NCC as presented in Fig. 3, with a cubic shape given as follows:

\begin{align}
y &= a_1 x^3 + a_2 x^2 + a_3 x + a_4 \\
\theta &= \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad \text{(7)}
\end{align}

where $\theta = [a1 \ a2 \ a3 \ a4]^T$ is a parameter vector to be determined according to LSE principle from a suitable sample $\{x(k), y(k)\}$ of $y(k) = N(x(k))$ with $k = 1, 2, \ldots, N$. In these conditions, the optimal
value $\theta^*$ of $\theta$ according to the LSE principle is given as follows [23-24]:

$$\theta^*(N) = \left( \sum_{k=1}^{N} h(x(k)) h^T(x(k)) \right)^{-1} \left( \sum_{k=1}^{N} h(x(k)) y(k) \right)$$

(8)

In Matlab-based graphical simulation, it is easy to compute (7) within the cftool design framework, i.e., the nature of results to be obtained when computing graphically the LSE solution $\theta^* = [a_1^* \ a_2^* \ a_3^* \ a_4^*]^T$, is illustrated in Fig. 3. Therefore, the suboptimal class of Van Der Pol oscillators obtained from the original piecewise NRO could be modelled from (6) given $\theta^*$ as follows:

Given that:

$$N^*(x(t)) = a_1^* x^3(t) + a_2^* x^2(t) + a_3^* x(t) + a_4^*$$

$$\frac{\partial N^*(x(t))}{\partial x(t)} = 3 a_1^* x^2(t) + 2 a_2^* x(t) + a_3^*$$

(9)

Then (8) becomes:

$$\frac{d^2 x(t)}{dt^2} = -\frac{1}{L} \left( R + \frac{\partial N^*(x(t))}{\partial x(t)} \right) \frac{dx(t)}{dt}$$

$$-\frac{1}{L C} x(t) + \frac{1}{L} \frac{dV_s(t)}{dt}$$

$$= -\frac{1}{L} (R + 3 a_1^* x^2(t) + 2 a_2^* x(t) + a_3^*) \frac{dx(t)}{dt}$$

$$-\frac{1}{L C} x(t) + \frac{1}{L} \frac{dV_s(t)}{dt}$$

$$= -\frac{1}{L} (R + 3 a_1^* x^2(t) + 2 a_2^* x(t) + a_3^*) \frac{dx(t)}{dt}$$

(11)

Using the new notations:

$$A = \frac{(a_1^* + R)}{L}, \ B = \frac{3 a_1^*}{a_1^* + R}, \ C = \frac{2 a_2^*}{a_1^* + R}, \ \omega^2 = \frac{1}{L C},$$

(12)

Then, (11) becomes:

$$\frac{d^2 x(t)}{dt^2} + A \left( 1 + B x^2(t) + C a_2^* x(t) \right) \frac{dx(t)}{dt}$$

$$+ \omega^2 x(t) = \frac{1}{L} \frac{dV_s(t)}{dt}$$

(13)

As a first relevant finding, if the external excitation $V_s(t)$ is a constant, then Equation (12) visually becomes the dynamic model of a family of Van Der Pol NRO. In addition, given the set of parameters:
\[ R_1 = R_2 = R_3 = 1.71 \, k\Omega \] in (2), \( a_1 = 4.043e+07, \ a_2 \approx 0, \ a_3^* = -1710, \ a_4^* \approx 0 \) in (7), \( R = 1 \, k\Omega, \ L = 100 \, mH, \ C = 22 \, nF \) in Fig. 2(a), \( A = 27100, \ B = 4.4756e+04, \ \omega_2 = 4.549e+08 \), the second relevant finding relies on a basic strange attractor obtained associated with (12).

That basic strange attractor is displayed in the x-y plane as shown in Fig. 4. Because of both relevant findings, the feasibility of chaotic behaviour within the original class of piecewise linear NRO is quite established.

![Basic attractor obtained in the x-y plane](image)

**4 Chaotic strange attractors in Piecewise Linear NRO/NCO**

The main goal of this section is to show that, more attractive chaotic strange attractors exist within the class of piecewise linear NRO. This is possible from suitable parameterization strategies in autonomous as well as in controlled operating conditions. Table 1 presents the set of data used to explore the expected sample of strange chaotic phenomena as presented in Table 2. In the first colon of Table 1, the notations \( V_s = 0 \) and \( V_s \neq 0 \) stand for autonomous and controlled operating conditions respectively (See Fig. 2 for better clarity), whereas \( ABC_j \) in Table 1 is related to line \( j = 1, 2, \ldots, 7 \) in Table 2 where a rich palette of novel 2D/3D chaotic strange attractors are presented.

Given Table 1 and following Table 2, it is important to observe that the search strategy adopted in this work for creating chaotic strange attractors with arbitrary 2D/3D shapes, relies on the identification and adjustment of key parameters of the same 2\textsuperscript{nd} order LC oscillators, e.g., the negative slop \( p \) of the NRC and the external excitation \( V_s(t) \). Hence, the minimum dimension (2\textsuperscript{nd} order) of the chaotic dynamic space is unchanged and independent of the topological strangeness of the chaotic attractor. These findings are straightforward merits and challenge compared to most existing chaotic electronic schemes in which the topological strangeness of attractors grows according to the increasing complexity of the whole systems in terms of volume, dynamic order, building costs.

**5 Comparative Study With A Sample of Other Chaotic NRO/NCO Schemes**

As shown in Table 3, the optimality of simple piecewise linear NRO/NRC presented in this paper, relies on a comparative study with a sample of existing chaotic NRO schemes encountered in the literature.

The comparison criteria retained for OPAM (operational amplifier) implementation technology, consists of the circuit architecture, the hardware complexity (in terms of number of: DC sources, operational amplifiers, diodes), the dynamic order and the topological strangeness of attractor(s).

It is a great challenge to observe that, the proposed class of piecewise NRO is optimal for almost all comparative criteria.

These main relevant findings and further work improvement lead to the conclusion of the paper.
Table 2  Sample of novel chaotic phenomena created in the proposed piecewise linear NRO/NCO
The discovery of a wide variety of novel chaotic phenomena, relies in this case on a better parameterization strategy within the optimal class of piecewise linear RNO/NCO, with minimum number of building parts, minimum state space dimension (equal to 2), and high topological strangeness of a wide variety of chaotic attractors involved.

**Table 3 Comparative study between existing chaotic NRONCO and the proposed chaotic NR**

<table>
<thead>
<tr>
<th>References</th>
<th>Electronic circuit architecture</th>
<th>Structural complexity</th>
<th>Topological strangeness of attractor</th>
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<tr>
<td>Proposed class of simple chaotic piecewise linear NRONCO</td>
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**Autonomous**

**Voltage controlled**
6 Conclusion

This research paper can be thought of as a key user guide for better trade off strategies, between structural simplicity and topological strangeness of relevant chaotic phenomena when creating chaotic NRO/NCO. A sample of novel 2D/3D strange chaotic attractors obtained from parameterization exploration and virtual simulations, have shown the merits of a simple methodology for effectively building virtual NRO/NCO circuits and systems.

However, the research work as presented in this paper, is limited to the feasibility analysis associated with well tested chaotic NRO schemes in the virtual world. Therefore, testing and characterizing in a real world novel chaotic strange attractors created and presented here, would be an additional relevant contribution in future research works.

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