Hybrid AC-DC System Power Flow Calculations Based on Modified Newton-Raphson Method

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Abstract: -HVDC system is necessary for very long power transmission lines and interconnections between networks of different frequencies. Therefore, it is required to model HVDC transmissions lines and combined them in the AC power flow calculation of the whole AC-DC system. In this paper, the Newton-Raphson method used in AC power flow calculations is modified to be suitable to model the AC-DC systems with inserting the HVDC line in the AC network. The elements of the unknown vector of the AC system are kept unchanged and are extended by a new unknown vector of the DC system variables. The Jacobian matrix of the AC system is modified to include the effect of the DC system which represents the modifications due to the DC line. The modified Jacobian matrix includes the effect of the active and reactive power absorbed at the DC buses, and their dependency on the AC system variables. The modified Newton-Raphson method was implemented on the AC-DC test systems with a power flow calculations using MATLAB script file and the results were presented and discussed for the hybrid AC-DC systems.


1 Introduction
The industrial growth of countries needs large amount of energy consumption, especially electrical energy, so that, the generation and transmission services should be increased to convene the increasing of demand. The energy demand was doubled every ten years in modern countries while it was doubled every only seven years in the developing countries which required considerable investment in electrical power sector [1].

There are significant increases in electrical power generations using renewable energy resources; however, they are in far away from load centers and the tendency towards transfer a large amount of active power for long distances. Therefore, the high voltage direct current (HVDC) system is a favorable solution for the transmitting these large amount of power for long distant between generations and load centers. Due to the benefits of generation and utilization of electrical power with the AC current compared to the DC current, the HVDC transmission systems need a conversion process at its two sides or ends, from AC to DC at its rectifier side and reverse to AC at its inverter side. The HVDC technology is still undergoing many developments applied to improve the system reliability and to reduce the large costs of converter stations at its ends. The latest development is subjected to the multi-terminal HVDC system configurations that increase the scope of application of HVDC systems [2, 3].

The new voltage source converters (VSC)-based HVDC system is more flexible than the conventional current source converter (CSC)-based HVDC system. But the conventional CSC-based HVDC system used today due to its ability to transfer large amount, greater than 500MW, power compared to the VSC-based HVDC system [4].

The power flow studies are very important, because it gives large importance information required for some power system studies such as: system plan study, stability study and ensure finally the correct operation of the electrical supply networks [4]. The AC-DC power flow is carried by two methods known as unified and sequential methods [4-10].
The unified method is the most reliable for solution convergence when the AC-DC power flow built with the fast decoupled technique as explained in [5], while the sequential method diverged in some mode of operation. But the unified method is more complicated compared to the sequential method when they built with the fast decoupled technique. Thus, the two methods and the proposed modified method are built using the more accurate Newton-Raphson methods.

In this paper, the different methods that used in AC-DC power flow calculations for hybrid AC-DC system are discussed and analyzed. Moreover, the AC-DC power flow with unified, sequential and proposed modified method was discussed and analyzed.

The system model of AC-DC power flow for hybrid AC-DC systems was introduced and analyzed in section 2. The modified AC-DC load flow method was presented in section 3. Finally, the proposed modified method is discussed and compared with the other the two methods.

2 AC-DC System Model for Hybrid Systems

In this section, the AC-DC system model for hybrid systems is presented and discussed. The AC-DC system model is divided into the parts: the DC system model and the AC system model. Accordingly, the interaction between them can be modeled and represented using the proposed power flow calculation method.

2.1 DC System Model

Figure 1 shows the monopolar HVDC system. It is modeled considering constant current, constant tapping of converter transformer at the rectifier side as well as constant extinction angle and constant tapping of converter transformer at the inverter side. The residual equations (R1 to R9) of the system are expressed as follows [4-7, 11-15]:

\[ R_1 = V_{d1} - \frac{3\sqrt{2}}{\pi} a_2 k V_{t1} \cos \phi_2 \]  
\[ R_2 = V_{d1} - \frac{3\sqrt{2}}{\pi} a_2 V_{t1} \cos \alpha + R_{d2} I_d \]  
\[ R_3 = V_{d1} - V_{d2} - R_e I_d \]  
\[ R_4 = I_d - I_2^o \]  
\[ R_5 = a_1 - a_2^o \]  
\[ R_6 = V_{d2} - \frac{3\sqrt{2}}{\pi} a_2 k V_{t2} \cos \phi_2 \]  
\[ R_7 = V_{d2} - \frac{3\sqrt{2}}{\pi} a_2 V_{t2} \cos \gamma + R_{d2} I_d \]  
\[ R_8 = \cos(\pi - \gamma) - \cos(\pi - \gamma^o) \]  
\[ R_9 = a_2 - a_2^o \]

where: \( V_{d1}, V_{d2} \): are the DC line voltages at the rectifier and inverter converter buses respectively. \( I_d \) is DC line current. \( V_{t1}, V_{t2} \): are the line to line terminal voltages at the rectifier and inverter converter buses respectively. \( \delta_1 \) and \( \delta_2 \) are the phase angle of the line voltage at both rectifier and inverter converter buses respectively. \( \alpha_1 \) and \( \alpha_2 \) are the transformer tapping ratios at rectifier and inverter converter buses respectively. \( \alpha \) and \( \gamma \) are the rectifier side firing angle and the inverter side extinction angle respectively. \( R_{d1} \) and \( R_{d2} \) are rectifier and inverter side commutated resistance respectively. \( R_e \): is DC line per unit resistance.

2.2 AC System Model

The AC system model consists of sets of mismatches of active power and reactive power equations. The mismatch of active power and reactive power at any AC bus are [3, 5, 12-15]:

\[ \Delta P_i = P_i^{op} - \sum_{j=1}^{n} P_{ij} \]  
\[ \Delta Q_i = Q_i^{op} - \sum_{j=1}^{n} Q_{ij} \]

where; \( P_i^{op} = P_{Gi} - P_{Li} \)  
\( Q_i^{op} = Q_{Gi} - Q_{Li} \)
\[ \sum_{j=1}^{n} P_j = V_i^2 Y_e \cos \theta_i + \sum_{j=1}^{n} V_j V'_j Y'_e \cos (\theta_j - \delta_i + \delta_j) \]

\[ \sum_{j=1}^{n} Q_j = -V_i^2 Y_e \sin \theta_i + \sum_{j=1}^{n} V_j V'_j Y'_e \sin (\theta_j - \delta_i + \delta_j) \]

(12)

And, \(i\) is the bus number

\[ n\text{ is the total number of AC buses} \]

\[ \bar{Y}_i = y_{ij} + \sum_{j=1}^{n} \frac{1}{z_{ij}} \]

\[ \bar{Y}_j = -\frac{1}{z_{ij}} \]

Equations (10) and (11) are modified for the converter AC buses as follows:

\[ \Delta P_i = P_i^{mp} - \sum_{j=1}^{n} P_j - P_{dc} \]

\[ \Delta Q_i = Q_i^{mp} - \sum_{j=1}^{n} Q_j - Q_{dc} \]

(13)

where:

\[ P_{dc} = V_{d1} I_{d1} \]

\[ Q_{dc} = V_{d1} I_{d1} \tan \phi_i \]

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\[ \Delta Q_i = Q_i^{mp} - \sum_{j=1}^{n} Q_j - Q_{dc} \]

(14)

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\[ \Delta P_i = P_i^{mp} - \sum_{j=1}^{n} P_j - P_{dc} \]

\[ \Delta Q_i = Q_i^{mp} - \sum_{j=1}^{n} Q_j - Q_{dc} \]

3 Solution Methodology

The solution methodology of AC-DC power flow can be classified into three methods:

1. Unified method
2. Sequential method
3. Proposed modified method

The description of these methods is as follows:

3.1 The Unified Method

The AC and DC equations are solved together in the unified method. In this method, the nonlinear AC and DC equations for both AC and DC system equations are combined into one set. The set of nonlinear equations of both AC and DC systems can be written as follows the Newton-Raphson method:

\[ \begin{bmatrix} \Delta P \\ \Delta P_i \\ \Delta Q \\ \Delta Q_i \\ R \end{bmatrix} = J \begin{bmatrix} \Delta \theta \\ \Delta \theta_i \\ \Delta \theta_f \\ \Delta V \\ \Delta V_i \\ \Delta x \end{bmatrix} \]

(15)

where:

\[ R \text{ is the residual DC equations which can be expressed as follows:} \]

\[ R = [R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9] \]

\[ \Delta x \text{ is the DC system variables which can be introduced as follows:} \]

\[ \Delta x = [V_{d1} I_{d1} \alpha \phi_1 V_{d2} a_2 y \phi_2] \]

\[ J \text{ is the Jacobian matrix of the hybrid AC-DC system which can be written as follows:} \]

\[ J = \begin{bmatrix} H & L & J_{1dc} \\ M & N & J_{2dc} \\ J_{3dc} & J_{4dc} & J_{5dc} \end{bmatrix} \]

where:

\[ H = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial \theta_i} \\ \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial \theta_i} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial \theta_i} \end{bmatrix}, \quad L = \begin{bmatrix} \frac{\partial \Delta P}{\partial V} & \frac{\partial \Delta P}{\partial V_i} \\ \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial V_i} \end{bmatrix} \]

\[ M = \begin{bmatrix} \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial \theta_i} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\partial \theta_i} \end{bmatrix}, \quad N = \begin{bmatrix} \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial V_i} \\ \frac{\partial \Delta Q}{\partial V} & \frac{\partial \Delta Q}{\partial V_i} \end{bmatrix} \]

\[ J_{1dc} = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} \\ \frac{\partial \Delta P}{\partial \theta_i} \\ \frac{\partial \Delta Q}{\partial \theta} \\ \frac{\partial \Delta Q}{\partial \theta_i} \end{bmatrix}, \quad J_{2dc} = \begin{bmatrix} \frac{\partial \Delta P}{\partial V} \\ \frac{\partial \Delta P}{\partial V_i} \\ \frac{\partial \Delta Q}{\partial V} \\ \frac{\partial \Delta Q}{\partial V_i} \end{bmatrix} \]

\[ J_{3dc} = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} \\ \frac{\partial \Delta P}{\partial \theta_i} \\ \frac{\partial \Delta Q}{\partial \theta} \\ \frac{\partial \Delta Q}{\partial \theta_i} \end{bmatrix}, \quad J_{4dc} = \begin{bmatrix} \frac{\partial \Delta P}{\partial V} \\ \frac{\partial \Delta P}{\partial V_i} \\ \frac{\partial \Delta Q}{\partial V} \\ \frac{\partial \Delta Q}{\partial V_i} \end{bmatrix} \]

\[ J_{5dc} = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} \\ \frac{\partial \Delta P}{\partial \theta_i} \\ \frac{\partial \Delta Q}{\partial \theta} \\ \frac{\partial \Delta Q}{\partial \theta_i} \end{bmatrix} \]

In order to obtain the new values of system variables for iteration \(k\), the following steps can be used:

\[ \begin{bmatrix} \Delta \theta \\ \Delta \theta_i \\ \Delta V \\ \Delta V_i \\ \Delta x \end{bmatrix} = J^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta \theta_f \\ \Delta V_f \end{bmatrix} \]

(17)

\[ \begin{bmatrix} \theta \\ \theta_i \\ V \\ V_i \end{bmatrix} = \begin{bmatrix} \theta \\ \theta_i \\ V \\ V_i \end{bmatrix} + \begin{bmatrix} \Delta \theta \\ \Delta \theta_i \\ \Delta V \\ \Delta V_i \end{bmatrix} \]

(18)
3.2 The Sequential Method
The AC and DC equations are solved together in the Unified method which takes it as a sophisticated method and difficult implementation while in the sequential method, the AC and DC equations are solved individually which takes it easy to implement. The Newton-Raphson method is also used here to solve these two sets of algebraic equations. The DC system residuals equations can be written as follows:

\[ [R] = [J_d][\Delta x] \]  

(19)

where

\[ [J_d] = [J_{sk}] \]  

(20)

In order to obtain the new values of the DC system variables for iteration k, the following steps are:

\[ [\Delta x]^k = [J_d]^{-1}[R]^k \]  

(21)

\[ [x]_{k+1} = [x]_k + [\Delta x]^k \]  

(22)

While, the AC system equations can be written as follows:

\[ \begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta \phi \end{bmatrix} = \begin{bmatrix} J_2 \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta \theta_i \\ \Delta V \end{bmatrix} \]  

(23)

where \( [J_2] \) is the Jacobian matrix which can be evaluated as follows:

\[ [J_2] = \begin{bmatrix} H & L \\ M & N \end{bmatrix} \]  

(24)

The new values of the AC system variables for iteration k, can be implemented using the following steps:

\[ \begin{bmatrix} \Delta \theta \\ \Delta \theta_i \\ \Delta V \end{bmatrix}_{k+1} = \begin{bmatrix} J_2 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q_i \\ \Delta V \end{bmatrix}_k \]  

(25)

\[ \begin{bmatrix} \theta \\ \theta_i \\ V \end{bmatrix} = \begin{bmatrix} \theta \\ \theta_i \\ V \end{bmatrix}_k + \Delta \begin{bmatrix} \theta \\ \theta_i \\ V \end{bmatrix} \]  

(26)

3.3 The Proposed Modified Method
In the sequential method, the AC and DC equations are solved individually. So that it is easy to implement but the divergence problem can be occurred in some control mode of operations of the DC system. The proposed modified method is firstly presented by [16]. The DC converter bus is considered as a voltage dependent load to take its effect on the AC Jacobian of the sequential method.

The proposed technique is applied on the sequential method for AC-DC power flow depending on the fast-decoupled method. In this work, the modified approach is applied on the Newton-Raphson method for the AC-DC power flow. In the proposed modified method takes the effect of DC system on the AC Jacobian matrix by considering the DC system as a voltage dependent load on the AC side at both DC system sides. This can be carried out by representing the active and reactive power of the DC system as a function of the AC terminal line voltage as follows [5]:

\[ P_{dc} = \frac{3\sqrt{2}}{\pi}a_i k_l v_t l_i \cos \phi_i \]  

(27)

\[ Q_{dc} = \frac{3\sqrt{2}}{\pi}a_i k_l v_t l_i \sin \phi_i \]  

(28)

where, \( i=1 \) for rectifier and \( i=2 \) for inverter. Therefore, the DC system effect is seen in the AC system Jacobian and this decreases the number of iterations of AC-DC power flow solutions and prevent the divergence in different control modes of operations of the DC system as well as it avoid the complexity of the unified method.

4 Application Examples
The three AC-DC power flow methods presented in section 3 are applied to two test systems. The first system is a simple AC-DC 4-bus system shown in Figure 2, while the second system is the IEEE 30-bus test system with replacing the AC line between buses 6 and 28 by a DC line.

4.1 Case Study 1:
The three power flow methods are first applied to a suggested simple 4-bus AC-DC system shown in Figure 2. The per unit system data of the 4-bus for both AC and DC is introduced in Table 1.
TABLE 1. DATA OF SIMPLE 4-BUS AC-DC SYSTEM

<table>
<thead>
<tr>
<th>AC system data</th>
<th>( E_1 = 1.05 )</th>
<th>( E_2 = 1.05 )</th>
<th>( Q_L = 0.3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{l1} = P_{l2} = 0.5 )</td>
<td>( P_{l3} = 2.0 )</td>
<td>( Q_{L2} = 0.3 )</td>
<td></td>
</tr>
<tr>
<td>( y_{10} = y_{20} = 0.3 )</td>
<td>( Z_1 = Z_2 ) = 0.1</td>
<td>( Z_{12} = j0.05 )</td>
<td></td>
</tr>
<tr>
<td>DC system data</td>
<td>( R_L = 0.05 )</td>
<td>( x_{c1} = x_{c2} = 0.1 )</td>
<td>( a_1^{sp} = a_2^{sp} = 0.9 )</td>
</tr>
</tbody>
</table>

The MATLAB script file is built for different control modes of operation of HVDC system for different AC-DC power flow methods, unified, sequential and proposed modified. The AC-DC power flow program results for base case, constant current at the rectifier side and constant extinction angle at the inverter side as well as constant transformer tapping at both sides, are tabulated in Table 2 for both AC and DC systems. The results obtained with the suggested modified method after 4 iterations with error of 0.0000678.

Figure 3 shows the converter bus voltages at both rectifier and inverter sides versus iteration numbers with different three AC-DC methods: unified, sequential and proposed modified respectively. It illustrated that the unified method converged in 4 iterations; the modified method converged in 4 iterations while the sequential method converged in 5 iterations.

Table 3 shows the AC-DC power flow results of simple 4-bus AC-DC system shown in Figure 2 at different control modes and different AC-DC power flow methods: unified (U), sequential (S) and proposed modified (M). The control mode of operation of AC-DC power flow is defined by specified DC system variables. For example, in the first control mode of operation in Table 3, \( I_d, \gamma, a_1, a_2 \) that are: the DC line current, extinction angle in the inverter side, and the transformer tapping ratios at the rectifier and inverter sides are specified or fixed at a certain value. The other control modes of operations can be defined in the same manner.

4.2 Case Study 2:
The system model is tested using the standard IEEE 30-bus system. The AC line between buses 6 and 28 is replaced by a DC line. The system data of the DC line used in this case is introduced in Table 4.
sides respectively with the three AC-DC power flow methods with \( (I_d, a_1, \beta, a_2) \) control mode. It illustrates that the unified method reach to final solution after 6 iterations while the sequential method reach to final solution after 7 iterations. However, the final solution is obtained after 5 iterations using the proposed modified method.

Fig. 4 AC converter voltages versus iteration numbers with \( (I_d, a_1, \beta, a_2) \) control mode.

Fig. 5 DC line voltages versus iteration numbers with \( (I_d, a_1, \beta, a_2) \) control mode.

The convergence of the AC-DC power flow with the three AC-DC power flow methods with different control operation modes of the DC system are summarized in Table 5. It illustrates that the convergence of the modified method is better than the sequential method and very close to the unified method. Furthermore, the speed of convergence using the suggested modified method is less than that of the unified method which means that it is a good method for AC-DC power flow calculations. In the other wards, the results confirm that the conversion time for the proposed modified method is smaller than that of the unified and sequential methods for most suggested control mode of operations. Also the iteration number required for proposed modified method is smaller than that required for both unified and sequential methods for most suggested control modes of operations.

<table>
<thead>
<tr>
<th>( Z )</th>
<th>Control Mode</th>
<th>( U )</th>
<th>( S )</th>
<th>( M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( I_d, \gamma, a_1, a_2 )</td>
<td>6</td>
<td>0.282</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>( P_{dc1}, \gamma, a_1, a_2 )</td>
<td>6</td>
<td>0.236</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>( I_d, V_{d2}, a_1, a_2 )</td>
<td>6</td>
<td>0.223</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>( a_1, V_{d2}, \gamma, \alpha )</td>
<td>7</td>
<td>0.247</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>( a_1, V_{d1}, \gamma, P_{dc2} )</td>
<td>6</td>
<td>0.246</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>( I_d, \beta, a_1, a_2 )</td>
<td>6</td>
<td>0.230</td>
<td>7</td>
</tr>
</tbody>
</table>

where, \( \beta \) is the inverter side advance angle.

5. Conclusions

This paper has presented a system model of AC-DC load flow solved by Newton-Raphson method. The first sentence means that there is no contribution. The DC line has been incorporated into the AC load flow study modeled by the unified and sequential methods. In the proposed modified method, the DC system has been considered as a voltage dependent load in the AC system model and then the AC Jacobian has been modified to include the DC line effect on the AC system. The proposed modified method overcame the slow of convergence of the sequential method and the implementation sophistication of the unified method.

The three AC-DC power flow methods have been tested by two different hybrid AC-DC systems. The first system was the 4-bus AC-DC system while the second test system was the standard IEEE 30-bus with replacing the AC section between buses 6 and 28 by a DC line. The results attained that the proposed modified method was very close to that of the unified method but without complexity of implementations. Furthermore, the modified method convergence time and iteration numbers were less than the other two methods for most suggested control modes of operations of the DC system especially with large AC-DC systems.
References: