A Magnetic Resonant Coupling Printed Spiral Coil System under Different Geometric Values

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Abstract: Recently, the wireless power transfer (WPT) technology is attracting attention widely because of the expanding use of portable devices. In WPT system, the coil formed on printed circuit board is used because it could be modified easily and captured in printed circuit board conveniently. A lot of studied have been proceeded to improve the power transmission efficiency. However, the optimum of coils geometric parameters on performance of coil system has not been fully explored yet. Consequently, three geometric parameters of printed square coil have been investigated for the effect on the power transmission efficiency coils while the other geometric parameters are remaining constant. As a result, we found that the optimum transmission efficiency improved by adjusting the width of copper wire, the pitch between adjacent wire and the thickness of wire. Moreover, a printed spiral coil board system has been designed and simulated based on the analysis above. The total power transfer efficiency is over 85% at 10mm equaling to half of coils diameter.

Key–Words: Magnetic resonant, Wireless power transmission, Q factor, Optimum power transmission efficiency, Geometric value.

1 Introduction

Wireless Power Transmission (WPT) technology is a method of transmitting electrical energy without contact. With growing electronics in recent years, messy electric wiring takes up a lot of the space, a wide range of devices often makes users feel disturbed. Therefore, WPT technology has been applied to multiple applications such as wireless charging of mobile cars, note PCs and other handheld devices [1].

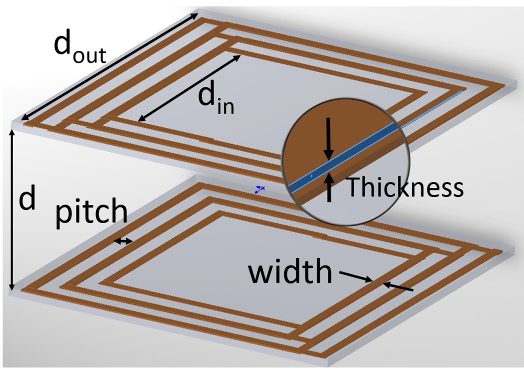
In low-power transmission field, so called "inductive coupling" has been implied successfully to power cell phones and desktop peripheral applications. Although attractive, this mechanism is limited to transferring energy over a very small distance [2]. Magnetic resonant coupling wireless power transmission, which has been used in inductive power transmission by Nicola Tesla more than hundred years ago and developed by Andr Kurs et al. from MIT(Massachusetts Institute of Technology), can achieve power transfer over a large distance [3, 4].

Simplified diagram of a WPT system consist of five parts as power source, DC-AC converter, PSC system, AC-DC rectifier and load. The total power efficiency is determined by efficiency of each part [5].The quality factor Q for each coil and the coupling coefficient k between coils are two critical parameters for long range and highly efficient WPT between two coils [6]. The quality factor Q, determined by self-inductance and parasitic resistance, does not relate to coupling coefficient k, and most determined by size of coil and distance between coils. The size and geometry of the primary and secondary coils significantly impact both Q and k independently. Hence to impact the transmission efficiency [7].

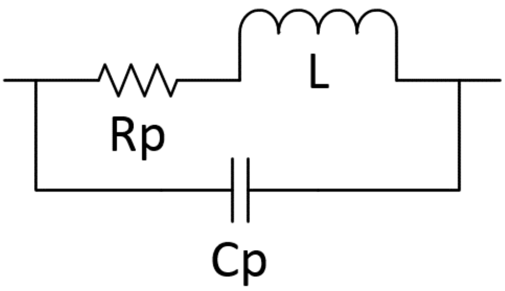
Compared with traditional helix coil, printed spiral coil (PSC) is more convenient and the performance of coil can be optimized by adjusting its geometric parameters. Furthermore, PSC can be fabricated on flexible dielectric substrate because the size of PSC is changeable through design [8]. It is to be noted that double-layer spiral coils can transmit power more efficiently than single-layer coils do at longer distance [9]. Characteristic analysis and optimal coils size ratios of coils for WPT have been discussed [**?**,7]. very high Q factors and strong coupling over significant ranges are achieved on [10]. However, even at strong coupling regime, the optimum power transmission efficiency is about 40%. Besides, the impact of the geometries, such as width, pitch and thickness of coil, on Q factors and coupling efficient, has not been fully explored.

In this work, we have proposed a double-layer printed spiral coil with size of $2 \times 2cm^2$, and formed a parallel-parallel compensation system. The relationship between its geometric parameters and the power transmission efficiency of PSC system is simulated. And the recommend requirements of geometric values are proposed. Based on the results, the printed spiral coils with several different geometric values have been designed and tested. The optimum power transmission efficiency is over 85% at the distance equaling to half of the diameter of coils.

2 Formulations and Equations



(a)



(b)

Figure 1: Geometry of spiral coil and lumped equivalent circuit

Fig. 3 shows a square-shaped PSC and its lumped equivalent circuit. The inductance L, parasitic resistance R_P and parasitic capacitance C_P of coil are determined by several geometries, such as width ***w***, pitch ***s*** of adjacent copper wire and thickness ***t*** of copper wire.

2.1 Inductance

The inductance of square coil is calculated by using the Eq. 1 from [8].

$$L = \frac{1.27\mu n^2 D_{avg}}{2}[ln\frac{2.07}{\varphi} + 0.18\varphi + 0.13\varphi^2] \quad (1)$$

Where n is the number of turns, μ is the permeability, $D_{avg} = (d_{out} + d_{in})/2$ and $\varphi = (d_{out} - d_{in})/(d_{out} + d_{in})$.

2.2 Parasitic Resistance

The parasitic resistance is given by Eq. 2 from [8] with respect to skin effect.

$$R_s = \frac{\rho \cdot length}{w \cdot \delta \cdot (1 - e^{-t/\delta})} \quad (2)$$

Where $length = 4nD_{avg} - n(0.5w + s)$ and $\delta = \sqrt{\rho/(\pi f \mu)}$, in which, ρ, t and $length$ are the resistivity, thickness and length of the copper wire. δ is the skin depth.

2.3 Parasitic Capacitance

The capacitance of the printed spiral coil is founded by using an empirical approach [8]. A parallel plate parasitic capacitor forms between the spiral conductor sidewalls through air and FR4 substrate dielectrics. Therefore, the parasitic capacitance can be calculated from Eq. 3

$$C_p = (\alpha\varepsilon_{air} + \beta\varepsilon_{FR4})\frac{\varepsilon_0 t L_{gap}}{s} \quad (3)$$

Where ε_{air} and $0\varepsilon_{FR4}$ are the relative permittivity of air and FR4 substrate. α and β are the empirical parameters, t is the thickness of copper wire. $L_{gap} = 4D_{avg}(n-1)$ is the length of spiral coil gap.

As the number of turns increases, the parasitic capacitance becomes increasingly difficult to calculate accurately due to the nonlinear adjacent winding capacitance. Moreover, the parasitic capacitance is on the order of few pF, and is negligible compared to the required tuning capacitor for resonance at 13.56MHz [7]. Therefore, the required tuning capacitance is calculated as Eq. 4 in terms of the inductance and resonant frequency f.

$$C_r = \frac{1}{(2\pi f)^2 L} \quad (4)$$

2.4 Equivalent Circuit for Power Efficiency

Applying Norton equivalent transformation to the input source, the voltage source with serial resistor is transformed to the current source i_s and source admittance g_s for simplification as shown in Fig. 2. Serial-parallel transformation of inductor is used as shown in Fig. 3(a). Fig. 3(b) shows an equivalent circuit diagram of magnetic wireless power transmission system, which operates like a transformer. L_1 is the inductance of primary coil, g_1 is the parasitic resistance and C_1 is the tuning capacitance. L_2, g_2 and C_2 are those of secondary coil. Usually, the load is battery, which is sensitive to current. Therefore, we apply the PP compensation in this system. In this case, compared with tuning capacitances, the parasitic capacitance of printed spiral coil is negligible. The power

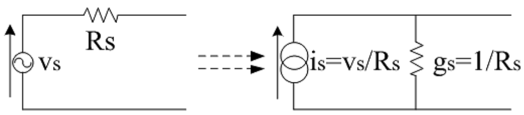
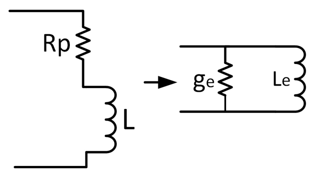
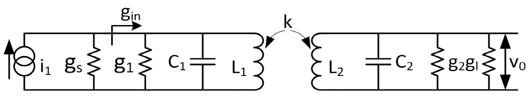


Figure 2: Norton equivalent circuit of the input source.



(a)



(b)

Figure 3: Equivalent of magnetic resonant coupling WPT system.

transmission efficiency η is the ration of the output power and input power. The input power is defined under a approach that the circuit working on optimum conditions, in other words, the input resistance g_{in} equal to source resistance g_s. at resonant frequency is given by Eq. 3 from [11].

$$P_{in} = |\frac{v_s}{2}|^2 g_{in} = (\frac{i_s}{2g_{in}})^2 g_{in} = \frac{i_s^2}{4g_{in}} = \frac{i_s^2}{4g_s} \quad (5)$$

$$P_{out} = |v_o|^2 g_l \quad (6)$$

$$\eta = \frac{P_{out}}{P_{in}} = 4\frac{v_o^2}{v_s^2} = \frac{g_s g_l}{g_{t1} g_{t2}} \frac{(2K)^2}{(1+K^2)^2 - 2(K^2 - \frac{b}{2})x^2 + x^4} \quad (7)$$

In the above equations,

$$g_{t1} = g_s + g_1, \quad (8)$$

$$g_{t2} = g_l + g_2, \quad (9)$$

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}}, \quad (10)$$

$$Q_{L1} = \frac{\omega_0 C_1}{g_{t1}} = \frac{1}{g_{t1}\omega_0 L_1}, \quad (11)$$

$$Q_{L2} = \frac{\omega_0 C_2}{g_{t2}} = \frac{1}{g_{t2}\omega_0 L_2}, \quad (12)$$

$$Q_L = \sqrt{Q_{L1} Q_{L2}}, \quad (13)$$

$$K = k\sqrt{Q_{L1} Q_{L2}} = kQ_L, \quad (14)$$

$$b = \frac{Q_{L1}}{Q_{L2}} + \frac{Q_{L2}}{Q_{L1}}, \quad (15)$$

and

$$x = Q_L(\frac{f}{f_0} - \frac{f_0}{f}). \quad (16)$$

Power transmission efficiency comprise two types of loss. One is transmission loss l_t defined as Eq. 17, the other is the insertion loss l_i defined by Eq. 18.

$$l_t = \frac{(2K)^2}{(1+K^2)^2 - 2(K^2 - \frac{b}{2})x^2 + x^4} \quad (17)$$

$$l_i = \frac{g_s g_l}{g_{t1} g_{t2}} = (1 - \frac{Q_{L1}}{Q_{U1}})(1 - \frac{Q_{L2}}{Q_{U2}}) \quad (18)$$

Where $Q_{U1} = 1/g_1\omega_0 L_1$ and $Q_{U2} = 1/g_2\omega_0 L_2$ are unloaded quality factors of the transmitting and receiving circuit, and k is the coupling coefficient between two coils. It is obvious that x has two solutions for optimum power transmission efficiency when $K^2 > b$, in this case, strong coupling regime formed between two coils. When $K^2 = b$, x has one solution which imply that two coils work on critical coupling regime. If $K^2 < b$, there is no x exist for optimum power transmission efficiency, which means two coils work on weak coupling regime. Therefore, in order to obtain the optimum power transmission efficiency we have to increase K. Meanwhile the insertion loss can be reduced by decrease ration of loaded quality factor and unloaded quality factor.

Then, the power efficiency at center frequency $\eta_{(x=0)}$ can be obtained by setting frequency f equal to center frequency f_0, meanwhile, x equal to zero.

$$\eta_{(x=0)} = \frac{g_s g_l}{g_{t1} g_{t2}} \frac{(2K)^2}{(1+K^2)^2} = (1 - \frac{Q_{L1}}{Q_{U1}})(1 - \frac{Q_{L2}}{Q_{U2}}) \frac{(2K)^2}{(1+K^2)^2} \quad (19)$$

3 Electromagnetic Simulation

Numerical simulation of electromagnetic computed by using MAXWELL with adjustable width, pitch and thickness. Each coil has double layers and three turns for one layer. Load resistance and source resistance are selected to be 500Ω based on load resistance analysis [11]. Tuning capacitance is chosen from Eq. 4.

3.1 Simulation in different width

At first, the proposed model is simulated under different widths of copper wire when the other parameters remaining constant. In the right side of Fig. 4 labeling Q_L/Q_U, the left side is K. Fig. 4 indicates that K is increasing with increase of width, and Q_L/Q_U decrease firstly and then increase. According to the analysis above, the K bigger than one and Q_L/Q_U as small as possible are preferable for high power transmission efficiency. Therefore, the width equaling to 1mm is preferable for high power transmission efficiency.

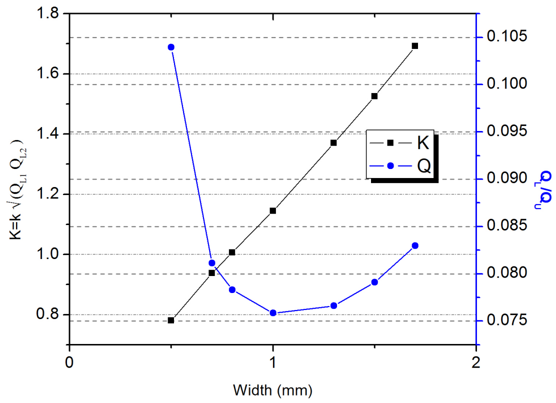


Figure 4: Simulation results of K and Q_L/Q_U with recept of different width

3.2 Simulation in various pitch

Assuming the other parameters are constant, the proposed model under different pitches of adjacent copper wires is simulated. Fig. 5 indicates that K and Q_L/Q_U are both increasing with the increase of pitch. According to the principle that K should bigger than one and Q_L/Q_U is as small as possible, the pitch equaling to 0.6mm is desired for high power transmission efficiency.

3.3 Simulation in various thickness

Even there is no symbol in Eq. 1 represent thickness of wire, it is quite obvious that increasing the thickness of copper wire pull parasitic resistance down, furthermore, the power transmission efficiency is affected. Fig. 6 shows the simulation result of different thickness. Increasing thickness brings K up and pulls Q_L/Q_U down. The large thickness require extra PCB technics, besides, Q_L/Q_U smaller than 0.05 is enough for over 90% power transmission efficiency.

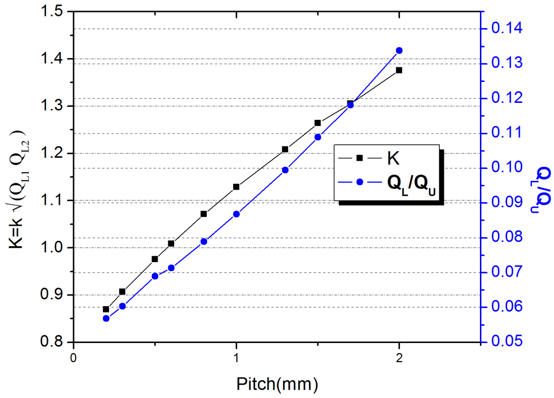


Figure 5: Simulation results of K and Q_L/Q_U with recept of different pitch.

Therefore, the thickness equal to 0.035mm is enough for high power transmission efficiency.

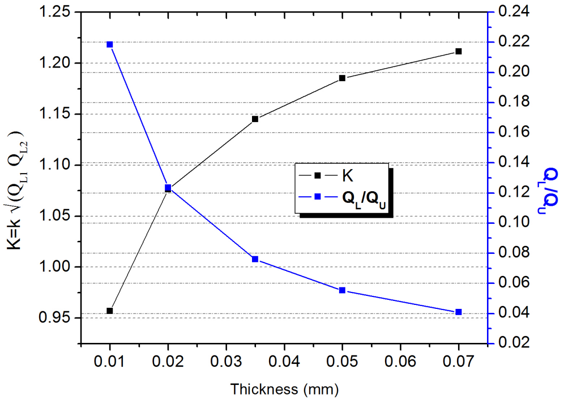


Figure 6: Simulation results of K and Q_L/Q_U with recept of different thickness.

4 Printed Spiral Coil Design and Measurement

To verify the simulation results, five kinds of prototype PSC are built as shown in Fig. 7. The width of PSC altered from 0.5mm, 1mm and 2mm while the pitch of PSC is fixed to 1mm, and the pitch altered from 0.6mm, 1mm and 2mm while the width is fixed to 1mm. In Fig. 8, a standard signal generator is used to supply v_s equal to 5V to the whole WPT system on the right side, and the output voltage is detected by a oscilloscope on the left side. The source resistor, load resistor and tuning capacitor are wielded in

PSC as SMD. Frequency response of power transmission efficiency calculated by Eq. 7 as shown in Fig. 9 and Fig. 10. A SPICE simulation is carried on for comparison with measurements. The distance of two coil is fixed at 10mm which is half of the diameter of coil. Measurements are performed for several different width of PSC, with the results plotted in Fig. 8. When the width is large, such as w=0.5mm shown in Fig. 7, there exist two resonance frequencies that permit maximum power transmission efficiency. It should be noted that the system work on strong coupling regime [12], which means the K is larger than one. In this regime, the peak power transmission efficiency also called maximum power transmission efficiency is limited by values of K and Q_L/Q_U. In the lower frequency mode, the current in the primary coil should be in phase with current in the secondary coil. Conversely, in the higher frequency mode, the coil currents should be antiphase. The antiphase current create a counter magnetic field contributes negatively to transmitter current, which is called parasitic couplings [6]. As the width becomes smaller, the resonance frequencies move closer and eventually becomes one and the peak power transmission efficiency improved slightly. The reason is that K become smaller and get closer to one with Q_L/Q_U diminishing. When the width get even smaller, the K inferior to one and large Q_L/Q_U resulting in decrease of peak transmission efficiency, which means the system works on weak coupling regime.It is obvious width=1mm is preferable for high power transmission efficiency. Simulation results are almost the same as experimental results.

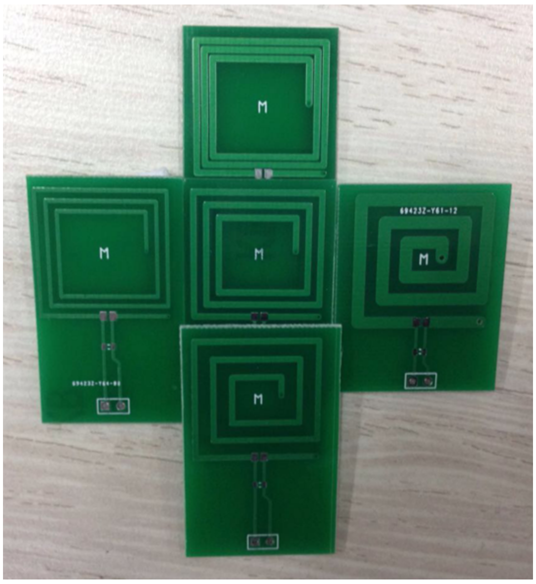


Figure 7: Five kinds of Prototype PSC.

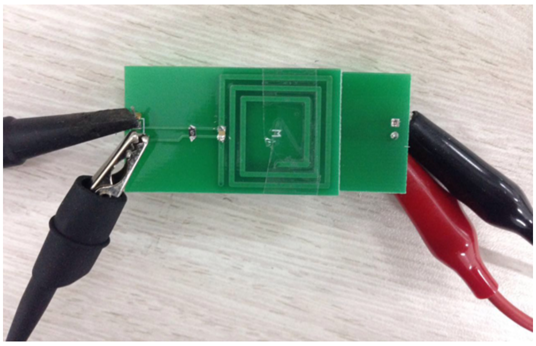


Figure 8: Practical WPT system with power input on the right side and voltage measurement on the left side.

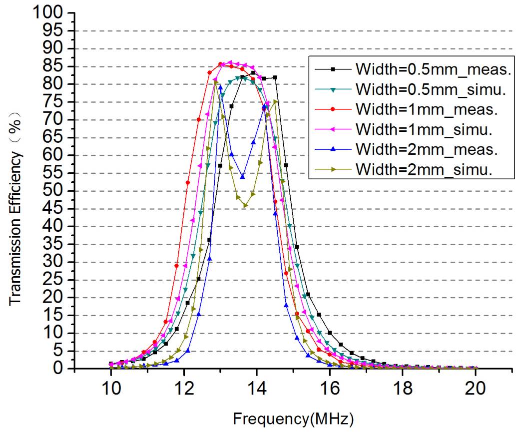


Figure 9: Frequency response of power transmission efficiency under different width.

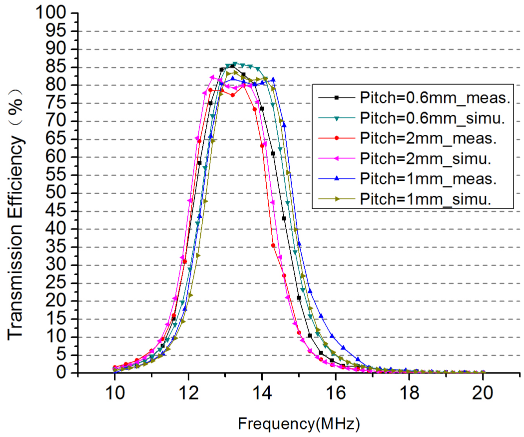


Figure 10: Frequency response of power transmission efficiency under different pitch.

Simulation and experimental of several different pitch of PSC are proceeded with results plotted in Fig. 8. When the pitch is 2mm, two resonance frequencies exist that permit maximum power transmission efficiency. The system is working on strong coupling regime, which means K superior to one. The peak transmission efficiency is pulled down by transmission loss and insertion loss. Smaller K and Q_L/Q_U are concomitant of decrease of pitch, causing the peak power efficiency improve until K equal to one. When pitch get smaller, implying K inferior to one, the system working one weak coupling regime have larger transmission loss. The optimum peak power transmission efficiency is achieved at pitch=0.6mm. Simulation results show good match to the experimental results.

5 conclusions

In this work, a pair of double-layer printed spiral coils used in low power transmission device are designed. Three geometric values, including copper wire width, pitch between adjacent wire and thickness of copper wire, are considered for improving optimum transmission efficiency at resonant frequency. A magnetic simulation is conducted to show the influence of geometric values on the two major parameters,$K = k\sqrt{Q_{L1}Q_{L2}}$ and the Q_L/Q_U, for improving optimum power transmission efficiency. In the design for PSC system, power transmission efficiency of coil with several different geometric values are measured respectively. The SPICE simulation and experimental results show that the power transmission efficiency is over 85% by adopting the optimal width and pitch of coil on our PSC system. Therefore, adjusting geometric values of PSC is an effective way to increase the optimum transmission efficiency. Future work include improve the power transmission efficiency for PSC designed in terms of number of turns.

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