Global State-Feedback Stabilization for a Class of Uncertain Nonholonomic Systems with Partial Inputs Saturation

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Abstract: This paper investigates the problem of global stabilization by state feedback for a class of uncertain nonholonomic systems in chained form with partial inputs saturation. By using input-state-scaling technique and backstepping recursive approach, a state feedback control strategy is presented. With the help of a switching control strategy, the designed controller renders that the states of closed-loop system are globally asymptotically regulated to zero. A simulation example is provided to illustrate the effectiveness of the proposed approach.

Key–Words: Nonholonomic systems, Partial inputs saturation, Backstepping, State feedback, Global stabilization

1 Introduction

In the past decades, there has been a rapidly growing research interest in nonholonomic systems, which can be found frequently in the real world, such as mobile robots, car-like vehicle, under-actuated satellites, the knife-edge and so on. As pointed out in [1], such a class of nonlinear systems cannot be asymptotically stabilized at the origin by only using continuous state feedback control signal. In order to overcome this obstruction, several approaches have been proposed for the problem, such as discontinuous feedback [4, 8], time-varying feedback [2, 5, 9] and hybrid stabilization [3, 6, 7]. Using these valid approaches, the asymptotic stabilization or exponential regulation for nonholonomic systems has been extensively studied [10-20]

However, no matter the system is precisely known or with the parametric (and non-parametric) uncertainty, a common characteristic of these designs of controllers above is that the bounds of control inputs were not considered. As a matter of fact, any actuator always has a limitation of the physical inputs and its existence often severely limits system performance, giving rise to undesirable inaccuracy or leading to instability[21,22]. Hence, from a practical point of view, it is more interesting and important to design a saturated controller for nonholonomic systems. In this regard, some interesting results have also been reported for some specific systems. For example, the saturated feedback controllers were constructed in [23-26] for nonholonomic wheeled mobile robots in different types. For the so-called standard chained form system, the saturated asymptotic stabilizers were constructed in [27,28]. Nevertheless, the above-mentioned control methods are unavailable for the general nonholonomic systems.

Motivated by the example presented in Section 2.1, this paper addresses the global stabilization by state feedback for a class of uncertain nonholonomic systems in chained form with partial inputs saturation. A constructive method in designing global stabilizing controller for such uncertain systems is proposed. The contributions of this paper are listed as follows:(i) by uing the nested saturation to handle the technical problem of input saturation, and based on a combined application of tthe input-state-scaling technique and backstepping recursive approach, a systematic control design procedure is developed for all plants in the considered class, including the ideal chained form system; (ii) the saturated control based switching strategy is adopted to handle the technical problem of uncontrollability at $x_0(0) = 0$, which prevents the finite escape of system and guarantees thatthe states of closed-loop system are globally asymptotically regulated to zero.

The remainder of this paper is organized as follows. Section 2 presents a motivating example and describes the systems to be studied and formulates the control problem. Section 3 presents the input-state-scaling technique and the backstepping design procedure, the switching control strategy and the main results. Section 4 gives the simulation of the motivating example to illustrate the theoretical finding of this paper. Finally, concluding remarks are proposed in Section 5.

2 Motivating example and problem formulation

2.1 Motivating example

Consider a tricycle-type mobile robot with nonholonomic constraints on the linear velocity, which has often been used as a benchmark example in the literature on nonholonomic control systems design. The kinematics of the robot can be modeled by the following differential equation [9]:

$$\begin{aligned} \dot{x}_c &= v\cos\theta \\ \dot{y}_c &= v\sin\theta \\ \dot{\theta} &= \omega \end{aligned} \tag{1}$$

where (x_c, y_c) denotes the position of the center of mass of the robot, θ is the heading angle of the robot, v is the forward velocity while ω is the angular velocity of the robot.

For system (1), by taking the following state and input transformation

$$\begin{aligned} &x_0 = \theta,\ x_1 = x_c\sin\theta - y_c\cos\theta, \\ &x_2 = x_c\cos\theta + y_c\sin\theta,\ u_0 = \omega,\ u_1 = v \end{aligned} \tag{2}$$

one obtains

$$\begin{aligned} \dot{x}_0 &= u_0 \\ \dot{x}_1 &= x_2u_0 \\ \dot{x}_2 &= u_1 - x_1u_0 \end{aligned} \tag{3}$$

It is evident that, system (3) is a third-order chained form system which has been extensively studied in the literature when the inputs saturation was not taken into consideration. However, in the process of actual movement, the unbounded angular velocity of the robot is impermissible. That is because the overquick rotation will result in robot overturned. Therefore, it is more practical to consider the stabilization problem of the nonholonomic mobile robot subject to saturated angular velocity, that is, consider the stabilization problem of the nonholonomic system (3) with input u_0 saturation. The presence of input u_0 saturation and nonlinear term $-x_1u_0$ leads to the existing unsaturated control methods [10-20] and saturated control methods [23-28] are inapplicable to nonholonomic system (3). This motivates us to investigates the global stabilization of a broad class of uncertain nonholonomic systems with input u_0 saturation.

2.2 Problem formulation

Considering that many nonlinear mechanical systems with nonholonomic constraints can be transformed, either locally or globally, to a nonholonomic system in the so-called chained form[10], in this paper we focus our attention on the global state feedback stabilization for the following class of nonholonomic systems in chain form:

$$\begin{aligned} \dot{x}_0 &= u_0 \\ \dot{x}_i &= x_{i+1}u_0 + \phi_i(x_0, x, u_0), \quad i = 1, \cdots, n-1 \\ \dot{x}_n &= u_1 + \phi_n(x_0, x, u_0) \end{aligned} \tag{4}$$

where $x_0 \in R$ and $x = (x_1, \cdots, x_n)^T \in R^n$ are system states, $u_0 \in R$ and $u_1 \in R$ are control inputs; and ϕ_i's denote the input and states driven uncertainties, which are called as the nonlinear drifts of the system (4).

The objective of this paper is to design a state feedback controller of the form

$$u_0 = u_0(x_0), \quad |u_0(x_0)| \le M, \quad u_1 = u_1(x_0, x) \tag{5}$$

where M is a known bound of u_0, such that the states of closed-loop system are globally asymptotically regulated to zero.

To this end, the following assumption is imposed on system (4).

Assumption 1. For $i = 1, \cdots, n$, there are nonnegative smooth functions $\varphi_i(x_0, x_1, \cdots, x_i)$ such that

$$|\phi_i(x_0, x, u_0)| \le (|x_1| + \cdots + |x_i|)\varphi_i(\cdot)$$

Remark 1. Assumption is common and similar to the one usually imposed on the nonholonimic systems [10,17,18]. It implies that the origin is the equilibrium point of system (4).

The following definitions and lemmas will serve as the basis of the coming control design and performance analysis.

3 Robust controller design

In this section, we proceed to design a robust controller based on backstepping technique. For clarity, the case that $x_0(t_0) \neq 0$ is considered first. Then the case where the initial $x_0(t_0) = 0$ is dealt later. The inherently structure of system (4) suggests that we should design the control inputs u_0 and u_1 in two separate stages.

3.1 Design u_0 for x_0-subsystem

For x_0-subsystem, we take the following control law

$$u_0(x_0) = -k_0\sigma(x_0) \tag{6}$$

where $k_0 > 0$ is a design constant and

$$\sigma(x_0) = \begin{cases} sign(x_0), & |x_0| > \varepsilon \\ x, & |x_0| \le \varepsilon \end{cases} \tag{7}$$

for a small constant $\varepsilon > 0$ to be determined later.

Remark 2. From (6) and (7), it can clearly be seen that the control law u_0 is bounded by a constant $k_0\varepsilon$, this is, by choosing design parameters k_0 and ε as $k_0\varepsilon < M$, the control law $|u_0(x_0)| \le M$ is guaranteed.

Under (6), the first result of this paper is established, which is crucial for the input-state-scaling transformation in what follows.

Lemma 1. For any initial condition $x_0(t_0) \neq 0$, where $t_0 \ge 0$, the corresponding solution $x_0(t)$ exists and globally asymptotically converges to zero. Furthermore, the control u_0 given by (4) also exists and does not cross zero.

Proof. Taking the Lyapunov function $V_0 = x_0^2/2$, a simple computation gives

$$\begin{aligned} \dot{V}_0 &\le \begin{cases} -k_0|x_0|, & |x_0| > \varepsilon \\ -k_0x_0^2, & |x_0| \le \varepsilon \end{cases} \\ &\le \begin{cases} -k_0V_0^{1/2}, & |x_0| > \varepsilon \\ -2k_0V_0, & |x_0| \le \varepsilon \end{cases} \end{aligned} \tag{8}$$

from which, we can conclude that $x_0(t)$ exists and $x_0(t) \to 0$ as $t \to \infty$.

Next, we will show that $x_0(t)$ does not cross zero. Obviously, it suffices to prove the statement in the case where $|x_0(t)| \le \varepsilon$. In this case, under the control law (6), the x_0-subsystem becomes

$$\dot{x}_0 = -k_0x_0 \tag{9}$$

Therefore, the solution of x_0- subsystem can be expressed as

$$x_0(t) = x_0(t_0)e^{-k_0(t-t_0)}$$

Consequently, x_0 can be zero only at $t = t_0$, when $x(t_0) = 0$ or $t = \infty$. Since $x_0(t_0) \neq 0$ is assumed, it is concluded that x_0 does not cross zero for all $t \in (t_0, \infty)$ provided that $x_0(t_0) \neq 0$. Furthermore, we can see from (6) that the u_0 exists, does not cross zero for all $t \in (t_0, \infty)$ independent of the x-subsystem and satisfies $\lim_{t\to\infty} u_0(t) = 0$. Thus, the proof of Lemma 1 is completed.

3.2 Input-state-scaling transformation

From Lemma 1, we can see the x_0-state in (4) can be globally regulated to zero via u_0 in (6) as $t \to \infty$. However, in the limit case, x_0 will converge to the origin, which will cause serious trouble in controlling the x-subsystem via the control input u_1. This difficulty can be well addressed by utilizing the following discontinuous input-state scaling transformation:

$$z_i = \frac{x_i}{u_0^{n-i}} \quad i = 1, \cdots, n \tag{10}$$

Under the new z-coordinates, the x-subsystem is transformed into

$$\begin{aligned} \dot{z}_i &= z_{i+1} + f_i(x_0, z) \\ \dot{z}_n &= u_1 + f_n(x_0, z) \end{aligned} \tag{11}$$

where

$$f_i(x_0, z) = \frac{\phi_i(x_0, x, u_0)}{u_0^{n-i}} - (n-i)z_i\frac{\dot{u}_0}{u_0} \tag{12}$$

By Assumption 1 and transformation (10), we easily obtain the following estimation for nonlinear function f_i.

Lemma 2. For $i = 1, \cdots, n$, there are nonnegative smooth functions γ_i such that

$$|f_i(x_0, z)| \le (|z_1| + \cdots + |z_i|)\gamma_i(x_0, z_1, \cdots, z_i) \tag{13}$$

Proof. In view of (10) and (12), we have

$$\begin{aligned} &|f_i(x_0, z)| \\ &\le \frac{(|x_1| + \cdots + |x_i|)}{|u_0^{n-i}|}\varphi_i(\cdot) + (n-i)|z_i|\left|\frac{\dot{u}_0}{u_0}\right| \\ &= \frac{(|z_1u_0^{n-1}| + \cdots + |z_iu_0^{n-i}|)}{|u_0^{n-i}|}\varphi_i(\cdot) \\ &\quad +(n-i)|z_i| \times \begin{cases} |\frac{\dot{u}_0}{u_0}|, & |x_0| > \varepsilon \\ |\frac{\dot{u}_0}{u_0}|, & |x_0| \le \varepsilon \end{cases} \\ &= \frac{(|z_1u_0^{n-1}| + \cdots + |z_iu_0^{n-i}|)}{|u_0^{n-i}|}\varphi_i(\cdot) \\ &+(n-i)|z_i| \times \begin{cases} 0, & |x_0| > \varepsilon \\ k_0, & |x_0| \le \varepsilon \end{cases} \\ &\le (|z_1| + \cdots + |z_i|)\gamma_i(x_0, z_1, \cdots, z_i) \end{aligned} \tag{14}$$

3.3 Backstepping Design for u_1

In this subsection, the controller u_1 will be recursively constructed by applying backstepping technique to system (11).

Step 1. Consider the Lyapunov function $V_1 = z_1^2/2$. From (11) and (13), it follows that

$$\dot{V}_1 \le z_1z_2 + z_1^2\gamma_1(x_0, z_1) \tag{15}$$

With the choice of the virtual controller

$$z_2^* = -z_1(n + \gamma_1(x_0, z_1)) := -z_1\beta_1(x_0, z_1) \tag{16}$$

we have

$$\dot{V}_1 \le -nz_1^2 + z_1(z_2 - z_2^*) \tag{17}$$

Step i $(i = 2, \cdots, n)$**.** Suppose at step $i-1$, we have designed a set of smooth virtual controllers $z_1^*, \cdots, z_i^*$ defined by

$$\begin{array}{rclrcl} z_1^* & = & 0 & \xi_1 & = & z_1 - z_1^* \\ z_2^* & = & -\xi_1\beta_1(\cdot) & \xi_2 & = & z_2 - z_2^* \\ \vdots & & & \vdots & & \\ z_i^* & = & -\xi_{i-1}\beta_{i-1}(\cdot) & \xi_i & = & z_i - z_i^* \end{array} \tag{18}$$

with $\beta_1(x_0, \xi_1) > 0, \cdots, \beta_{i-1}(x_0, \xi_1, \cdots, \xi_{i-1}) > 0$ being smooth, such that

$$\begin{aligned} \dot{V}_{i-1} \le & -(n-i+2)(\xi_1^2 + \cdots + \xi_{i-1}^2) \\ & + \xi_{i-1}(z_i - z_i^*) \end{aligned} \tag{19}$$

We intend to establish a similar property for $(z_1, \cdots, z_i)$-subsystem. Consider the Lyapunov function

$$V_i(\xi_1, \cdots, \xi_i) = V_{i-1}(\xi_1, \cdots, \xi_{i-1}) + \frac{1}{2}\xi_i^2 \tag{20}$$

Clearly

$$\begin{aligned} \dot{V}_i \le & -(n-i+2)(\xi_1^2 + \cdots + \xi_{i-1}^2) + \xi_{i-1}(z_i - z_i^*) \\ & + \xi_i\Big(z_{i+1} + f_i - \frac{\partial z_i^*}{\partial x_0}u_0 - \sum_{j=1}^{i-1}\frac{\partial z_i^*}{\partial z_j}(z_{j+1} + f_j)\Big) \end{aligned} \tag{21}$$

Now we estimate each term on the right-hand side of (21). First, it follows (18) that

$$\xi_{i-1}(z_i - z_i^*) \le \frac{1}{4}\xi_{i-1}^2 + \xi_i^2\sigma_{i1} \tag{22}$$

where σ_{i1} is a positive constant.

Noting that $z_i^* = \xi_{i-1}\beta_{i-1}$, it implies that z_i^* satisfies

$$z_i^*(x_0, 0, \cdots, 0) = 0, \quad \frac{\partial z_i^*}{\partial x_0}(x_0, 0, \cdots, 0) = 0 \tag{23}$$

from (23), (13) and (18), after lengthy but simple calculations based on the completion of squares, there is a smooth nonnegative function σ_{i2} such that

$$\begin{aligned} & \xi_i\Big(f_i - \frac{\partial z_i^*}{\partial x_0}u_0 - \sum_{j=1}^{i-1}\frac{\partial z_i^*}{\partial z_j}(z_{j+1} + f_j)\Big) \\ & \le \frac{3}{4}\sum_{j=1}^{i-1}\xi_j^2 + \xi_i^2\sigma_{i2}(x_0, \xi_1, \cdots, \xi_i) \end{aligned} \tag{24}$$

Substituting(22) and (24) into (21) gives

$$\begin{aligned} \dot{V}_i \le & -(n-i+1)(\xi_1^2 + \cdots + \xi_{i-1}^2) \\ & + \xi_i z_{i+1} + \xi_i^2(\sigma_{i1} + \sigma_{i2}) \end{aligned} \tag{25}$$

Now, it easy to see that the smooth virtual controller

$$\begin{aligned} z_{i+1}^* = & -\xi_i(n-i+1+\sigma_{i1}+\sigma_{i2}) \\ := & -\xi_i\beta_i(x_0, \xi_1, \cdots, \xi_i) \end{aligned} \tag{26}$$

renders

$$\dot{V}_i \le -(n-i+1)(\xi_1^2 + \cdots + \xi_i^2) + \xi_i(z_{i+1} - z_{i+1}^*) \tag{27}$$

As $i = n$, the last step, we can construct explicitly a change of coordinates $(\xi_1, \cdots, \xi_n)$, a positive-definite and proper Lyapunov function $V_n(\xi_1 \cdots, \xi_n)$ and a smooth controller z_{n+1}^* of form (26) such that

$$\dot{V}_n \le -(\xi_1^2 + \cdots + \xi_n^2) + \xi_n(u_1 - z_{n+1}^*) \tag{28}$$

Therefore, choosing the smooth actual control u_1 as

$$u_1 = z_{n+1}^* = \xi_n\beta_n(x_0, \xi_1 \cdots, \xi_n) \tag{29}$$

such that

$$\dot{V}_n \le -(\xi_1^2 + \cdots + \xi_n^2) \tag{30}$$

which implies $\lim_{t\to\infty} z(t) = 0$. According to the input-state-scaling transformation(10), we conclude that $\lim_{t\to\infty} x(t) = 0$.

The above analysis is summarized into the following theorem:

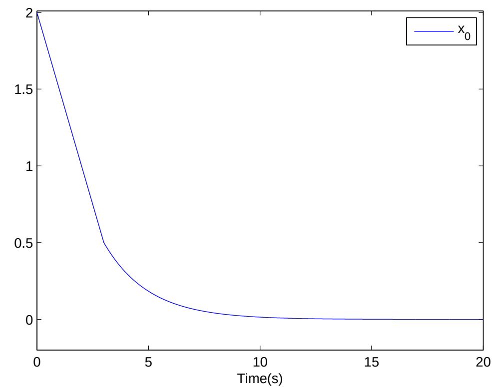
Theorem 1. For system (4), under Assumption 1, if control law (6) and the full feedback control law(29) are applied, the globally asymptotic regulation of the closed-loop system is achieved for $x_0(t_0) \ne 0$.

3.4 Switching controller and main results

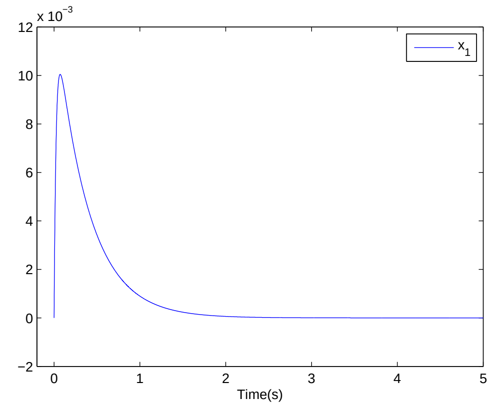
Without loss of generality, we assume that $t_0 = 0$. When the initial state $x_0(0) \ne 0$, we have given controller (6) and (29) for u_0 and u_1 of system (4). Now, we discuss how to select the control laws u_0 and u_1 when $x_0(0) = 0$. In the absence of disturbances, the most commonly used control strategy is using constant control $u_0 = u_0^* \ne 0$ in time interval $[0, t_s)$. In this paper, we also use this method when $x_0(0) = 0$, with u_0 chosen as

$$u_0 = u_0^* \tag{31}$$

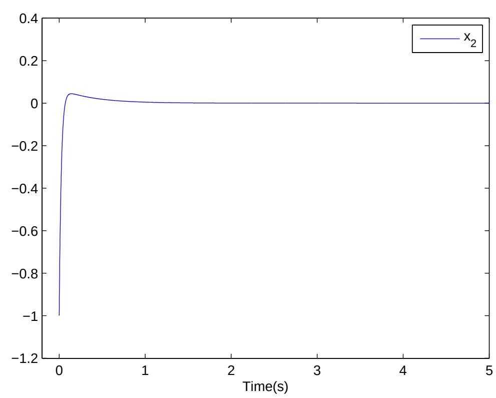
where $0 < u_0^* < M$ is a constant.



(a) x_0



(b) x_1



(c) x_2

Figure 1: System states.

Since $x_0(0) = 0$, under (31), the solution of x_0-subsystem can be expressed as

$$x_0(t) = u_0^* t \tag{32}$$

Obviously, we have x_0 does not escape and $x(t_s) \neq 0$, for given any finite $t_s > 0$. Thus, input-state-scaling transformation for the control design can be carried out.

During the time period $[0, t_s)$, using u_0 defined in (31), new control law $u_1 = u_1^*(x_0, x)$ can be obtained by applying the procedure described in Section 3.3 to the original x-subsystem in (4). Then we can conclude that the x-state of (4) cannot blow up during the time period $[0, t_s)$. Since $x(t_s) \neq 0$ at t_s, we can switch the control input u_0 and u_1 to (6) and (29), respectively.

We are now ready to state the main theorem of our paper.

Theorem 2. Under Assumption 1, if the proposed saturated control design procedure together with the above switching control strategy is applied to system (4), then the states of closed-loop system globally asymptotically regulated to zero.

4 Simulation example

In this section, we illustrate the proposed control design method by means of motivating example in Section 2.1.

Consider the tricycle-type mobile robot subject to saturated angular velocity(see Section 2.1), which is described by

$$\begin{aligned} \dot{x}_0 &= u_0 \\ \dot{x}_1 &= x_2 u_0 \\ \dot{x}_2 &= u_1 - x_1 u_0 \end{aligned} \tag{33}$$

For illustration purpose, in what follows we assume that the boundedness of u_0 is 0.5.

If $x_0(0) = 0$, controls u_0 and u_1 are set as in Section 3.4 in interval $[0, t_s)$, such that $x(t_s) \neq 0$, then we can adopt the controls developed below. Therefore, without loss of generality, we assume that $x_0(0) \neq 0$. For the x_0-subsystem, we can choose the control law

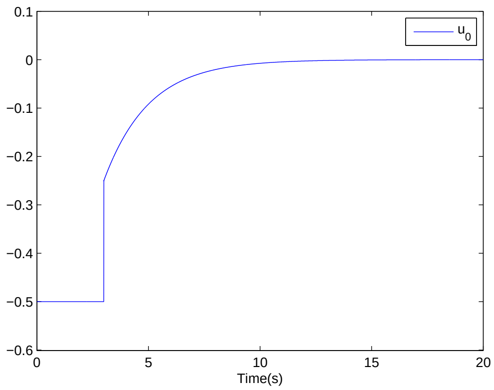
$$u_0(x_0) = \begin{cases} -sign(x_0), & |x_0| > 0.5 \\ -x, & |x_0| \leq 0.5 \end{cases} \tag{34}$$

and introduce the input-state-scaling transformation

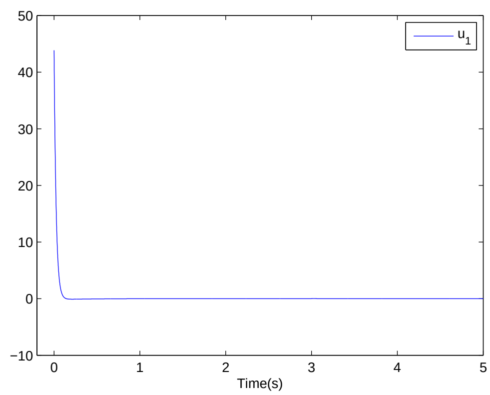
$$z_1 = \frac{x_1}{u_0}, \quad z_2 = x_2 \tag{35}$$

In new z-coordinates, the(x_1, x_2)-subsystem of (33) is rewritten as

$$\begin{aligned} \dot{z}_1 &= z_2 - \frac{\dot{u}_0}{u_0} z_1 \\ \dot{x}_2 &= u_1 - z_1 u_0^2 \end{aligned} \tag{36}$$



(a) u_0



(b) u_1

Figure 2: Control inputs.

Using (34), it is easy to verify that Lemma 2 holds with $\gamma_1 = \gamma_2 = 1$. By applying the design procedure shown in Section 3.3 to system (36), we can obtain the following controller

$$u_1 = -\beta_2(z_2 + \beta_1 z_1) \tag{37}$$

where $\beta_1 = 2.1$ and $\beta_1 = 23.45$. When $(x_0(0), x_1(0), x_2(0))= (2, -1, 1)$, the simulation results are shown in Figs. 1 and 2, from which, it can be seen that the system states are asymptotically regulated to zero and the amplitude of the control input u_0 is bounded by 0.5.

5 Conclusion

This paper has solved the problem of global stabilization by state feedback for a class of uncertain nonholonomic systems in chained form with partial inputs saturation. With the help of the input-state-scaling transformation and backstepping technique, a constructive design procedure for global state feedback control is given. Together with a novel switching control strategy, the designed controller can guarantee that the closed-loop system states are globally asymptotically regulated to zero and the amplitude of the control input u_0 is bounded.

There are some related problems to investigate, e.g., how to design a state feedback stabilizing controller for nonholonomic systems when both input u_0 and input u_1 are bounded. Furthermore, if only partial state vector being measurable, how to design an output feedback stabilizing controller for nonholonomic systems with inputs saturation.

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