

# Stabilization of differential linear repetitive processes saturated systems by state feedback control

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*Abstract:* The stabilization of linear differential linear repetitive processes subject to saturating controls is addressed. Sufficient conditions obtained via a linear matrix inequality (LMI) formulation are stated to guarantee both the local stabilization and the satisfaction of some performance requirements. The method of synthesis consists in determining simultaneously a state feedback control law and an associated domain of safe admissible states for which the stability of the closed-loop system is guaranteed. Two cases are considered: the first one, the control may saturate and limits may be attained. The second one, the control does not saturate and limits are avoided.

*Key-Words:* Differential linear repetitive processes, State-feedback control, Lyapunov functions, Saturation, LMIs.

## 1 Introduction

As is well known, many practical systems can be modeled as two-dimensional 2D systems [18, 19], such as those in image data processing and transmission, thermal processes, gas absorption and water stream heating. During the last few decades, the investigation of 2D systems in the control and signal processing fields has attracted considerable attention and many important results have been reported to the literature. Among these results, the  $H_\infty$  filtering problem for two-dimensional 2D linear systems described by Roesser and FornasiniMarchesini (FM) models in [21, 25, 26, 22, 23, 8, 9, 38, 37, 30, 28, 29, 31, 32, 34], for 2D linear parameter-varying systems, the related work can be found in [36, 29], stability and stabilization of 2D systems in [7, 24, 35, 27],  $H_\infty$  control for 2-D nonlinear systems with delays and the nonfragile  $H_\infty$  and  $l_2 - l_1$  problem for Roesser-type 2D systems in [33]. However, because there is no systematic and general approach to analyze linear repetitive processes systems, many problems still remain.

On the other hand, Many physical systems complete the same finite duration operation over and over again. Repetitive processes have this characteristic where a series of sweeps or passes are made through dynamics defined over a finite duration known as the pass length. Once each pass is complete, the process resets to the original location and the next one begins. The output on each pass is termed the pass profile and the notation for scalar or vector valued

variables is  $y^k(t)$ ,  $0 \leq t \leq \alpha < \infty$ ,  $k \geq 0$ , where  $y$  is the scalar or vector valued variable, the integer  $k$  is the pass number and  $\alpha$  is the pass length. Also the previous pass profile contributes to dynamics of the next one and the result can be oscillations in the pass profile sequence  $\{y\}_k$  that increase in amplitude from pass-to-pass ( $k$ ) and cannot be controlled by standard systems theory.

This paper studies the stability of differential linear repetitive processes with input saturation where the dynamics along the pass are governed by a linear matrix differential equation and the pass-to-pass dynamics by a discrete linear matrix equation. The stability theory ([1]) for linear repetitive processes is of the bounded-input bounded-output (BIBO) type and is based on an abstract model in a Banach space setting that includes a large range of examples as special cases.

On the other hand, an important problem which is always inherent to all dynamical systems is the presence of actuator saturations. The class of systems with saturations has attracted great interest over the three last decades. Even for linear systems, this problem has been an active area of research for many years. In general, nonlinear control has to be used, as only simple cases may be handled via linear control laws,([2]) and the references therein. Two main approaches have been developed in the literature:

The first, the so-called positive invariance approach, is based on the design of controllers which work inside a region of linear behavior where saturations

do not occur, ([3]), ([4]), ([5]), ([6]) and the references therein. One can also cite the work of ([7]), ([10]), ([11]), where the synthesis of the controller is presented as a technique of partial eigenstructure assignment. This resolution was also associated to the constrained regulator problem. In this work, the controller designed with this technique will be referred as an "unsaturating controller".

The second approach allows saturations to take effect while guaranteeing asymptotic stability, ([12]), ([13]) and the references therein. This approach, allowing the control to be saturated, leads to a bounded region of stability along the pass which is ellipsoidal and symmetric. This region can easily be obtained by the resolution of a set of LMIs. In ([14]), besides the saturated character of the control, additional constraints on the increment or rate are taken into account. Further, its results combine this technique with the former one. The output feedback problem is also studied in ([7]), ([15]) using the tools of this approach. In this work, the designed controller with this technique will be referred as a saturating controller.

The main challenge in these two approaches is to obtain a large enough domain of initial states which ensures stability for the closed-loop system, despite the presence of saturations, ([16]), ([17]), ([12]), ([13]). More precisely, this paper investigates the differential linear repetitive processes systems. The obtained the differential linear repetitive processes is then described as a convex combination of  $2^l$  differential linear repetitive processes systems. The aim of this work is the design of stabilizing state-feedback controllers for this class of systems. To this end, sufficient conditions of stabilizability under LMI form are presented. This formulation enables saturating state-feedback controllers to be derived. Furthermore, the unsaturating controller case for differential linear repetitive processes systems is also considered in this paper. In fact, stabilizability conditions are derived such that the linear behavior is always guaranteed. These conditions are also given under LMI form.

**Notation :** we use standard notation throughout this paper. The notation  $P > 0 (< 0)$  is used for positive (negative) definite matrices.  $*$  stands for the symmetric term of the diagonal elements of square symmetric matrix.  $I$  denotes the identity matrix with appropriate dimension. the superscript "T" represents the transpose.  $sym(A)$  indicates  $A^T + A$ ,  $diag(...)$  stands for a block-diagonal matrix.

## 2 Problem formulation

Consider the differential linear repetitive processes described by the following state-space model over  $0 \leq t \leq \beta, k \geq 0$ :

$$\begin{aligned} \dot{x}_{k+1}(t) &= Ax_{k+1}(t) + B_0y_k(t) + Bsat(u_{k+1}(t)) \\ y_{k+1}(t) &= Cx_{k+1}(t) + D_0y_k(t) + Dsat(u_{k+1}(t)) \end{aligned} \quad (1)$$

where on pass  $k$ ,  $x_k(t) \in \mathbb{R}^n$  is the state vector and  $y_k(t) \in \mathbb{R}^m$  is the pass profile vector, and  $u_{k+1}(t) \in \mathbb{R}^l$  is the control vector. respectively;  $A, B_0, B, C, D_0, D$  are time-invariant real matrices with appropriate dimensions.

the state initial vector on each pass and the initial pass profile (on pass 0). The form of these considered here is

$$\begin{aligned} x_{k+1}(0) &= d_{k+1}, k \geq 0 \\ y_0(t) &= f(t) \end{aligned} \quad (2)$$

The saturation function used here is the standard symmetric one defined as follows for  $i = 1, \dots, l$ :

$$sat(u) = (sat(u_i)) = \begin{cases} 1 & \text{if } u_i > 1 \\ u_i & \text{if } -1 \leq u_i \leq 1 \\ -1 & \text{if } u_i < -1 \end{cases} \quad (3)$$

Note that in the general case of saturations, when maximums are any positive real numbers (i.e., the  $i$ th element of the real control vector  $w$  saturates at  $Q_i > 0$ ), the change of variables  $u_i = \frac{w_i}{Q_i}$  can be used: to use directly the definition of the saturation function in (3), it is then only necessary to replace the original matrix  $B_u = \begin{bmatrix} B \\ D \end{bmatrix}$  with  $B_u diag(Q_1, \dots, Q_l)$  for the differential linear repetitive processes system (1).

Further, the state-feedback control is used such that:

$$u_{k+1}(t) = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x_{k+1}(t) \\ y_k(t) \end{bmatrix} \quad (4)$$

where matrix  $K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$  is the state-feedback gain to be designed.

Furthermore, define the sets  $\varepsilon(P, \rho)$  and  $\mathcal{L}(H)$  as follows:

$$\varepsilon(P, \rho) = \{x \in \mathbb{R}^n / x^T P x \leq \rho; P = P^T > 0\} \quad (5)$$

$$\mathcal{L}(H) = \{x \in \mathbb{R}^n / |(H)_{ix}| \leq 1\} \quad (6)$$

where  $\rho$  is a positive scalar.

The problem we are addressing thereafter is to find stabilizing state-feedback controllers for the differential linear repetitive processes system (1) with saturation on the control (3) by using state-feedback control (4). We address the problem from two points of view: first, saturating controls are allowed, so nonlinear behavior may occur (Thus, a saturating controller is used). Second, the behavior is limited to be linear, so saturating controls are not allowed (an un saturating controller is designed).

### 3 Preliminaries

This section is devoted to some preliminaries useful to the development in the sequel: the first lemma makes it possible to write the saturated closed-loop system as a convex combination of  $2^l$  linear systems. Besides, conditions of stability for the differential linear repetitive processes systems are then presented. Finally, a technical lemma giving a sufficient stability condition is provided.

**Lemma 3.1** ([12]) *Let  $u \in \mathbb{R}^l$  and  $v \in \mathbb{R}^l$ , suppose that  $|v_i| \leq 1, i = 1, \dots, l$  then*

$$sat(u) \in co\{D_s u + D_s^- v\}, s \in [1, N],$$

where the  $D_s$  are all the different diagonal matrices with elements either 1 or 0,  $D_s^- = I_l - D_s, N = 2^l$  and  $co$  stands for the convex hull: in such a case, there exist  $\delta_1 \geq 0, \dots, \delta_N \geq 0$ , with  $\sum_{s=1}^N \delta_s = 1$ , such that

$$sat(u) = \sum_{s=1}^N \delta_s (D_s u + D_s^- v) \quad (7)$$

This lemma is used to rewrite the saturated control problem using an auxiliary control  $v$  that fulfills  $|v_i| \leq 1$ . Hence, in state-feedback control, for two given feedback matrices  $K$  and  $H$  with  $u = Kx$  and  $v = Hx$ , such that  $|(Hx)_i| \leq 1$ , by Lemma 1, there exist  $\delta_1 \geq 0, \dots, \delta_N \geq 0$ , with  $\sum_{s=1}^N \delta_s = 1$ , such that

$$sat(Kx) = \sum_{s=1}^N \delta_s (D_s Kx + D_s^- Hx) \quad (8)$$

The result of Lemma 1 can be extended to differential linear repetitive processes systems since the reasoning depends on the saturation function and not on the number of dimensions (independent variables on

which the control depends). Thus, (7) can be written as follows:

$$sat(u_{k+1}(t)) = \sum_{s=1}^N \delta_s(k, t) (D_s u_{k+1}(t) + D_s^- v_{k+1}(t)) \quad (9)$$

Hence, using the state-feedback control (4) and the fact that  $v_{k+1}(t) = H \begin{bmatrix} x_{k+1}(t) \\ y_k(t) \end{bmatrix}$  with  $H = [H_1 \ H_2]$  and  $\xi_k(t) = \begin{bmatrix} x_{k+1}(t) \\ y_k(t) \end{bmatrix} \in \mathcal{L}(H)$  the closed-loop saturated differential linear repetitive processes system can be rewritten as

$$\begin{bmatrix} \dot{x}_{k+1}(t) \\ y_{k+1}(t) \end{bmatrix} = \mathcal{A} \xi_k(t) + \mathcal{B} \sum_{s=1}^N \delta_s(k, t) (D_s K + D_s^- H) \xi_k(t)$$

where matrices  $\mathcal{A}$  and  $\mathcal{B}$  are given by

$$\mathcal{A} = \begin{bmatrix} A & B_0 \\ C & D_0 \end{bmatrix}; \mathcal{B} = \begin{bmatrix} B \\ D \end{bmatrix} \text{ That is}$$

$$\begin{bmatrix} \dot{x}_{k+1}(t) \\ y_{k+1}(t) \end{bmatrix} = \sum_{s=1}^N \delta_s(k, t) \tilde{A}_s \xi_k(t) = \tilde{A}(\delta) \xi_k(t) \quad (10)$$

where matrices  $\tilde{A}(\delta)$  and  $\tilde{A}_s$ , are given by

$$\begin{aligned} \tilde{A}(\delta) &= \sum_{s=1}^N \delta_s(k, t) \tilde{A}_s; \tilde{A}_s = \begin{bmatrix} \tilde{A}_s^s & \tilde{B}_0^s \\ \tilde{C}^s & \tilde{D}_0^s \end{bmatrix} \\ &= \begin{bmatrix} A + B(D_s K_1 + D_s^- H_1) & B_0 + B(D_s K_2 + D_s^- H_2) \\ C + D(D_s K_1 + D_s^- H_1) & D_0 + D(D_s K_2 + D_s^- H_2) \end{bmatrix} \end{aligned} \quad (11)$$

Consider now the following differential linear repetitive processes autonomous system:

$$\begin{bmatrix} \dot{x}_{k+1}(t) \\ y_{k+1}(t) \end{bmatrix} = \mathcal{A} \xi_k(t) \quad (12)$$

**Theorem 1** [20] *A differential linear repetitive processes described by (12) is stable along the pass if there exist matrices  $0 < P_1 \in \mathbb{R}^{n \times n}$  and  $0 < P_2 \in \mathbb{R}^{m \times m}$  such that the following LMI holds*

$$\begin{bmatrix} P_1 A + A P_1 & P_1 B_0 & C P_2 \\ \star & -P_2 & D_0 P_2 \\ \star & \star & -P_2 \end{bmatrix} < 0, \quad (13)$$

In this case, the following is a Lyapunov function of system (12):

$$\begin{aligned} V(k, t) &= V_1(k, t) + V_2(k, t) \\ &= x_{k+1}^T(t) P_1 x_{k+1}(t) + y_k^T(t) P_2 y_k(t) \end{aligned} \quad (14)$$

**Definition 1 :**The increment of function  $V(k, t)$  given by:

$$\Delta V(k, t) = \dot{V}_1(k, t) + \Delta V_2(k, t) \quad (15)$$

**Lemma 3.2** [39] *A differential linear repetitive processes described by (12) and is stable along the pass if*

$$\Delta V(k, t) < 0, \quad (16)$$

## 4 Main results

With the background of the previous section, sufficient conditions are now given for the stabilization of differential linear repetitive processes saturated systems. The conditions presented so far are only useful for analysis, but not for synthesis. In order to allow the synthesis of stabilizing controllers some transformations to LMI form are then worked out. Thus, this section presents sufficient conditions of stabilizability of the differential linear repetitive processes saturated systems expressed as LMIs. The two cases are considered separately: saturating controller and unsaturating controller.

### 4.1 The saturating controller

**Theorem 2** *For a given scalar  $\rho > 0$ , if there exist matrices  $H_1 \in \mathbb{R}^{l \times n}$ ,  $H_2 \in \mathbb{R}^{l \times m}$ ,  $K_1 \in \mathbb{R}^{l \times n}$ ,  $K_2 \in \mathbb{R}^{l \times m}$ , and symmetric positive definite matrices  $0 < P_1 \in \mathbb{R}^{n \times n}$  and  $0 < P_2 \in \mathbb{R}^{m \times m}$  such that the following LMI conditions hold true, for  $s = 1, \dots, N$ :*

$$\Upsilon(s) = \begin{bmatrix} P_1 \tilde{A}^s + \tilde{A}^{sT} P_1 & P_1 \tilde{B}_0^s & \tilde{C}^{sT} P_2 \\ \star & -P_2 & \tilde{D}_0^{sT} P_2 \\ \star & \star & -P_2 \end{bmatrix} < 0, \quad (17)$$

where matrices  $\tilde{A}^s$ ,  $\tilde{B}_0^s$ ,  $\tilde{C}^s$  and  $\tilde{D}_0^s$  are given by (11), and

$$\varepsilon(P, \rho) \subset \mathcal{L}(H) \quad (18)$$

with  $P = \text{diag}(P_1, P_2)$ , then the differential linear repetitive processes system (12) is stable along the pass  $\forall \xi^0 = \begin{bmatrix} x_{k+1}(0) \\ y_0(t) \end{bmatrix} \in \varepsilon(P, \rho)$ .

**Prof:** Assume that condition (18) holds true; using the condition of stability (13) for the closed loop system given by (10), one obtains  $\Upsilon(s) < 0$ , for  $s = 1, \dots, N$ .  $\square$

The previous result states the stabilizability condition along the pass for the closed-loop system. In the next, the LMI formulation of these conditions is derived.

The state-feedback saturating controller can then be synthesized.

**Corollary 1:**For a given scalar  $\rho > 0$ , if there exist matrices  $Z_1, Z_2, U_1, U_2, W_1 = W_1^T > 0$ , and  $W_2 = W_2^T > 0$  such that the following LMIs hold true:

$$\Psi(s) = \begin{bmatrix} \Psi_{11}^s + \Psi_{11}^{sT} & \Psi_{12}^s & \Psi_{13}^s \\ \star & -W_2 & \Psi_{23}^s \\ \star & \star & -W_2 \end{bmatrix} < 0, s = 1, \dots, N \quad (19)$$

$$\begin{bmatrix} \mu & (U_1)_i & (U_2)_i \\ \star & W_1 & 0 \\ \star & \star & W_2 \end{bmatrix} > 0, i = 1, \dots, l \quad (20)$$

where  $(U_1)_i$  and  $(U_2)_i$  hold for the  $i$ th row of matrices  $U_1$  and  $U_2$  respectively;  $\mu = 1/\rho$  while matrices  $\Psi_{11}^s, \Psi_{12}^s, \Psi_{13}^s, \Psi_{23}^s$  are given by

$$\Psi_{11}^s = AW_1 + B(D_s Z_1 + D_s^- U_1)$$

$$\Psi_{12}^s = B_0 W_2 + B(D_s Z_2 + D_s^- U_2)$$

$$\Psi_{13}^s = W_1 C^T + (Z_1^T D_s^T + U_1^T D_s^{-T}) D^T$$

$$\Psi_{23}^s = W_2 D_0^T + (Z_2^T D_s^T + U_2^T D_s^{-T}) D^T$$

then the differential linear repetitive processes system (12) is stable along the pass  $\forall \xi^0 = \begin{bmatrix} x_{k+1}(0) \\ y_0(t) \end{bmatrix} \in \varepsilon(P, \rho)$  with  $P = \text{diag}(P_1, P_2)$ , when the controller gain is given by

$$K = [ Z_1 W_1^{-1} \quad Z_2 W_2^{-1} ] \quad (21)$$

Moreover, the set  $\mathcal{L}(H)$  is given by (8) with

$$H = [ U_1 W_1^{-1} \quad U_2 W_2^{-1} ] \quad (22)$$

**Prof:**Post- and pre-multiply  $\Upsilon(s)$  by the following matrix:

$$\Theta = \text{diag}(P_1^{-1}, P_2^{-1}, P_2^{-1})$$

Replacing matrices  $\tilde{A}^s, \tilde{B}_0^s, \tilde{C}^s$  and  $\tilde{D}_0^s$  by their expressions given by (11)  $\forall s \in [1; N]$ , one obtains (19) with  $W = \text{diag}(W_1, W_2)$ ,  $W_i = P_i^{-1}$ ,  $Z_i = K_i W_i$  and  $U_i = H_i W_i$ , for  $i = 1, 2$ .

On the other hand, the inclusion (18) is equivalent to  $\rho(H)_i P^{-1}(H)_i^T \leq 1, i = 1, \dots, l$ . Developing equivalently as follows:  $\rho(HW)_i W^{-1}(HW)_i^T \leq 1$ , that is  $\rho(U)_i W^{-1}(U)_i^T \leq 1$ , Using Schur complement, one obtains:

$$\begin{bmatrix} \mu & (U)_i \\ \star & W \end{bmatrix} > 0, i = 1, \dots, l \quad (23)$$

Finally, using  $\mu = 1/\rho$ ,  $W = \text{diag}(W_1, W_2)$  and  $U = [U_1, U_2]$ , the LMIs (20) follows.  $\square$

**Example 1:**

As an example, the metal rolling process is considered. This process is an extremely common industrial process where, in essence, deformation of the workpiece takes place between two rolls with parallel axes revolving in opposite directions.

Thus the following linear differential equation represent Metal Rolling dynamics: ([40]).

$$\ddot{S}_k(t) + \frac{\alpha}{M}S_k(t) = \frac{\alpha}{\alpha_1}\ddot{S}_{k-1}(t) + \frac{\alpha}{M}S_{k-1}(t) - \frac{\alpha}{M\alpha_2}F_M(t) \tag{24}$$

Where:

$S_k(t)$ :current passes through the rolls.

$S_{k-1}(t)$ :previous passes through the rolls.

$M$ : is the lumped mass of the roll-gap adjusting mechanism

$\alpha_1$ : the stiffness of the adjustment mechanism spring

$\alpha_2$ : the hardness of the metal strip

$\alpha = \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$ :the composite stiffness of the metal strip and the roll mechanism

$F_M(t)$ :the force developed by the motor. is the input function which is assumed here to be constrained as  $|F_M(t)| \leq F_{Mmax}$  (with  $F_{Mmax}$  represent the maximum driving force)

The linear differential equation (17) can be modeled as a system (1) by imposing :

$$x_{k+1}(t) = \begin{pmatrix} s_k(t) \\ \dot{s}_k(t) \end{pmatrix}, y_k(t) = \begin{pmatrix} s_{k-1}(t) \\ \ddot{s}_{k-1}(t) \end{pmatrix}$$

Thus The linear differential equation (23) can be writhed:

$$\dot{x}_{k+1}(t) = \begin{pmatrix} 0 & 1 \\ -\frac{\alpha}{M} & 0 \end{pmatrix} x_{k+1}(t) + \begin{pmatrix} 0 & 0 \\ \frac{\alpha}{M} & \frac{\alpha}{\alpha_1} \end{pmatrix} y_k(t) + \begin{pmatrix} 0 \\ -\frac{\alpha}{M\alpha_2} \end{pmatrix} F_M(t)$$

$$y_{k+1}(t) = \begin{pmatrix} 1 & 0 \\ -\frac{\alpha}{M} & 0 \end{pmatrix} x_{k+1}(t) + \begin{pmatrix} 0 & 0 \\ \frac{\alpha}{M} & \frac{\alpha}{\alpha_1} \end{pmatrix} y_k(t) + \begin{pmatrix} 0 \\ -\frac{\alpha}{M\alpha_2} \end{pmatrix} F_M(t)$$

In these design studies, the data used are  $\alpha_1 = 600$ ,  $\alpha_2 = 2000$  and  $M = 100$  This yields  $\alpha = 461.54$ , resulting in the state-space model matrices in (1):

$$A = \begin{pmatrix} 0 & 1 \\ -4.61538 & 0 \end{pmatrix}, B_0 = \begin{pmatrix} 0 & 0 \\ 4.61538 & 0.76923 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 \\ -4.61538 & 0 \end{pmatrix}, D_0 = \begin{pmatrix} 0 & 0 \\ 4.61538 & 0.76923 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ -0.00231 \end{pmatrix}, D = \begin{pmatrix} 0 \\ -0.00231 \end{pmatrix}$$

For this data and  $\rho = 100$ , This model is both unstable along the pass (A has eigenvalues  $(0 + 2.1483i, 0 - 2.1483i)$ ) Corollary 1 can be successfully applied here since the LMIs (19) and (20) are feasible, with a solution give

$$\begin{aligned} K_1 &= [542.6908 \quad 434.9946] \\ K_2 &= 10^3[1.6313 \quad 0.2719] \\ H_1 &= [189.7717 \quad 143.6449] \\ H_2 &= [810.1543 \quad 135.0257] \end{aligned}$$

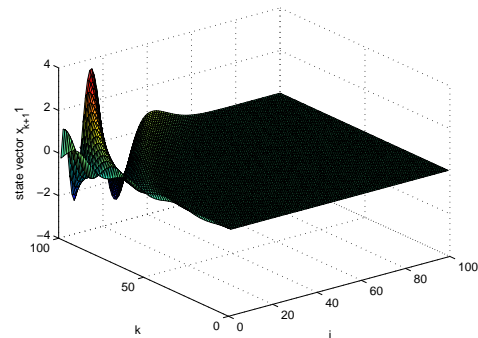


Figure 1. The evolution of the first component of  $x_{k+1}(t)$  using the saturating controller

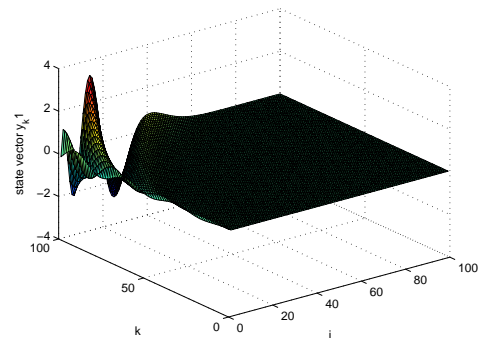


Figure 2. The evolution of the first component of  $y_k(t)$  using the saturating controller

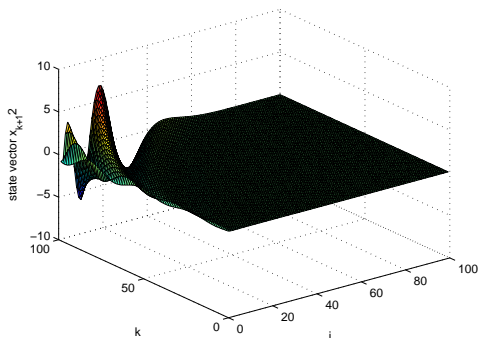


Figure 3. The evolution of the second component of  $x_{k+1}(t)$  using the saturating controller

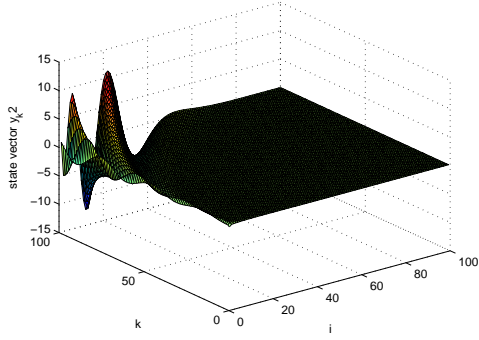


Figure 4. The evolution of the second component of  $y_k(t)$  using the saturating controller

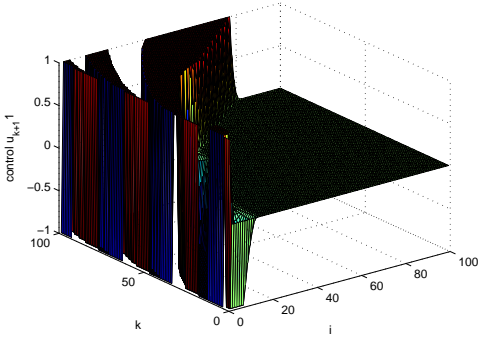


Figure 5. The evolution of the saturating controller  $u_{k+1}(t)$

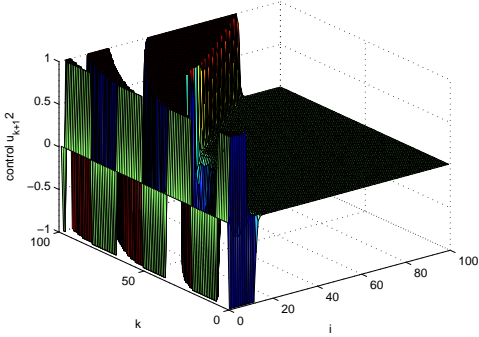


Figure 6. The evolution of the saturating controller  $u_{k+1}(t)$

### 4.2 The unsaturating controller

Consider now the differential linear repetitive processes system (1) with constrained control (3). In the previous section, the design of a saturating controller was studied (saturation of control was allowed), whereas in this section, saturation is not allowed and the synthesis will guarantee that the state evolves inside a region of linear behavior given by  $\mathcal{L}(F)$  (F being the controller gain). Thus, this case can be seen as a particular case of the saturating one.

**Theorem 3** For a given scalar  $\rho > 0$ , if there exist

matrices  $F_1 \in \mathbb{R}^{l \times n}$ ,  $F_2 \in \mathbb{R}^{l \times m}$ , and symmetric positive definite matrices  $0 < P_1 \in \mathbb{R}^{n \times n}$  and  $0 < P_2 \in \mathbb{R}^{m \times m}$  such that the following LMI conditions hold true:

$$\Upsilon(s) = \begin{bmatrix} P_1 \tilde{A} + \tilde{A}^T P_1 & P_1 \tilde{B}_0 & \tilde{C}^T P_2 \\ \star & -P_2 & \tilde{D}_0^T P_2 \\ \star & \star & -P_2 \end{bmatrix} < 0, \quad (25)$$

$$\varepsilon(P, \rho) \subset \mathcal{L}(H) \quad (26)$$

where

$$\begin{bmatrix} \tilde{A} & \tilde{B}_0 \\ \tilde{C} & \tilde{D}_0 \end{bmatrix} = \begin{bmatrix} A + BF_1 & B_0 + BF_2 \\ C + DF_1 & D_0 + DF_2 \end{bmatrix}$$

and  $P = \text{diag}(P_1, P_2)$ , then the differential linear repetitive processes system (12) is stable along the pass  $\forall \xi^0 = \begin{bmatrix} x_{k+1}(0) \\ y_0(t) \end{bmatrix} \in \varepsilon(P, \rho)$ .

**Proof:** The proof follows readily if one replaces K by F in the proof of Theorem 4.1 and removes the saturated convex writing of the control. This can be done, as in this case, the state is restricted to evolve inside the linear region of behavior given by condition (26).  $\square$

In the next result, the LMI formulation of these conditions, that enables the unsaturating state-feedback control to be derived, is given:

**Corollary 1:** For a given scalar  $\rho > 0$ , if there exist matrices  $Z_1, Z_2, W_1 = W_1^T > 0$ , and  $W_2 = W_2^T > 0$  such that the following LMIs hold true:

$$\begin{bmatrix} \Psi_{11} + \Psi_{11}^T & \Psi_{12} & \Psi_{13} \\ \star & -W_2 & \Psi_{23} \\ \star & \star & -W_2 \end{bmatrix} < 0, \quad (27)$$

$$\begin{bmatrix} \mu & (Z_1)_i & (Z_2)_i \\ \star & W_1 & 0 \\ \star & \star & W_2 \end{bmatrix} > 0, i = 1, \dots, l \quad (28)$$

where  $(Z_1)_i$  and  $(Z_2)_i$  hold for the  $i$ th row of matrices  $Z_1$  and  $Z_2$  respectively;  $\mu = 1/\rho$  while matrices  $\Psi_{11}, \Psi_{12}, \Psi_{13}, \Psi_{23}$  are given by

$$\Psi_{11} = AW_1 + BZ_1$$

$$\Psi_{12} = B_0W_2 + BZ_2$$

$$\Psi_{13} = W_1C^T + Z_1^TD^T$$

$$\Psi_{23} = W_2D_0^T + Z_2^TD^T$$

then the differential linear repetitive processes system (12) is stable along the pass  $\forall \xi^0 = \begin{bmatrix} x_{k+1}(0) \\ y_0(t) \end{bmatrix} \in \varepsilon(P, \rho)$  with  $P = \text{diag}(W_1^{-1}, W_2^{-1})$ , when the controller gain is given by

$$F = \begin{bmatrix} Z_1 W_1^{-1} & Z_2 W_2^{-1} \end{bmatrix} \quad (29)$$

**Prof:** The proof follows readily from Corollary 1.

**Example 2:**

Consider the same system studied in the saturating controller case of Example 1. If no saturation is allowed in the control signal, Corollary 2 can be used to synthesize the controller. In this case, the LMIs (27) and (28) are feasible, with a solution given by:

$$F_1 = \begin{bmatrix} 231.9679 & 166.5527 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 627.1912 & 104.5319 \end{bmatrix}$$

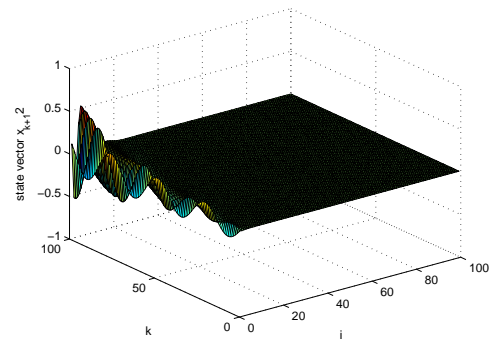


Figure 9. The evolution of the second component of  $x_{k+1}(t)$  using the saturating controller

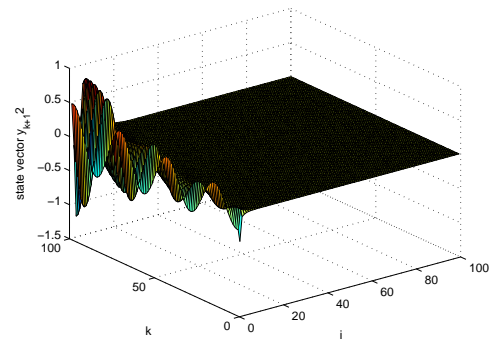


Figure 10. The evolution of the second component of  $y_k(t)$  using the saturating controller

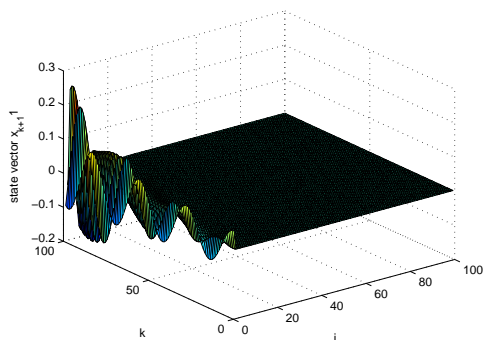


Figure 7. The evolution of the first component of  $x_{k+1}(t)$  using the saturating controller

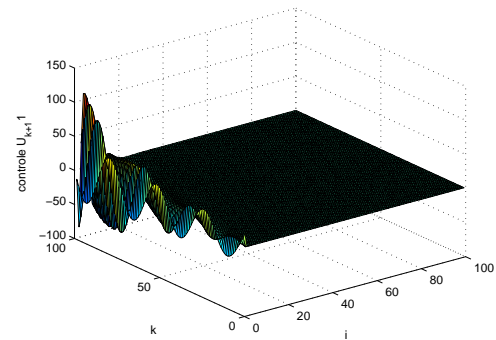


Figure 11. The evolution of the saturating controller  $u_{k+1}(t)$

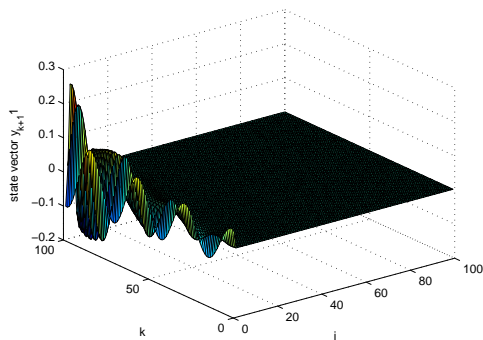


Figure 8. The evolution of the first component of  $y_k(t)$  using the saturating controller

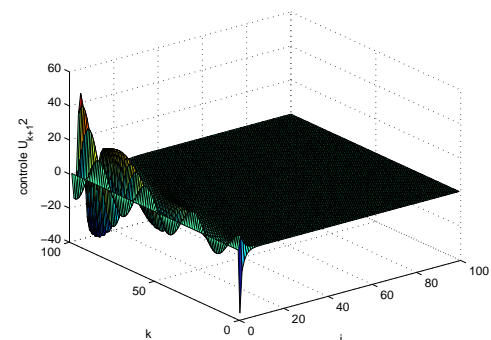


Figure 12. The evolution of the saturating controller  $u_{k+1}(t)$

## 5 Conclusions

In this paper, the problem of the stabilizability of the differential linear repetitive processes saturated systems has been studied using state-feedback control. Two different cases are considered guaranteeing stability along the pass: the design of saturating and unsaturating controllers. The first one allows saturation to take effect, while the second limits the systems evolution to the region of linear behavior. Sufficient conditions of stability along the pass are derived for each case. The synthesis of the required controllers are also given under LMI form. Numerical examples are provided to illustrate the results.

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