# Transient Analysis of Track Circuit Based on Finite Differential Method

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*Abstract:*-In allusion to the research on track circuit are mostly focused on spatial domain at present, the finite differential method is put forward to the time domain analysis of track circuit in this paper, in the theory of partial differential equation numerical solution. A new differential formation of track circuit based on Lax format is established and according to the lumped parameter equivalent circuit, the boundary conditions at the beginning and terminal are confirmed. Through example, the time domain solution conform to the transmission characteristic of track circuits, and the accuracy of the method is validated by comparing the simulation result with that of ADI-FDTD method. Therefore, the finite differential method can be used to transient analysis of track circuit.

*Key-Words:*-finite differential method, lumped parameter equivalent circuit, track circuit, time domain solution, transient analysis, ADI-FDTD.

## 1 Introduction

Track circuit can transmit train movement authority and check the track occupancy, it is the most important equipment for traffic safety and operation efficiency [1]. Track circuit analysis is to solve the corresponding responses under a set of given initial track circuit parameters and a specific excitation source, in order to make a reasonable explanation to the signal transmission phenomena. This is depend on the accurate analysis of the transient process of track circuit, therefore in the track circuit analysis, solving the track circuit transmission line equation is the main research content. At present, research on track circuit are mostly concentrated on spatial domain, the research results in time domain analysis are very little. In the spatial domain analysis of track circuits, their working states are distinguished by power values at the receiving end, but when the environment of ballast is very terrible, the working states of track circuits are difficult to determine in spatial domain [2-4]. To apply this principle, it is necessary to distinguish track circuits' state in time domain.

As a result of the limitation of existing mathematical tool, it is very difficult to solving its analytical solution. Therefore, in recent years, many scholars used different methods to research the numerical solutions of transmission line equation from different sides [5], but there is no common analysis method which can apply to any situations for the research of time response of transmission line [6]. In the actual numerical transient analysis, time domain methods were widely adopted, and among these time domain methods, finite differential method has a wide versatility. It is a general way of directly obtaining the time domain response, has been applied to the lossy transmission lines, high voltage transmission line and other scientific fields [7-11]. This method is well suited to the modeling of transient electrical phenomena [12] and wide band analysis, it can be used to analyze all kinds of electromagnetic structures. And compared with the matrix method or the other time domain method, it takes up a small storage space [13]. In addition, it's physical concept is clear, easy to realized by programming.

In this paper, finite differential method is put forward to establish the differential formation based on Lax format and to obtain the time domain solution of track circuit. Through example and compared with

ADI-FDTD method, their simulation results are almost consistent. It show that using finite difference method to solve the time domain solution of track circuit is feasible, it will become a new way to analyze transient process of track circuit.

## 2 Time domain analysis of track circuits based on finite differential method

Equivalent circuit of piece $\Delta x$ of rail lines is shown in Fig.1. The whole track circuit can be considered as a cascade of countless equivalent circuit.

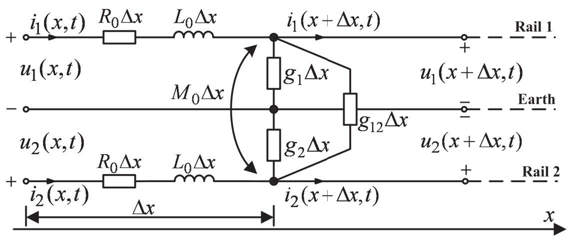


Fig.1 Equivalent circuit of piece $\Delta x$ of rail lines

Each circuit parameters of track circuit distribute uniformly throughout the entire rail lines and are regarded as constants, for different types of rails corresponding to different values respectively. Rail line is uniform distributed parameter circuit, there are two asymmetric leakage currents, the one leakage into the earth through $g_1$ and $g_2$, the other one flow from one rail to another rail through $g_{12}$ which across the surface of ballast and sleeper. The current of rail 1 and rail 2 at distance $x$ and time $t$, respectively described by $i_1(x,t)$ and $i_2(x,t)$. The voltage of rail 1 and rail 2 at distance $x$ and time $t$, respectively described by $u_1(x,t)$ and $u_2(x,t)$.

According to the kirchhoff's law, the transmission equations of track circuits can be written as

$$\begin{cases} -\dfrac{\partial u_1(x,t)}{\partial x} = R_0 i_1(x,t) + L_0\dfrac{\partial i_1(x,t)}{\partial t} + M_0\dfrac{\partial i_2(x,t)}{\partial t} \\ -\dfrac{\partial u_2(x,t)}{\partial x} = R_0 i_2(x,t) + L_0\dfrac{\partial i_2(x,t)}{\partial t} + M_0\dfrac{\partial i_1(x,t)}{\partial t} \\ -\dfrac{\partial i_1(x,t)}{\partial x} = (g_1+g_{12})u_1(x,t) - g_{12}u_2(x,t) \\ -\dfrac{\partial i_2(x,t)}{\partial x} = (g_2+g_{12})u_2(x,t) - g_{12}u_1(x,t) \end{cases} \tag{1}$$

Eq.1 expresses the relationship between voltage and current in two rail ways. It is the basic foundation to research transient response of track circuits.

The basic principle of finite differential method is disperses the partial differential equations to get the differential formation of track circuits firstly, then according to the boundary conditions at the beginning and terminal, values of all the grid points will be work out by iteration process [14].

Compared with other transmission lines, track circuits have leakage currents to the ground, and at present there is no literature acquired a common time domain solution for track circuit. So we must establish a new differential formation according to the theory of partial differential equation numerical solution. Considering that pulse voltage jump may appeared, it will often caused parasitic oscillation to the variables discrete, while Lax differential formation is effective in smooth the pulse jump and eliminate the parasitic oscillation. Therefore, reference to Lax differential formation, build the transmission line equations differential formation of track circuits.

### 2.1 Differential formation

Establishment of differential formation is the key to the whole process. In the case of the solution of partial differential equations, the solution domain in the $x$-$t$ plane must be divided by equal grid of steps $h$ and $k$.

By using centered-difference approximations and forward-difference approximations instead of the first-order spatial derivatives and temporal derivatives. Then, by $\left(u_{m+1}^n + u_{m-1}^n\right)/2$ and $\left(i_{m+1}^n + i_{m-1}^n\right)/2$ instead of $u_m^n$ and $i_m^n$. And after a series of derivation, differential formation was derived

$$\begin{aligned} u_{1,m}^{n+1} &= \frac{u_{1,m+1}^n + u_{1,m-1}^n}{2} \\ &+ \frac{(g_2+g_{12})(i_{1,m+2}^n - i_{1,m-2}^n) + g_{12}(i_{2,m+2}^n - i_{2,m-2}^n)}{4hg} \\ &- \frac{g_2+g_{12}}{4h\left(L_0^2 - M_0^2\right)g}\left\{ \begin{aligned} &\frac{-L_0 k\left[\left(u_{1,m+2}^n - u_{1,m}^n\right) - \left(u_{1,m}^n - u_{1,m-2}^n\right)\right]}{h} \\ &+ \frac{M_0 k\left[\left(u_{2,m+2}^n - u_{2,m}^n\right) - \left(u_{2,m}^n - u_{2,m-2}^n\right)\right]}{h} \\ &+ b\left(i_{1,m+2}^n - i_{1,m-2}^n\right) + a\left(i_{2,m+2}^n - i_{2,m-2}^n\right) \end{aligned} \right\} \\ &- \frac{g_{12}}{4h\left(L_0^2 - M_0^2\right)g}\left\{ \begin{aligned} &\frac{M_0 k\left[\left(u_{1,m+2}^n - u_{1,m}^n\right) - \left(u_{1,m}^n - u_{1,m-2}^n\right)\right]}{h} \\ &- \frac{L_0 k\left[\left(u_{2,m+2}^n - u_{2,m}^n\right) - \left(u_{2,m}^n - u_{2,m-2}^n\right)\right]}{h} \\ &+ a\left(i_{1,m+2}^n - i_{1,m-2}^n\right) + b\left(i_{2,m+2}^n - i_{2,m-2}^n\right) \end{aligned} \right\} \end{aligned} \tag{2}$$

$$u_{2,m}^{n+1}=\frac{u_{2,m+1}^{n}+u_{2,m-1}^{n}}{2}+\frac{g_{12}(i_{1,m+2}^{n}-i_{1,m-2}^{n})+(g_1+g_{12})(i_{2,m+2}^{n}-i_{2,m-2}^{n})}{4hg}-\frac{g_{12}}{4h(L_0^2-M_0^2)g}\left\{\begin{array}{l}\frac{-L_0k\left[\left(u_{1,m+2}^{n}-u_{1,m}^{n}\right)-\left(u_{1,m}^{n}-u_{1,m-2}^{n}\right)\right]}{h}\\+\frac{M_0k\left[\left(u_{2,m+2}^{n}-u_{2,m}^{n}\right)-\left(u_{2,m}^{n}-u_{2,m-2}^{n}\right)\right]}{h}\\+b\left(i_{1,m+2}^{n}-i_{1,m-2}^{n}\right)+a\left(i_{2,m+2}^{n}-i_{2,m-2}^{n}\right)\end{array}\right\}-\frac{g_1+g_{12}}{4h(L_0^2-M_0^2)g}\left\{\begin{array}{l}\frac{M_0k\left[\left(u_{1,m+2}^{n}-u_{1,m}^{n}\right)-\left(u_{1,m}^{n}-u_{1,m-2}^{n}\right)\right]}{h}\\-\frac{L_0k\left[\left(u_{2,m+2}^{n}-u_{2,m}^{n}\right)-\left(u_{2,m}^{n}-u_{2,m-2}^{n}\right)\right]}{h}\\+a\left(i_{1,m+2}^{n}-i_{1,m-2}^{n}\right)+b\left(i_{2,m+2}^{n}-i_{2,m-2}^{n}\right)\end{array}\right\}\tag{3}$$

$$i_{1,m}^{n+1}=\frac{-L_0k(u_{1,m+1}^{n}-u_{1,m-1}^{n})+M_0k(u_{2,m+1}^{n}-u_{2,m-1}^{n})}{2h\left(L_0^2-M_0^2\right)}+\frac{b(i_{1,m+1}^{n}+i_{1,m-1}^{n})+a(i_{2,m+1}^{n}+i_{2,m-1}^{n})}{2\left(L_0^2-M_0^2\right)}\tag{4}$$

$$i_{2,m}^{n+1}=\frac{M_0k(u_{1,m+1}^{n}-u_{1,m-1}^{n})-L_0k(u_{2,m+1}^{n}-u_{2,m-1}^{n})}{2h\left(L_0^2-M_0^2\right)}+\frac{a(i_{1,m+1}^{n}+i_{1,m-1}^{n})+b(i_{2,m+1}^{n}+i_{2,m-1}^{n})}{2\left(L_0^2-M_0^2\right)}\tag{5}$$

Where, $g=g_1g_2+g_1g_{12}+g_2g_{12}$,

$a=R_0M_0k$,

$b=L_0^2-M_0^2-R_0L_0k$ 。

Eq.2 and Eq.3 show voltage differential formation is a two layers six points iterative formula, and Eq.4 and Eq.5 show current differential formation is a two layers three points iterative formula. The basic principle is that voltage and current values in a certain layer can be iterated by voltage and current values in the layer below.

## 2.2 Error analysis

In the derivation process, derivative quotient was insteaded by difference quotient on their grid points, and truncation error was produced .

Truncation error refers to the error between the true solution of differential equation and the true solution of difference equation after neglected some minor items [15].

According to the differential formation of track circuit, under the second order Taylor expansion of $u(x,t)$ about time and the fifth order Taylor expansion of $u(x,t)$ about space in point $(mh,nk)$.

When $t=(n+1)k$, $x=(m+2)h$, $(m+1)h$, $(m-1)h$, $(m-2)h$, we got

$$\frac{u_m^{n+1}-u_m^n}{k}=\left(\frac{\partial u}{\partial t}\right)_m^n+o(k)$$

$$\frac{u_{m+1}^n+u_{m-1}^n}{2}=u_m^n+o\left(h^2\right)$$

$$\frac{u_m^{n+1}-\frac{1}{2}\left(u_{m+1}^n+u_{m-1}^n\right)}{k}=\left(\frac{\partial u}{\partial t}\right)_m^n+o(k)+ro(h),\ r=\frac{h}{k}\tag{6}$$

In the same way, $i(x,t)$ also has the similar expressions in point $(mh,nk)$.

From the above formulas, truncation error of transmission line equation is $R_m^n=o(h+k)$. When $h\to 0$, $k\to 0$, truncation error is also tends to zero. So, this differential formation is valid. It explains that the differential formation is a first-order accuracy, and it is a consistent approximation to partial differential equations.

## 2.3 Convergence and stability analysis

According to Lax equivalence theorem, in the case of a compatible format, the necessary and sufficient condition for convergence is the differential formation is stability. So, the problem can be simplified, only needs to discuss the stability of the differential equations of track circuits.

Assuming that $V_m^n=V^ne^{i\alpha m}$, $W_m^n=W^ne^{i\alpha m}$, they are errors of voltage and current in point $(m,n)$, $\alpha$ is arbitrary parameter, then put them into the differential equations, and written in matrix form after reduction,

$$\begin{bmatrix}V_1^{n+1} & V_2^{n+1} & W_1^{n+1} & W_2^{n+1}\end{bmatrix}^{-1}=G\cdot\begin{bmatrix}V_1^n & V_2^n & W_1^n & W_2^n\end{bmatrix}^{-1}$$

where, $G$ is called growth matrix.

$$G=\begin{pmatrix}a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\\ a_{41} & a_{42} & a_{43} & a_{44}\end{pmatrix}\tag{7}$$

$$a_{11}=a_{22}=\cos\alpha-\frac{\left[(g_2+g_{12})L_0-g_{12}M_0\right]k\sin^2\alpha}{h^2\left(g_1g_2+g_1g_{12}+g_2g_{12}\right)\left(L_0^2-M_0^2\right)},$$

$$a_{12}=a_{21}=\frac{\left[(g_2+g_{12})M_0-g_{12}L_0\right]k\sin^2\alpha}{h^2\left(g_1g_2+g_1g_{12}+g_2g_{12}\right)\left(L_0^2-M_0^2\right)},$$

$$a_{13} = a_{24} = \frac{i[(g_2+g_{12})L_0 - g_{12}M_0]R_0 k \sin\alpha\cos\alpha}{h(g_1g_2+g_1g_{12}+g_2g_{12})(L_0^2-M_0^2)},$$

$$a_{14} = a_{23} = \frac{i[g_{12}L_0 - (g_2+g_{12})M_0]R_0 k \sin\alpha\cos\alpha}{h(g_1g_2+g_1g_{12}+g_2g_{12})(L_0^2-M_0^2)},$$

$$a_{31} = a_{42} = -\frac{iL_0 k\sin\alpha}{h(L_0^2-M_0^2)},$$

$$a_{32} = a_{41} = \frac{iM_0 k\sin\alpha}{h(L_0^2-M_0^2)},$$

$$a_{33} = a_{44} = \frac{(L_0^2-M_0^2-R_0L_0k)\cos\alpha}{(L_0^2-M_0^2)},$$

$$a_{34} = a_{43} = \frac{R_0M_0k\cos\alpha}{(L_0^2-M_0^2)}.$$

$G$ has four characteristic roots, in order to get the sufficient condition for difference equation to keep the differential formation stable, make the absolute values of these 4 roots less than 1, there is

$$\begin{cases} 0 < k < 2\min\left(\frac{L_0-M_0}{R_0}, \frac{L_0+M_0}{R_0}\right) \\ h > \max\left(\sqrt{\frac{k}{(g_1+2g_{12})(L_0-M_0)}}, \sqrt{\frac{k}{g_1(L_0+M_0)}}\right) \end{cases} \tag{8}$$

When satisfies Eq.8, the approximate solution of differential equation will tend close to the accurate solution of difference equation.

## 3 ADI-FDTD method

In order to validate the trust of the time domain solution of track circuit solved by finite differential method, an unconditionally stable FDTD method using the principle of alternating direction implicit (ADI) is adopted. It has the same distribution grid with previous method.

In order to guarantee the stability of this algorithm, the distribution form of voltages and currents are crossed. The space between voltage point and adjacent current point is $\Delta x/2$. Compared with finite differential equations, ADI-FDTD method divide transmission line equations into two parts to calculate the iteration equations from step $n$ to step ($n$+1) [16].

For $u_1(x,t)$, there is

$$-\frac{u_{1,m+1}^{n+1/2}-u_{1,m}^{n+1/2}}{h} = 2L_0\frac{\left(i_{1,m+1/2}^{n+1/2}-i_{1,m+1/2}^{n}\right)}{k} + R_0\frac{\left(i_{1,m+1/2}^{n+1/2}+i_{1,m+1/2}^{n}\right)}{2} + 2M_0\frac{\left(i_{2,m+1/2}^{n+1/2}-i_{2,m+1/2}^{n}\right)}{k} \tag{9}$$

$$-\frac{u_{1,m+1}^{n+1/2}-u_{1,m}^{n+1/2}}{h} = 2L_0\frac{\left(i_{1,m+1/2}^{n+1}-i_{1,m+1/2}^{n+1/2}\right)}{k} + R_0\frac{\left(i_{1,m+1/2}^{n+1}+i_{1,m+1/2}^{n+1/2}\right)}{2} + 2M_0\frac{\left(i_{2,m+1/2}^{n+1}-i_{2,m+1/2}^{n+1/2}\right)}{k} \tag{10}$$

For $u_2(x,t)$, there is

$$-\frac{u_{2,m+1}^{n+1/2}-u_{2,m}^{n+1/2}}{h} = 2L_0\frac{\left(i_{2,m+1/2}^{n+1/2}-i_{2,m+1/2}^{n}\right)}{k} + R_0\frac{\left(i_{2,m+1/2}^{n+1/2}+i_{2,m+1/2}^{n}\right)}{2} + 2M_0\frac{\left(i_{1,m+1/2}^{n+1/2}-i_{1,m+1/2}^{n}\right)}{k}$$

$$-\frac{u_{2,m+1}^{n+1/2}-u_{2,m}^{n+1/2}}{h} = 2L_0\frac{\left(i_{2,m+1/2}^{n+1}-i_{2,m+1/2}^{n+1/2}\right)}{k} + R_0\frac{\left(i_{2,m+1/2}^{n+1}+i_{2,m+1/2}^{n+1/2}\right)}{2} + 2M_0\frac{\left(i_{1,m+1/2}^{n+1}-i_{1,m+1/2}^{n+1/2}\right)}{k}$$

For $i_1(x,t)$, there is

$$-\frac{i_{1,m+1/2}^{n+1/2}-i_{1,m-1/2}^{n+1/2}}{h} = (g_1+g_{12})\frac{\left(u_{1,m}^{n+1/2}+u_{1,m}^{n}\right)}{2} - g_{12}\frac{\left(u_{2,m}^{n+1/2}+u_{2,m}^{n}\right)}{2} \tag{11}$$

$$-\frac{i_{1,m+1/2}^{n+1/2}-i_{1,m-1/2}^{n+1/2}}{h} = (g_1+g_{12})\frac{\left(u_{1,m}^{n+1}+u_{1,m}^{n+1/2}\right)}{2} - g_{12}\frac{\left(u_{2,m}^{n+1}+u_{2,m}^{n+1/2}\right)}{2} \tag{12}$$

For $i_2(x,t)$, there is

$$-\frac{i_{2,m+1/2}^{n+1/2}-i_{2,m-1/2}^{n+1/2}}{h} = (g_2+g_{12})\frac{\left(u_{2,m}^{n+1/2}+u_{2,m}^{n}\right)}{2} - g_{12}\frac{\left(u_{1,m}^{n+1/2}+u_{1,m}^{n}\right)}{2}$$

$$-\frac{i_{2,m+1/2}^{n+1/2}-i_{2,m-1/2}^{n+1/2}}{h} = (g_2+g_{12})\frac{\left(u_{2,m}^{n+1}+u_{2,m}^{n+1/2}\right)}{2} - g_{12}\frac{\left(u_{1,m}^{n+1}+u_{1,m}^{n+1/2}\right)}{2}$$

In the above formulas, Eq.9 and Eq.11 can be used to calculate voltage and current components of rail 1 from step $n$ to step ($n$+1/2), then Eq.10 and Eq.12 can be used to calculate voltage and current components from step ($n$+1/2) to step ($n$+1). Finally, calculate the differential equations from step $n$ to step ($n$+1). It is same to $u_2(x,t)$ and $i_2(x,t)$ of rail 2.

In order to simplified calculation, ignore the wastage and the mutual impedance of the track circuit. For $u_1(x,t)$ and $i_1(x,t)$, equations can be simplified as

$$u_{1,m}^{n+1/2} = -u_{1,m}^{n} - P_2\left(i_{1,m+1/2}^{n+1/2} - i_{1,m-1/2}^{n+1/2}\right) \tag{13}$$

$$u_{1,m}^{n+1} = -u_{1,m}^{n+1/2} - P_2\left(i_{1,m+1/2}^{n+1/2} - i_{1,m-1/2}^{n+1/2}\right) \tag{14}$$

$$i_{1,m+1/2}^{n+1/2} = i_{1,m+1/2}^{n} - P_1\left(u_{1,m+1}^{n+1/2} - u_{1,m}^{n+1/2}\right) \tag{15}$$

$$i_{1,m+1/2}^{n+1} = i_{1,m+1/2}^{n+1/2} - P_1\left(u_{1,m+1}^{n+1/2} - u_{1,m}^{n+1/2}\right) \tag{16}$$

Assuming that $u_m^n = u_0 e^{-j(ik_x\Delta x)}\beta^n$, $i_m^n = i_0 e^{-j(ik_x\Delta x)}\beta^n$, $\beta$ is time growth parameter, then put them into the above equations.

From Eq.13 and Eq.15, we got

$$u_0\left(1+\beta_1^{-1/2}\right)=2jP_2 i_0\sin\left(\frac{k_x\Delta x}{2}\right)$$

$$i_0\left(1-\beta_1^{-1/2}\right)=2jP_1 u_0\sin\left(\frac{k_x\Delta x}{2}\right)$$

So, the time growth parameter in first part is

$$\beta_1=\left(1+4P_1P_2\sin^2\left(\frac{k_x\Delta x}{2}\right)\right)^{-1}$$

From Eq.14 and Eq.16, we got

$$u_0\left(1+\beta_2^{1/2}\right)=2jP_2 i_0\sin\left(\frac{k_x\Delta x}{2}\right)$$

$$i_0\left(1-\beta_2^{1/2}\right)=2jP_1 u_0\sin\left(\frac{k_x\Delta x}{2}\right)$$

So, the time growth parameter in second part is

$$\beta_2=1+4P_1P_2\sin^2\left(\frac{k_x\Delta x}{2}\right)$$

Thus, the total time growth parameter is $|\beta|=|\beta_1||\beta_2|=1$ .This condition can be satisfied all the time, so the solution solved by ADI-FDTD method is stable under any circumstance.

# 4 Boundary conditions

Boundary conditions of transmission line equations were obtained by using the lumped parameter circuit model of track circuits. In Fig.4, the lines are driven by a voltage source $u_s$, with internal resistance $Z_s$, and terminated at $x = l$ with resistance $Z_f$.

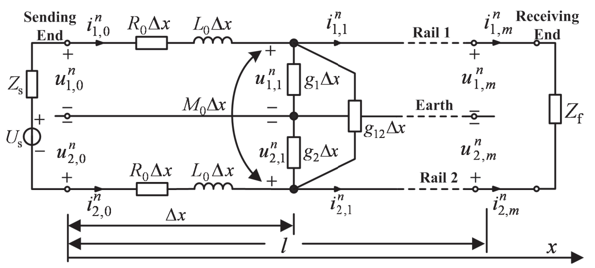


Fig.2 Lumped parameter equivalent circuit of track circuit

The iteration was started from the point 3 in space coordinate, so it need the expressions of point 1 and point 2 to launch the whole calculative process.

The spatial first point in each layers are set directly, they would not change over time.

$$\begin{cases} i_{1,0}^{n+1}=i_{2,0}^{n+1}=i_s(n+1) \\ u_{1,0}^{n+1}=u_{2,0}^{n+1}=\dfrac{u_s(n+1)-Z_s i_s(n+1)}{2} \end{cases}$$

According to the kirchhoff's law, the second point in each layers were expressed as

$$\begin{cases} u_{1,0}^{n+1}-u_{1,1}^{n+1}=\left(z_1 i_{1,0}^{n+1}+z_M i_{2,0}^{n+1}\right)h \\ u_{2,0}^{n+1}-u_{2,1}^{n+1}=\left(z_2 i_{2,0}^{n+1}+z_M i_{1,0}^{n+1}\right)h \end{cases}$$

$$\begin{cases} i_{1,0}^{n+1}-i_{1,1}^{n+1}=\left[(g_1+g_{12})u_{1,1}^{n+1}-g_{12}u_{2,1}^{n+1}\right]h \\ i_{2,0}^{n+1}-i_{2,1}^{n+1}=\left[(g_2+g_{12})u_{2,1}^{n+1}-g_{12}u_{1,1}^{n+1}\right]h \end{cases}$$

The form of calculation formulas in point ($M$+1) is similar to the second point's.

In the case of the terminal open circuit, the ($M$+2) point were expressed as

$$\begin{cases} u_{1,M+1}^{n+1}-u_{1,M+2}^{n+1}=\left(z_1 i_{1,M+1}^{n+1}+z_M i_{2,M+1}^{n+1}\right)h \\ u_{2,M+1}^{n+1}-u_{2,M+2}^{n+1}=\left(z_2 i_{2,M+1}^{n+1}+z_M i_{1,M+1}^{n+1}\right)h \end{cases}$$

$$\begin{cases} i_{1,M+2}^{n+1}=0 \\ i_{2,M+2}^{n+1}=0 \end{cases}$$

Thus, the boundary condition at terminal of track circuit was got. The boundary conditions of voltage and current at beginning and terminal

# 5 Simulation analysis

## 5.1 Simulation calculation process

Fig.3 is the basic simulate flowchart by using finite differential method.

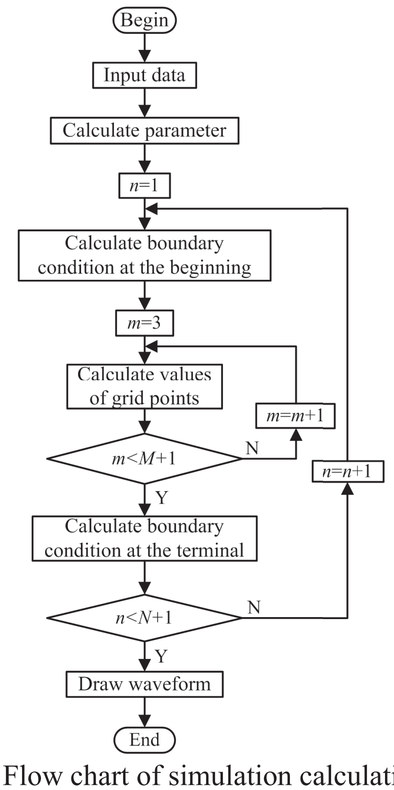


Fig.3 Flow chart of simulation calculation

The main process is

(1) Input data for simulation, including basic initial electrical parameters of track circuit, voltage in sending end and its internal resistance, time step length, space step length, overall computation time, line length.

(2) Calculate the total number of grids.

(3) Calculate boundary condition at the beginning in a certain moment.

(4) According to the boundary conditions at the beginning and terminal, through the cycle in space, calculate the voltage distribution along the road in a certain moment.

(5) Obtain the voltage change rule over time though the cycle in time.

(6) Draw the voltage response curves in the receiving end.

## 5.2 Example analysis

A track circuit, rail type is P60, the receiving end is open, where $u_s = 7.47$V, $i_s = 1$A, $Z_s = 0.1\Omega$, $L = 1.6$km, $T = 500$ns, $k = 5$ns, $h = 0.08$km.

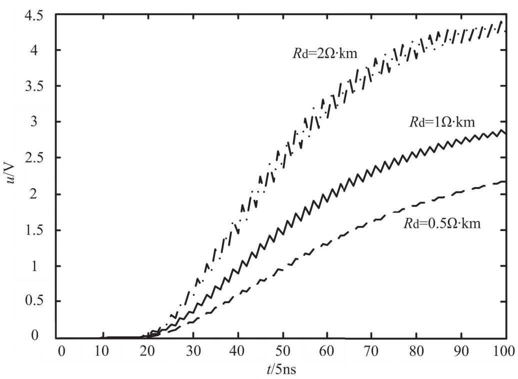


Fig.4 Voltage response waveforms of the receiving end in different ballast resistance

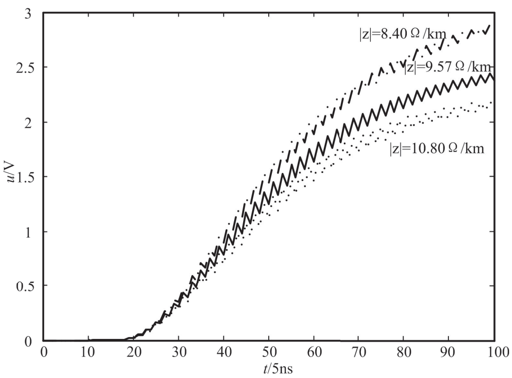


Fig.5 Voltage response waveforms of the receiving end in different rail impedance

Ballast resistance and rail impedance are the inherent parameters of track circuit, rail impedance is determined by the current frequency to a great extent [17]. So, when other parameters of track circuits are same, we simulated the influence of ballast resistance and rail impedance to voltage in the receiving end respectively. Voltage response waveforms of the receiving end in different ballast resistances is shown in Fig.4, voltage response waveforms of the receiving end in different rail impedance is shown in Fig.5.

As shown in Fig.4 and Fig.5, initial voltage values are both zero, and with the time increasing, the values increase and then tend toward stability. This is because receiving end has not received signals from the sending end, and it needs to pass a certain amount of time before the signals arrived. Due to the wave superposition in signals transmission process and the iteration structure of differential formation, the curves we obtained are fluctuated. But the incremental trend of curves is reasonable, it conform to the physical truth. In the process of analysis, track circuit is a kind of distortion transmission line in essence, thus voltage amplitudes are attenuate when they reached the receiving end in contract to the initial amplitudes of sending end.

The voltage response curses in different ballast resistance are presented in Fig.4. The module value of rail impedance choosed for Matlab simulation is 8.40Ω/km. It can be seen from the diagrams that, in the case of other parameters are fixed, along with the rising of the ballast resistance, the voltage values in receiving end will increase, this change conform to the physical truth. The reason is that when the ballast resistance increase, the leakage between rails and the earth reduce, and the attenuations in the transmission process of track circuits will decrease.

The voltage response curses in different rail impedance are shown in Fig.5. Ballast resistance is 1Ω·km and other parameters are unchanged. It can be seen from the diagrams that, along with the rising of the rail impedance, the voltage values in receiving end will decrease, it also conform to the physical truth. The reason is that when the rail impedance increase, the leakage between rails and the earth increase, and the attenuations in the transmission process of track circuits will decrease.

According to the response waveforms in this two figure,we can see that they all conforms with the transmission characteristic of track circuits.

In order to explain the trust of finite differential method further, compared its simulation curve with that of ADI-FDTD method. Also in the previous example, when $R_d$ = 1Ω·km, |z| = 8.40 Ω/km, the voltage response curses are obtained by different methods are presented in Fig.6.

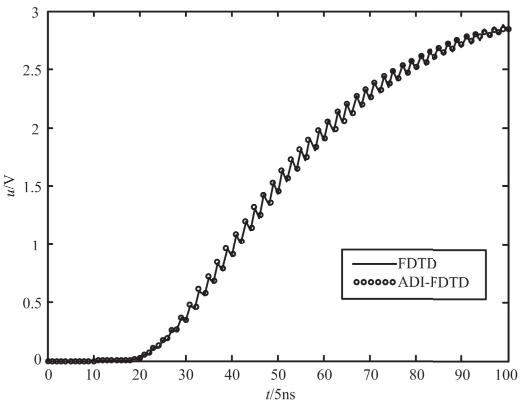


Fig.6 voltage response waveforms comparison between FDTD and ADI-FDTD method

As shown in Fig.6, two waveforms are almost overlapping, so results of numerical experiment confirm that the ADI-FDTD method is almost consistent with the finite differential method.

## 6 Conclusion

Finite differential method is used to calculate time domain solution of track circuit. According to the working characteristic of track circuits, an example was analyzed by Matlab simulation. The results show that the time domain solution conform to the general transmission characteristic of track circuits and compared with ADI-FDTD method, their simulation results are consistent. Therefore, finite differential method can be used to research time domain solution of track circuits.

## 7 ACKNOWLEDGEMENTS

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