Resonance and Efficiency in Wireless Power Transfer System

KAZUYA YAMAGUCHI
Department of Materials and Informatics
Interdisciplinary Graduate School
of Agriculture and Engineering
University of Miyazaki
1-1, Gakuen-kibanadai-nishi
Miyazaki 889-2192 Japan
nc13001@student.miyazaki-u.ac.jp

TAKUYA HIRATA
Department of Materials and Informatics
Interdisciplinary Graduate School
of Agriculture and Engineering
University of Miyazaki
1-1, Gakuen-kibanadai-nishi
Miyazaki 889-2192 Japan
nc14001@student.miyazaki-u.ac.jp

YUTA YAMAMOTO
Department of Materials and Informatics
Interdisciplinary Graduate School
of Agriculture and Engineering
University of Miyazaki
1-1, Gakuen-kibanadai-nishi
Miyazaki 889-2192 Japan
nc12006@student.miyazaki-u.ac.jp

ICHIJO HODAKA
Department of Environmental Robotics
School of Engineering
University of Miyazaki
1-1, Gakuen-kibanadai-nishi
Miyazaki 889-2192 Japan
hijhodaka@cc.miyazaki-u.ac.jp

Abstract: Many papers about wireless power transfer use resonant phenomena. In general if input frequency is adjusted to the resonant frequency, the amplitude of current or voltage will be maximal. Since the power of resistive load is determined by its current or voltage, the power will be maximal at the resonance. As for wireless power transfer, it is one of significant specifications to transfer power as much as possible. Then it is natural to try to maximize transmitted power with resonant phenomena. On the other hand, efficiency is another significant specification on wireless power transfer. However efficiency is defined as a ratio with two different powers, and it cannot be concluded that efficiency is always maximal when either power is maximal. Therefore efficiency has no straightforward relation with resonance in contrast with the relation between resonance and power. In this paper, we show an example which cannot realize the maximal efficiency, although resonant phenomena are caused and discuss why the situation is caused. An analytical method for wireless power transfer is clarified and how to choose an optimal frequency is scrutinized. The power of transmitting and receiving sides and efficiency are calculated by using mathematical models, e.g. a state equation, a transfer function.

Key–Words: wireless power transfer, highly efficient power transmission, electromagnetic resonance

1 Introduction

Most of devices that are driven by electric power are usually fed via electric wires connected to AC power supply. On the other hand, some of electric devices are required or designed to be fed without electric wires — in the way of called WPT (wireless power transfer). Since the amount of power provided by WPT is usually restricted compared with wired power transfer, WPT has not been used for many applications.

In 2007, a successful experiment that could wirelessly transfer practical amount of power away from sixty centimeters was reported[1]. The heart of the experiment was to generate resonant phenomena of electromagnetic coupling between transmitting and receiving coils. After the pioneering study[1], many researchers have tried to develop theory and experiments for WPT. In [2] a structure of circuit was devised in order to raise the factor Q of circuits, since better WPT needs higher values of the factor Q. In [3] devised antennas are proposed to be used for a directed energy radiation in order to implement efficient WPT. In [4] the effect of radiation energy to human body was discussed.

It is generally understood that resonant phenomena will be caused if one tunes the frequency of AC power supply to the natural frequency of a circuit. The natural frequency is determined by the values of circuit elements; especially, the mutual inductances which relate transmitting and receiving sides of the circuit play an important role. These values are determined by the radii, winding numbers, and relative position of two coils[5]. It is important to know the relation between the relative position of coils and mutual inductances because the position of the coils could be changed in various reasons.

For extending the usage of WPT, the most important specification for WPT is power and efficiency of transmission. In many literatures, causing resonant phenomena should lead to high power and efficient WPT. However, little is known about how resonance brings the best performance of WPT based on a concrete and mathematical aspects.

In this paper, we propose a series of procedures to analyze relations between the resonant frequency, obtained average power at the receiving side, and the ratio of the average power at the receiving side to the average power at the transmitting side that is called an efficiency of WPT. Our procedures are based on the differential equations in the form which commonly used in control theory. This gives us a benefit that we can manipulate different transfer functions and write the relations under investigation in a clear and unified manner. Consequently, we will assert that resonance does not imply the optimization of efficiency of WPT in general, by observing derived equations.

To illustrate this situation that should be carefully treated, we pick one of most common circuits for WPT, and put practical values into the circuit elements, and then we demonstrate the situation numerically. In fact, although we have the case when resonance is equivalent to maximization of efficiency of WPT, we have another case when resonance leads to loss of efficiency of WPT. Thus we propose to use the way of modelling by mathematical equations and to describe the targets into equations for WPT systems.

2 Analysis of Wireless Power Transfer Circuit

2.1 Wireless power transfer circuit and its mathematical model

We study a typical circuit for WPT depicted below[6]. Despite of the placement of two coils in the figure, we assume they have a common central axis.

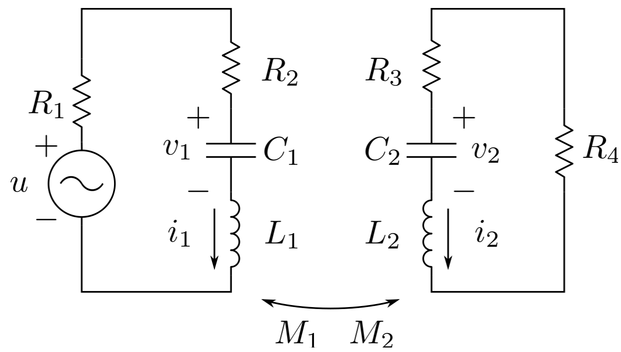

Figure 1 A Wirelss Power Transfer Circuit

The resistor R_1 is supposed to represent an internal impedance of the power supply, and R_4 the load. R_2, R_3, C_1, C_2 are parasitic factors of transmitting and receiving coils. This circuit is mathematically modelled as the following state equation. This type of expression is widely used in control theory. In particular one can write the model of WPT circuits in a compact form, and obtain a clear perspective to analysis of stability and responses with a sinusoidal input $u = \sin\omega t$.

$$\dot{x} = Ax + Bu,\ \dot{x} = \frac{dx}{dt} \tag{1}$$

$$x = \begin{bmatrix} v_1 \\ v_2 \\ i_1 \\ i_2 \end{bmatrix}$$

$$A = \frac{1}{\Delta}\begin{bmatrix} 0 & 0 & \frac{\Delta}{C_1} & 0 \\ 0 & 0 & 0 & \frac{\Delta}{C_2} \\ -L_2 & M_2 & -(R_1+R_2)L_2 & (R_3+R_4)M_2 \\ M_1 & -L_1 & (R_1+R_2)M_1 & -(R_3+R_4)L_1 \end{bmatrix}$$

$$B = \frac{1}{\Delta}\begin{bmatrix} 0 \\ 0 \\ L_2 \\ -M_1 \end{bmatrix}$$

$$\Delta = L_1L_2 - M_1M_2.$$

The derivation of equation (1) from the circuit is straightforward and omitted here. It is convenient to use a symbolic computing software when we try to find state equations such as the equation (1)[7].

2.2 Formulation of average power and efficiency

The purpose of this paper is to investigate the relation between the frequency of AC supply voltage, the average power delivered to the receiving side, and the efficiency of average power transmission. The efficiency is defined as the ratio of the average power obtained at the receiving side against the average power supplied at AC power supply. Thus we formulate equations of the average powers at transmitting and receiving sides, and of the efficiency in the following.

We discuss the stability of matrix A. First, the characteristic polynomial $f(s)$ of A is found as follows.

$$\begin{aligned} f(s) &= s^4 \\ &+ \frac{(R_1+R_2)L_2+(R_3+R_4)L_1}{\Delta}s^3 \\ &+ \frac{1}{\Delta}\left((R_1+R_2)(R_3+R_4)+\frac{L_2}{C_1}+\frac{L_1}{C_2}\right)s^2 \\ &+ \frac{1}{\Delta}\left(\frac{R_1+R_2}{C_2}+\frac{R_3+R_4}{C_1}\right)s \\ &+ \frac{1}{\Delta C_1 C_2} \end{aligned} \quad (2)$$

The roots of $f(s)=0$ are the eigenvalues of A. We can confirm that A is stable because these eigenvalues have all negative real parts and Hurwitz stability criterion is satisfied under $R_1+R_2>0, R_3+R_4>0$.

Since the matrix A in the equation (1) is stable, the solution to the equation (1) will be stationary after time have passed adequately. The stationary solution with a sinusoidal input $u=\sin\omega t$ will have the same frequency of the input. The stationary solution of equation (1) is given as follows.

$$x_s(t) = -[\omega I\cos\omega t + A\sin\omega t](\omega^2 I + A^2)^{-1}B \quad (3)$$

I is a 4-dimensional identity matrix. As for the equation (3), the following condition is satisfied.

$$\lim_{t\to\infty}|x(t)-x_s(t)| = 0 \quad (4)$$

In general, the average powers P_1 and P_4 which are at the AC supply and the load in the receiving side respectively, and then the efficiency η depends on the frequency of AC supply. These stationary values can be all expressed in terms of transfer functions. Here we set $G_1(s)$ and $G_2(s)$ as the transfer functions from the input u to i_1 and i_2, respectively. With the equation (1), these transfer functions are written in $G_1(s)=H_1(sI-A)^{-1}B, G_2(s)=H_2(sI-A)^{-1}B$, where $H_1=[0,0,1,0]$ and $H_2=[0,0,0,1]$.

The equations of i_{s1}, i_{s2}, which are the third and fourth rows of equation (3) are written as follows.

$$\begin{aligned} i_{s1}(t) &= |G_1(j\omega)|\sin(\omega t+\theta) \\ i_{s2}(t) &= |G_2(j\omega)|\sin(\omega t+\phi) \end{aligned} \quad (5)$$

θ, ϕ are the arguments of $G_1(j\omega), G_2(j\omega)$ respectively.

And the average powers P_1 and P_4 under stationary situation are expressed as below with a period $T=\frac{2\pi}{\omega}$.

$$\begin{aligned} P_1 &= \frac{1}{T}\int_0^T i_{s1}(t)(u(t)-R_1 i_{s1}(t))dt \\ P_4 &= \frac{1}{T}\int_0^T R_4 i_{s2}(t)^2 dt \end{aligned} \quad (6)$$

Therefore, if the equation (5) and $u=\sin\omega t$ are substituted in the equation (6), P_1, P_4 and efficiency η are written as below.

$$\begin{aligned} P_1 &= \frac{1}{2}\left(\mathrm{Re}[G_1(j\omega)] - R_1|G_1(j\omega)|^2\right) \\ P_4 &= \frac{1}{2}R_4\,|G_2(j\omega)|^2 \\ \eta &= \frac{P_4}{P_1}. \end{aligned} \quad (7)$$

2.3 Resonance and average power

In view of the equation (7), we see that if one puts an AC input voltage with the frequency which gives the maximal gain of the transfer function $G_2(s)$ (the gain is called H_∞-norm), one will have the maximal average power at the receiving side. $G_2(s)$ is found from the equation (1) and (2) as below.

$$G_2(s) = \frac{1}{\Delta}\times\frac{-M_1 s^3}{f(s)} \quad (8)$$

To illustrate the situation, we set values of circuit elements as below. These values are given by consulting a practical situation of WPT[2].

Table 1: parameters

elements	values	elements	values
R_1	50Ω	L_2	10μH
R_2	0.1Ω	M_1	0.5μH
R_3	0.1Ω	M_2	0.5μH
R_4	50Ω	C_1	1nF
L_1	10μH	C_2	1nF

The bode diagram of $G_2(s)$ is shown as below.

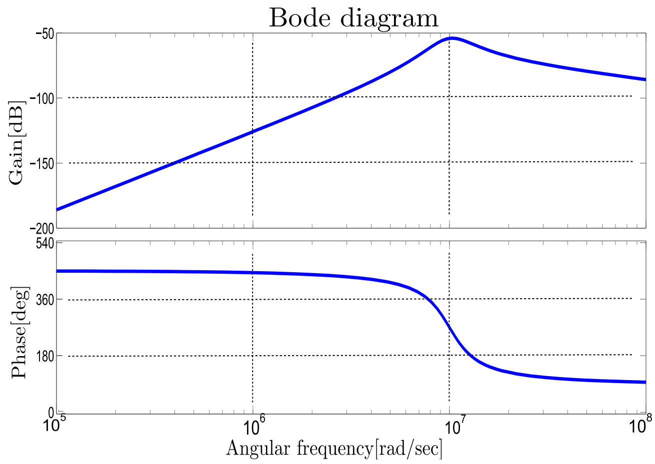

Figure 2 Bode Diagram

In this case the resonance will occur at the unique frequency $\omega = 1.00 \times 10^7 \mathrm{rad/sec}$ and we have the maximal average power at the receiving side if we use a sinusoidal wave with the resonant frequency.

And the graphs of instantaneous powers $p_1(t), p_4(t)$ with $\omega = 1.00 \times 10^7 \mathrm{rad/sec}$ are shown as follows.

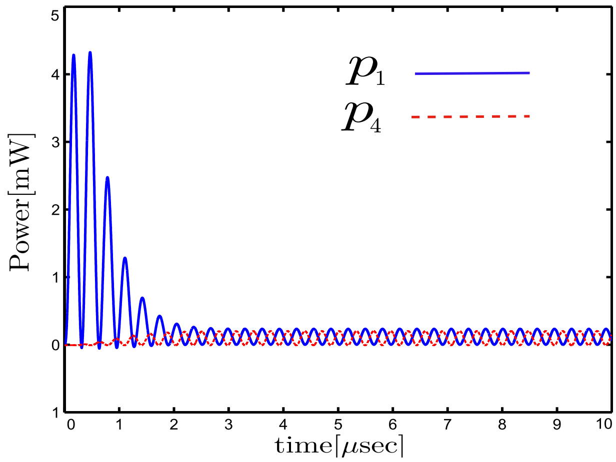

Figure 3 Instantaneous Powers $p_1(t), p_4(t)$

For the simulation, the numeric computer MATLAB is used. From the above figure, we can see that $p_1(t)$ and $p_4(t)$ become stationary waves when time passes sufficiently because the matrix A is stable[8]. We should notice it because WPT circuits have low resistances in general and they take a long time to become a stationary state.

2.4 Resonance and efficiency

Based on the equation (7), we can observe that the relation between the input frequency and the efficiency of WPT is not straightforward. Many other papers state that using an AC input voltage with a resonant frequency is optimal or better in the design of WPT. In the previous section, we have clarified that using the input with resonance maximizes the average power at the receiving side. However, the efficiency of power transmission can be maximized either when one use resonance or when one use nonresonance. This difficult situation is illustrated by numerical examples in the following.

First, power P_4 and efficiency η with the condition of Table 1 are shown as below.

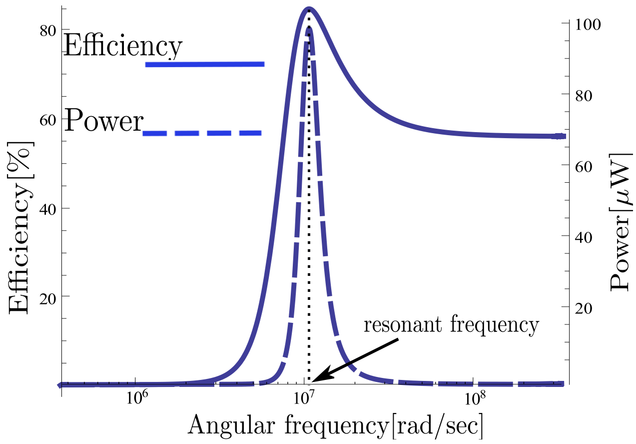

Figure 4 Power and Efficiency for Table 1

Then, η is maximized when

$$\omega = 1.07 \times 10^7 \mathrm{rad/sec} \tag{9}$$

as previously known in [9].

Another example is shown as below. The values of elements are set as Table 2. The only difference between Table 1 and Table 2 is the value of C_1.

Table 2: parameters

elements	values	elements	values
R_1	50Ω	L_2	10μH
R_2	0.1Ω	M_1	0.5μH
R_3	0.1Ω	M_2	0.5μH
R_4	50Ω	C_1	0.1nF
L_1	10μH	C_2	1nF

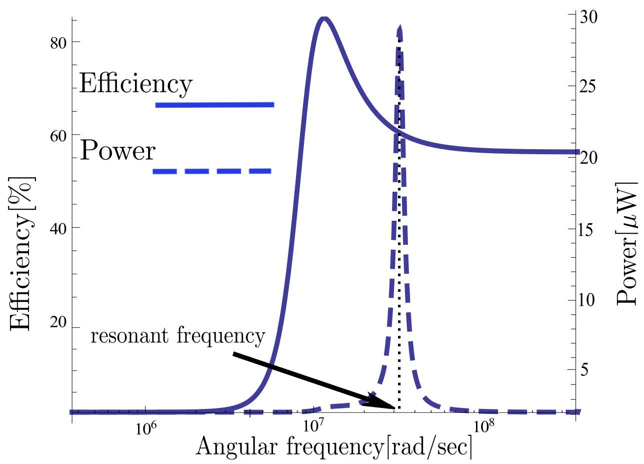

Figure 5 Power and Efficiency for Table 2

By comparing Figure 4 and Figure 5, we see that the frequencies which respectively maximize power and efficiency are different.

The variation of efficiency by changing the value of R_4 with resonance is shown as below. The values in Figure 1 are set as Table 1 except for R_4.

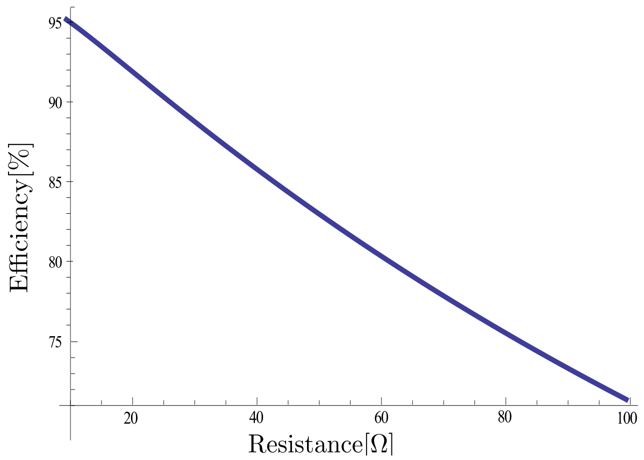

Figure 6 variation of efficiency by changing R_4

From Figure 6, If the value of R_4 is high, highly efficient WPT cannot be expected.

And the variation of efficiency by changing the values of mutual inductances ($M_1 = M_2 = M$) with resonance is shown as below. The values of elements in Figure 1 are set as Table 1 except for M_1 and M_2.

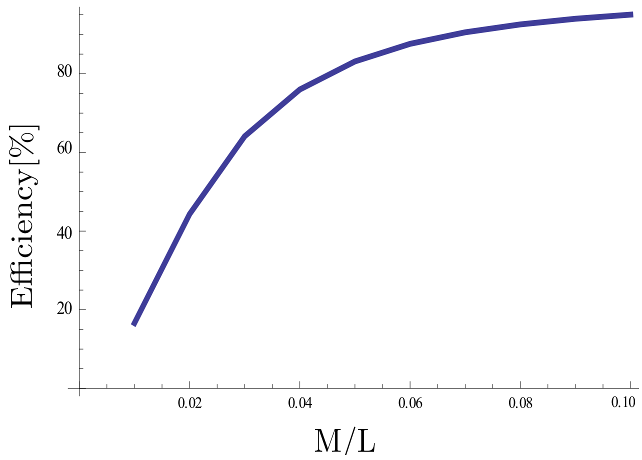

Figure 7 variation of efficiency by changing M_1 and M_2

From Figure 7, if the values of mutual inductances are high, highly efficient WPT with more than 90% efficiency can be expected. In this situation, it can be considered that the magnetic connection of two coils is strong and the energy loss between these is low.

3 Conclusion

In this paper, we have pointed out that using a resonant frequency of AC voltage input does not lead to maximize the efficiency of average power transmission, although resonance is equivalent to maximization of output average power, in general WPT systems. In fact, we have illustrated a situation that non-resonance maximizes the efficiency of power transmission by numerical examples. This suggests that we should take both of output power and efficiency into account and then decide a balanced working frequency, in the case that resonance is not equivalent to the maximal efficiency. For example, the maximum power is desirable if an AC power supply can serve enough power, but the maximum efficiency is best if the supply can serve less power. On the other hand, as another method, a feedback is added to a sinusoidal wave and it is used as a new input to improve power and efficiency[10].

Even one of the simplest WPT circuits treated in this paper may have disagreement between resonance and efficiency. Therefore, one will face on further difficulty to design better WPT when one tries more complex circuit with more elements in order to meet an increasing design specification.

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