Analysis of Lyapunov Function Features for Some Strategies of the Network Optimization Problem

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Abstract: – General methodology for system design was elaborated by means of the optimal control theory approach. The problem of analog system design can be formulated in this case as a classical problem of the optimal control for some functional minimization. In this context the aim of the optimal control is to result to minimum point a cost function of the design process and to minimize the total computer time. The minimal time system design algorithm was defined as the problem of functional minimization. By this definition the aim of the system design process with minimal computer time is presented as a transition process of some dynamic system that has the minimal transition time. The optimal sequence of the control vector switch points was determined as a principal characteristic of the minimal-time system design algorithm. The conception of the Lyapunov function was proposed to analyze the behavior of design process. The special function that is a combination of the Lyapunov function and its time derivative was proposed to predict the design time of any strategy by means of the initial time interval analysis.

Key Words: – Minimal-time system design, control theory application, Lyapunov function.

1 Introduction

The problem of the computer time reduction of a large system design is one of the essential problems of the total quality design improvement. Besides the traditionally used ideas of sparse matrix techniques and decomposition techniques [1]-[5] some another ways were proposed to reduce the total computer design time [6]-[7]. The generalized approach for the analog system design on the basis of control theory formulation was elaborated in some previous works, for example [8]. This approach serves for the minimal-time design algorithm definition. On the other hand this approach gives the possibility to analyze with a great clearness the design process while moving along the trajectory curve into the design space. The main conception of this theory is the introduction of the special control functions, which, on the one hand generalize the design process and, on the other hand, they give the possibility to control design process to achieve the optimum of the design cost function for the minimal computer time. This possibility appears because practically an infinite number of the different design strategies that exist within the bounds of the theory. The different design strategies have the different operation number and executed computer time. As shown in [8] the potential computer time gain that can be obtained by the new design problem formulation increases when the size and complexity of the system increase. However it is realized only in case when the algorithm for the optimal strategy of design is constructed.

We can define the formulation of the intrinsic properties and special restrictions of the optimal design strategy as one of the first problems that needs to be solved for the optimal algorithm construction.

2 Problem Formulation

The design process for any analog system design can be defined in discrete form [8] as the problem of the generalized cost function $F(X,U)$ minimization by means of the equation (1) with the constraints (2):

$$X^{s+1} = X^{s} + t_s \cdot H^{s} \tag{1}$$

$$\left(1-u_j\right) g_j(X) = 0, \quad j = 1,2,\ldots,M, \tag{2}$$

where $X \in R^N$, $X=(X',X'')$, $X' \in R^K$ is the vector of the independent variables and the vector $X'' \in R^M$ is the vector of dependent variables ($N = K + M$), $g_j(X)$ for all j presents the system model, s is the iterations number, t_s is the iteration parameter, $t_s \in R^1$, $H \equiv H(X,U)$ is the direction of the generalized cost function $F(X,U)$ decreasing, U is the vector of the special control functions $U = (u_1, u_2, \ldots, u_m)$, where $u_j \in \Omega$; $\Omega = \{0;1\}$. The generalized cost function $F(X,U)$ is defined as:

$$F(X,U) = C(X) + \psi(X,U) \tag{3}$$

where $C(X)$ is the non negative cost function of the design process, and $\psi(X,U)$ is the additional penalty function:

$$\psi(X,U) = \frac{1}{\varepsilon}\sum_{j=1}^{M} u_j \cdot g_j^2(X) \tag{4}$$

This formulation of the problem permits to redistribute the computer time expense between the solution of problem (2) and the optimization procedure (1) for the function $F(X,U)$. The control vector U is the main tool for the redistribution process in this case. Practically an infinite number of the different design strategies are produced because the vector U depends on the optimization procedure current step. The problem of the optimal design strategy search is formulated now as the typical problem for the functional minimization of the control theory. The functional that needs to minimize is the total CPU time T of the design process. This functional depends directly on the operations number and on the design strategy that has been realized. The main difficulty of this definition is unknown optimal dependencies of all control functions u_j.

The continuous form of the problem definition is more adequate for the control theory application. This continuous form replaces Eq. (1) and can be defined by the next formula:

$$\frac{dx_i}{dt} = f_i(X,U), \quad i = 0,1,\ldots,N \tag{5}$$

This system together with equations (2), (3) and (4) composes the continuous form of the design process. The structural basis of different design strategies that correspond to the fixed control vector includes 2^M design strategies. The functions of the right hand part of the system (5) are determined for example for the gradient method as:

$$f_i(X,U) = -\frac{\delta}{\delta x_i}F(X,U), \quad i = 1,2,\ldots,K \tag{6}$$

$$f_i(X,U) = -u_{i-K}\frac{\delta}{\delta x_i}F(X,U) + \frac{(1-u_{i-K})}{t_s}\left\{-x_i^s + \eta_i(X)\right\},$$
$$i = K+1, K+2, \ldots, N \tag{6'}$$

where the operator $\delta / \delta x$ hear and below means $\frac{\delta}{\delta x_i}\varphi(X) = \frac{\partial \varphi(X)}{\partial x_i} + \sum_{p=K+1}^{K+M}\frac{\partial \varphi(X)}{\partial x_p}\frac{\partial x_p}{\partial x_i}$, x_i^s is equal to $x_i(t-dt)$; $\eta_i(X)$ is the implicit function ($x_i = \eta_i(X)$) that is determined by (2).

The control variables u_j have the time dependency in general case. The equation number j is removed from (2) and the dependent variable x_{K+j} is transformed to the independent when $u_j = 1$. This independent parameter is defined by the formulas (5), (6'). In this case there is no difference between formulas (6) and (6'). On the other hand, the equation (5) with the right part (6') is transformed to the identity $\frac{dx_i}{dt} = \frac{dx_i}{dt}$, when $u_j = 0$, because $\eta_i(X) - x_i^s = x_i(t) - x_i(t-dt) = dx_i$. It means that at this time moment the parameter x_i is dependent one and the current value of this parameter can be obtained from the system (2) directly. This transformation of the vectors X' and X'' can be done at any time moment.

The function $f_0(X,U)$ is determined as the necessary time for one-step integration of the system (5). This function depends on the concrete design strategy. The additional variable x_0 is determined as the total computer time T for the system design. It is necessary to find the optimal behavior of the control functions u_j during the design process to minimize the total design computer time.

The idea of the system design problem formulation as the functional minimization problem of the control theory is not depend of the optimization method. This idea was implemented for the designing of different networks [8]. It was shown that any optimization procedures can be embedded like the gradient method, the Newton method and Davidon-Fletcher-Powell method.

Now the analog system design process is formulated as a dynamic controllable system. The time-optimal design process can be defined as the dynamic system with the minimal transition time in this case. So we need to find the special conditions to minimize the transition time for this dynamic system.

3 Lyapunov function of optimization process

On the basis of the analysis in previous section we can conclude that the minimal-time algorithm has one or some switch points in control vector where the switching is realize among different design strategies. As shown in [9] it is necessary to switch the control vector from like modified traditional design strategy to like traditional design strategy with an additional adjusting. Some principal features of the time-optimal algorithm were determined previously. These are: 1) an additional acceleration effect that appeared under special circumstances [9]; 2) the start point special selection outside the separate hyper-surface to guarantee the acceleration effect, at least one negative component of the start value of the vector *X* is can be recommended for this; 3) an optimal structure of the control vector with the necessary switch points. The two first problems were discussed in [9-10].

A Lyapunov function of dynamic system serves as a very informative object to any system analysis in the control theory. We suppose that the Lyapunov function can be used for the revelation of the optimal algorithm structure. First of all we can compare the behavior of the different design strategies by means of the Lyapunov function analysis.

There is a freedom of the Lyapunov function choice because of a non-unique form of this function. Let us define the Lyapunov function of the design process (2)-(6) by the following expression:

$$V(X) = \sum_i (x_i - a_i)^2 \tag{7}$$

where a_i is the stationary value of the coordinate x_i, in other words the set of all the coefficients a_i is the main objective of the design process. The function (7) satisfies all of the conditions of the standard Lyapunov function definition for the variables $y_i = x_i - a_i$. In fact the function $V(Y) = \sum_i y_i^2$ is the piecewise continue. Besides there are three characteristics of this function: i) *V(Y)*>0, ii) *V(0)*=0, and iii) $V(Y) \to \infty$ when $\|Y\| \to \infty$. Inconvenience of the formula (7) is an unknown point $a = (a_1, a_2, ..., a_N)$, because this point can be reached at the end of the design process only. We can use this form of the Lyapunov function if we already found the design solution someway. On the other hand, it is very important to control the stability of the design process during the optimization procedure. In this case we need to construct other form of the Lyapunov function that doesn't depend on the unknown stationary point.

Let us define the Lyapunov function of the design process (2)-(6) by the following expression:

$$V(X,U) = [F(X,U)]^r \tag{8}$$

$$V(X,U) = \sum_i \left(\frac{\partial F(X,U)}{\partial x_i} \right)^2 \tag{9}$$

where *F(X,U)* is the generalized cost function of the design process. The formula (8) can be used when the general cost function is non-negative and has zero value at the stationary point *a*. Formula (9) can be used always because all derivatives $\partial F / \partial x_i$ are equal to zero in the stationary point *a*.

We can define now the design process as a transition process for controllable dynamic system that can provide the stationary point (optimal point of the design procedure) during some time. The problem of the time-optimal design algorithm construction can be formulated now as the problem of the transition process searching with the minimal transition time. There is a well-known idea [11]-[12] to minimize the time of transition process by means of the special choice of the right hand part of the principal system of equations, in our case these are the functions $f_i(X,U)$. It is necessary to change the functions $f_i(X,U)$ by means of the control vector *U* election to obtain the maximum speed of the Lyapunov function decreasing. Normally the time derivative of Lyapunov function is non-positive for the stable processes. However we define more informative function as a relatively time derivative of the Lyapunov function $W = \dot{V}/V$. This function serves well to analyze a designing process. Below some practical examples were analyzed to support the ideas of developed methodology.

4 Numerical results

All examples were analyzed for the continuous form of the optimization procedure (5). Functions *V(t)* and *W(t)* were the main objects of the analysis and its behavior has been analyzed for different strategies that compose the structural basis of generalized methodology.

First example corresponds to a simple nonlinear voltage divider in Fig. 1.

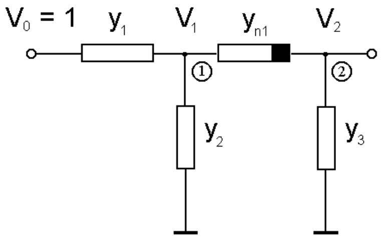


Fig. 1. Two-node nonlinear passive network.

The nonlinear element has the following dependency: $y_{n1} = y_0 + b(V_1 - V_2)^2$. The vector X includes five components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$. Defining the components x_1, x_2, x_3 by the above formulas automatically results in positive magnitudes of the conductance, which eliminates the issue of positive definiteness for each resistance and conductance and makes it possible to carry out the optimization in the whole space of magnitudes of these variables without any limitations. The cost function $C(X)$ has been determined by the formula $C(X) = (x_5 - m_1)^2$, where m_1 is a beforehand-defined output voltage of the divider.

This network is characterized by two dependent parameters (two nodal voltages) and the control vector includes two control functions: $U=(u_1, u_2)$. The structural basis of the design strategies includes four design strategies with the control vectors: (00), (01), (10), and (11). The functions $g_j(X)$ are defined by the next formulas:

$$g_1(X) \equiv (1 - x_4)x_1^2 - (x_4 - x_5)\left(y_0 + a(x_4 - x_5)^2\right) - x_4 x_2^2 = 0$$

$$g_2(X) \equiv (x_4 - x_5)\left(y_0 + a(x_4 - x_5)^2\right) - x_5 x_3^2 = 0 \quad (10)$$

The system (2) is transformed into the following one:

$$(1 - u_j)g_j(x_1, x_2, x_3, x_4, x_5) = 0, \; j = 1,2.$$

The optimization procedure (1), (5) includes five equations. The Lyapunov function was calculated by formula (8) for r=0.5.

The results of the analysis of complete structural basis of different strategies of designing for network in Fig. 1 and initial point $x_{i0} = 1$, i = 1,2,…,5 are shown in Table 1.

Table 1. Data of complete structural basis of designing strategies.

N	Control vector	Iterations number	Total design time (sec)
1	(0 0)	406308	8.52
2	(0 1)	455191	3.96
3	(1 0)	226909	3.31
4	(1 1)	451090	2.81

The behavior of the functions *V(t)* and *W(t)* for the network in Fig. 1 is shown in Fig. 2.

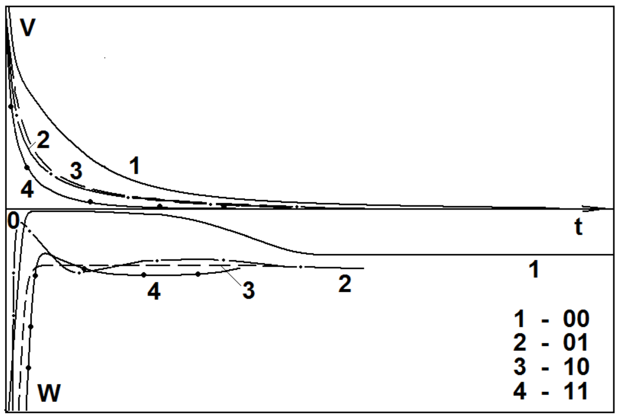


Fig. 2. Behavior of the functions *V(t)* and *W(t)* for four design strategies during the design process for network in Fig. 1.

As we can see from Fig. 2 the functions *V(t)* and *W(t)* can give an exhaustive explanation for the design process characteristics. A greater absolute value of the function *W(t)* corresponds to a more rapid decreasing of the function *V(t)*. We can state that the greater absolute value of the function *W(t)* on initial part of the design process provoke the lesser computer time. On the other hand the function *W(t)* is a normalized derivative and for this reason it is very sensitive.

Another passive nonlinear network with three nodes (Fig. 3) was analyzed below. The vector X includes seven components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5 = V_1$, $x_6 = V_2$, $x_7 = V_3$. The nonlinear elements have been defined by following dependencies: $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2$.

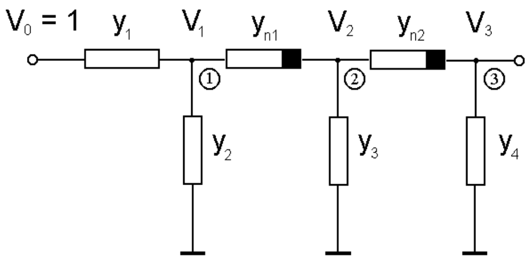


Fig. 3. Three-node nonlinear passive network.

The model of this network (2) includes three equations (M=3) and the optimization procedure (5) includes seven equations. This network is characterized by three dependent parameters and the control vector includes three control functions: $U=(u_1,u_2,u_3)$. In this case we have a system of seven equations playing the role of the optimization algorithm.

$$\frac{dx_i}{dt}=-\frac{\delta}{\delta x_i}F(X,U),\ i = 1,2,3,4$$

$$\frac{dx_i}{dt}=-u_{i-4}\cdot\frac{\delta}{\delta x_i}F(X,U)+\frac{(1-u_{i-4})}{dt}\{-x_i(t-dt)+\eta_i(X)\}$$
i = 5,6,7,

where $F(X,U)=C(X)+\sum_{j=1}^{3}u_j g_j^2(X)$.

The network model can be expressed by three nonlinear equations:

$$g_1(X)\equiv\left(x_1^2+x_2^2+a_{n1}+b_{n1}x_6^2\right)x_5-\left(a_{n1}+b_{n1}x_6^2\right)x_6-x_1^2=0$$

$$g_2(X)\equiv-\left(a_{n1}+b_{n1}x_6^2\right)x_5+\left(x_3^2+a_{n1}+b_{n1}x_6^2+a_{n2}+b_{n2}x_7^2\right)x_6-\left(a_{n2}+b_{n2}x_7^2\right)x_7=0$$

$$g_3(X)\equiv-\left(a_{n2}+b_{n2}x_7^2\right)x_6+\left(x_4^2+a_{n2}+b_{n2}x_7^2\right)x_7=0$$

This system can be transformed into the following one:

$$\left(1-u_j\right)g_j\left(x_1,x_2,x_3,x_4,x_5,x_6,x_7\right)=0,\ j=1,2,3.$$

The structural basis of design strategies includes eight design strategies: 000, 001, 010, 011, 100, 101, 110 and 111. The results of the analysis of complete structural basis of different strategies of designing for network in Fig. 3 and initial point $x_{i0}=1$, i = 1,2,…,7 are shown in Table 2.

Table 2. Data of complete structural basis of designing strategies.

N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0)	104961	5.721
2	(0 0 1)	270001	5.660
3	(0 1 0)	74428	1.652
4	(0 1 1)	80317	0.931
5	(1 0 0)	102500	2.534
6	(1 0 1)	253473	4.342
7	(1 1 0)	157583	2.633
8	(1 1 1)	246776	1.921

The behavior of the functions $V(t)$ and $W(t)$ for this network is shown in Fig. 4.

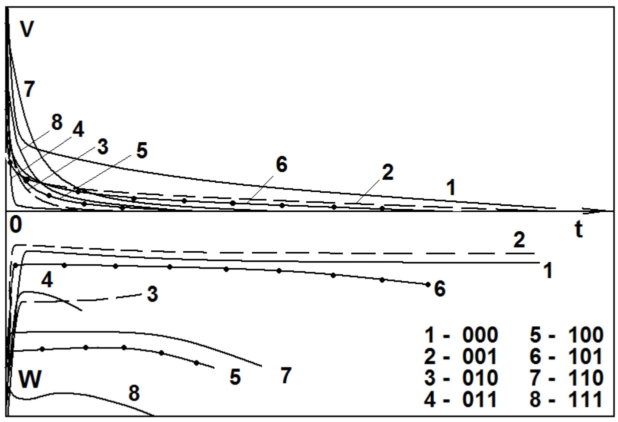


Fig. 4. Behavior of the functions $V(t)$ and $W(t)$ for eight design strategies during the design process for network in Fig. 3.

As for previous example for the network in Fig. 3 we also can conclude that the speed of decreasing of the Lyapunov function is inversely proportional to the design time. The minimal value of the Lyapunov function that corresponds to the maximum precision is in the limits from 1.2_{10}-5 for strategy 000 to 5.9_{10}-5 for strategy 111. Anew we can see from Fig. 4 that a large absolute value of the function $W(t)$ corresponds to a more rapid decreasing of the function $V(t)$ and a smaller computer design time. The strategies 3, 4, 5, 7 and 8 have a large value of the function $W(t)$ during all design process till a small value of the function $V(t)$. That is why these strategies have a relative little computer time.

Other example corresponds to the passive nonlinear network with four nodes (Fig. 5).

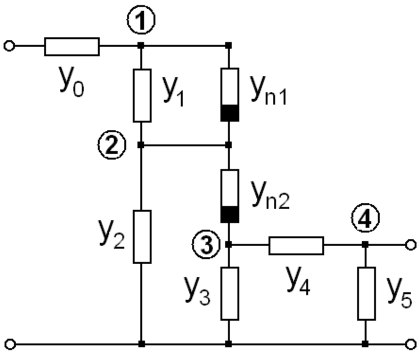


Fig. 5. Four-node nonlinear passive network.

The vector X includes nine components. Five components correspond to the admittances $(x_1, x_2, x_3, x_4, x_5)$, where $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, and four parameters are the nodal voltages (x_6, x_7, x_8, x_9), where $x_6 = V_1$, $x_7 = V_2$, $x_8 = V_3$, $x_9 = V_4$, The nonlinear elements are defined as: $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2$. The control vector U includes nine components $(u_1, u_2, ..., u_9)$. The model of circuit (2) includes 4 equations and functions $g_j(X)$ are defined by the next system:

$$g_1(X) \equiv y_0(V_0 - x_6) - \left[x_1^2 + a_{n1} + b_{n1}(x_6 - x_7)^2\right](x_6 - x_7) = 0$$

$$g_2(X) \equiv \left[x_1^2 + a_{n1} + b_{n1}(x_6 - x_7)^2\right](x_6 - x_7) - x_2^2 x_7 - \left[a_{n2} + b_{n2}(x_7 - x_8)^2\right](x_7 - x_8) = 0$$

$$g_3(X) \equiv \left[a_{n2} + b_{n2}(x_7 - x_8)^2\right](x_7 - x_8) - \left(x_3^2 + x_4^2\right)x_8 - x_4^2 x_9 = 0$$

$$g_4(X) \equiv x_4^2 x_8 - \left(x_4^2 + x_5^2\right)x_9 = 0$$

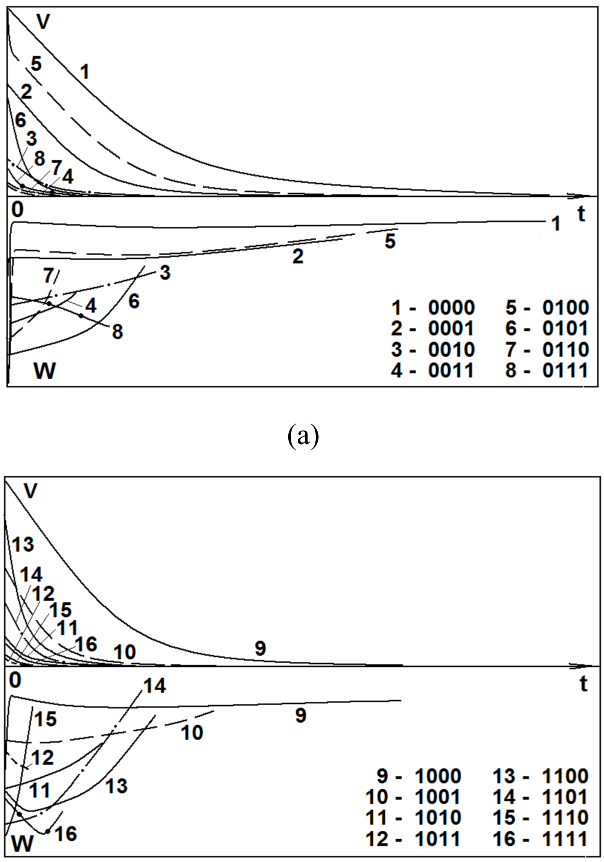
The optimization procedure (1) includes nine equations. The cost function $C(X)$ of the design process is defined by the following form: $C(X) = (x_9 - k_0)^2 + (x_6 - x_7 - k_1)^2 + (x_7 - x_8 - k_2)^2$.

The results of the analysis of complete structural basis of different strategies of designing for network in Fig. 3 are shown in Table 3.

Table 3. Data of complete structural basis of designing strategies for network in Fig. 5.

N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0 0)	32371	5.441
2	(0 0 0 1)	31726	2.970
3	(0 0 1 0)	11598	1.263
4	(0 0 1 1)	21486	0.611
5	(0 1 0 0)	33846	3.574
6	(0 1 0 1)	41960	1.162
7	(0 1 1 0)	18223	0.491
8	(0 1 1 1)	37651	0.885
9	(1 0 0 0)	33136	3.572
10	(1 0 0 1)	61377	1.762
11	(1 0 1 0)	27278	0.834
12	(1 0 1 1)	11582	0.271
13	(1 1 0 0)	44656	1.257
14	(1 1 0 1)	46412	1.113
15	(1 1 1 0)	19478	0.330
16	(1 1 1 1)	41384	0.553

The behavior of the functions $V(t)$ and $W(t)$ for the complete set of structural basis is shown in Fig. 6.



(a)

(б)

Fig. 6. Behavior of the functions $V(t)$ and $W(t)$ for all strategies of structural basis during the design process for network in Fig. 5.

We can see that the processor time of each strategy is inverse proportional of the absolute value of the function *W(t)*.

In Fig. 7 there is a passive nonlinear network with five nodes.

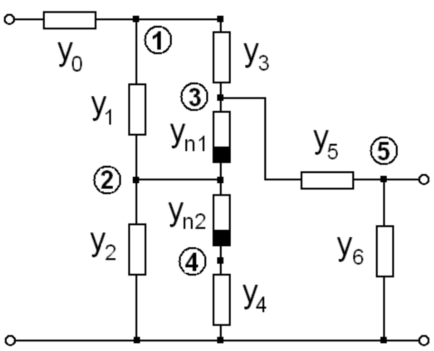


Fig. 7. Five-node nonlinear passive network.

The nonlinear elements have next dependencies: $y_{n1}=a_{n1}+b_{n1}\cdot(V_3-V_2)^2$, $y_{n2}=a_{n2}+b_{n2}\cdot(V_4-V_2)^2$. The vector X includes eleven components. The first six components are defined as: $x_1^2=y_1$, $x_2^2=y_2$, $x_3^2=y_3$, $x_4^2=y_4$, $x_5^2=y_5$, $x_6^2=y_6$. The others components are defined as: $x_7=V_1$, $x_8=V_2$, $x_9=V_3$, $x_{10}=V_4$, $x_{11}=V_5$. The control vector U includes eleven components too. The structural basis includes 32 strategies. The mathematical model (2) of this circuit is defined on the basis of nodal method and includes five equations in this case. The optimization procedure includes eleven equations and it is based on formulas (1) and (5). The cost function *C(X)* is defined by the formula similar a previous example:

$$C(X)=(x_{11}-kk_0)^2+\left[(x_8-x_9)^2-kk_1\right]^2+\left[(x_9-x_{10})^2-kk_2\right]^2.$$

The results of the analysis of some strategies of structural basis are shown in Table 4.

Table 4. Data of complete structural basis of designing strategies for network in Fig. 7.

N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0 0 0)	33456	14.121
2	(0 0 0 0 1)	10837	5.632
3	(0 0 1 1 0)	15490	5.164
4	(0 1 1 1 0)	35567	3.911
5	(0 1 1 1 1)	28360	2.415
6	(1 0 1 1 0)	20756	4.181
7	(1 1 1 1 0)	36049	3.460
8	(1 1 1 1 1)	29002	1.211

The behavior of the functions *V(t)* and *W(t)* for strategies corresponding the Table 4 is shown in Fig. 8.

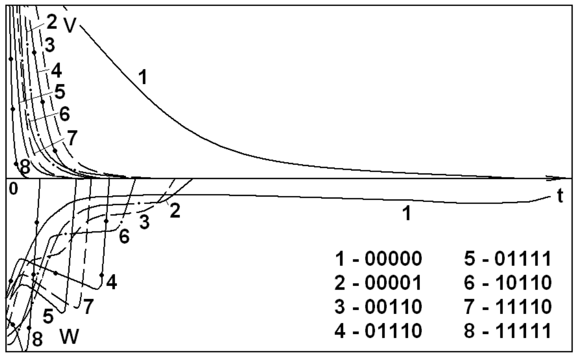


Fig. 8. Behavior of the functions *V(t)* and *W(t)* for some strategies of structural basis during the design process for network in Fig. 7.

Anew we can state that processor time for each strategy is inverse proportional of absolute value of the function *W(t)*.

Next example corresponds to the one-stage transistor amplifier in Fig. 9.

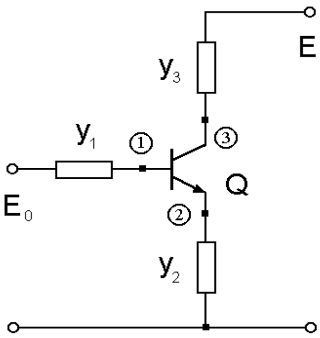


Fig. 9. One-stage transistor amplifier.

The vector X includes six components: $x_1^2=y_1$, $x_2^2=y_2$, $x_3^2=y_3$, $x_4=V_1$, $x_5=V_2$, $x_6=V_6$. The model of this network (2) includes three equations (M=3) and the optimization procedure (5) includes six equations. The total structural basis contains eight different strategies. The control vector includes three control functions: $U=(u_1,u_2,u_3)$. The Ebers-Moll static model of the transistor has been used [13].

The results of the analysis of complete structural basis of the design strategies are shown in Table 5. As for the previous example, Fig. 10 shows the behavior of the functions *V(t)* and *W(t)* for a time interval when the majority of the design strategies are finished.

Table 5. Data of complete structural basis of designing strategies for network in Fig. 9.

N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0)	7683758	518,22
2	(0 0 1)	45900	2,42
3	(0 1 0)	1151505	60,14
4	(0 1 1)	47464	2,53
5	(1 0 0)	109784	5,87
6	(1 0 1)	4753	0,25
7	(1 1 0)	303579	14,83
8	(1 1 1)	4940	0,08

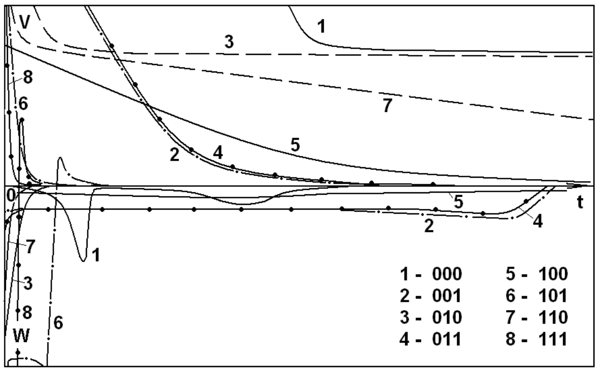


Fig. 10. Behavior of the functions *V(t)* and *W(t)* for some strategies of structural basis during the design process for network in Fig. 9.

The strategies with control vector 101 and 111 have extremely large value of the relative derivative *W(t)* from the beginning of the design process and that is why the Lyapunov function is decreases very rapidly. The relative design time is very small for two these strategies and it is equal to 0.00048 and 0.00015 accordingly. The strategies with the control vector 001, 011 and 100 have the sufficient level of the function *W* during the analyzed interval and the relative design time is equal to 0.0054, 0.0061 and 0.0114 accordingly. Nevertheless three other design strategies with the control vector 000, 010 and 110 are not finished during the presented interval.

It occurs because the function *W* for these strategies decreases rapidly while the Lyapunov function had a relatively large value. After this the Lyapunov function decreases very slowly and the relative design time is equal to 1.0, 0.116 and 0.029 accordingly.

Other example corresponds to the two-stage transistor amplifier in Fig. 11.

This network is characterized by five dependent parameters and the control vector includes five control functions: $U = (u_1, u_2, u_3, u_4, u_5)$. The structural basis consists of 32 design strategies. The results of the analysis of some design strategies from the structural basis are shown in Table 6.

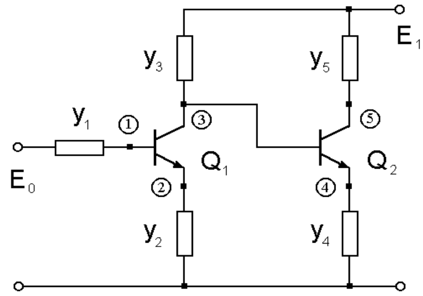


Fig. 11. Two-stage transistor amplifier.

Table 6. Data of some strategies of structural basis for network in Fig. 11.

N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0 0 0)	165962	299,56
2	(0 0 0 0 1)	337487	737,55
3	(0 0 1 0 0)	44118	68,87
4	(0 0 1 0 1)	14941	19,06
5	(0 0 1 1 1)	21971	22,03
6	(0 1 1 0 1)	4544	4,56
7	(1 0 1 0 1)	2485	1,65
8	(1 0 1 1 1)	7106	3,57
9	(1 1 1 0 1)	2668	1,32
10	(1 1 1 1 1)	79330	10,11

The behavior of the functions *V(t)* and *W(t)* for these strategies is shown in Fig. 12.

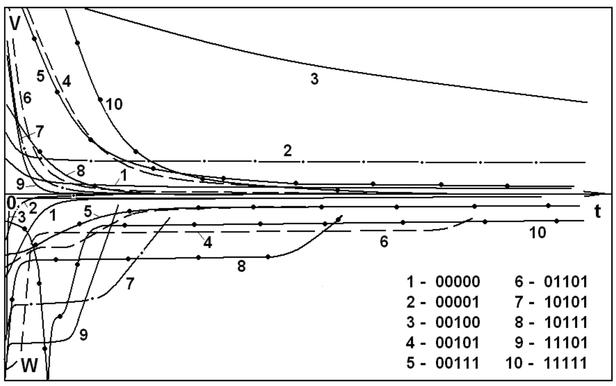


Fig. 12. Behavior of the functions *V(t)* and *W(t)* for some strategies for circuit in Fig. 11.

These graphs correspond to a time interval when the majority of the design strategies are finished. The strategies 6, 7, 8 and 9 have a large value of the relative derivative *W(t)* from the initial of the design process. This property provides extremely fast decreasing of the Lyapunov function. The design time for these design strategies is presented in Table 2. We can see that just these strategies 6, 7, 8 and 9 have the design time lesser than other strategies. The strategies 4, 5 and 10 have an average value of the function *W* in the initial part of the design process and these strategies have an average value of the design time. At last, the strategies 1, 2 and 3 have a large design time and just these strategies have a very fast decreasing of the function *W* during initial part of the design process.

The analysis of the three-stage amplifier of Fig. 13 shows very similar results.

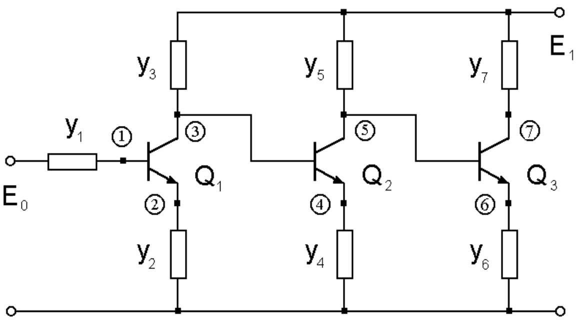


Fig. 13. Three-stage transistor amplifier

The results of the analysis of three-stage amplifier are presented below in Table 7 and Fig. 14. Functions *V(t)* and *W(t)* were the main objects of the analysis and have been analyzed for some strategies that compose the structural basis of the general methodology.

Table 7. Data of some design strategies for three-stage amplifier

N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0 0 0 0 0)	2354289	420.18
2	(0 0 1 0 1 0 1)	110889	117.15
3	(0 1 1 1 0 0 0)	1075433	272.01
4	(1 0 1 0 1 0 1)	102510	49.76
5	(1 0 1 1 1 0 1)	107541	43.99
6	(1 0 1 1 1 1 1)	38751	12.53
7	(1 1 1 0 1 1 1)	43387	13.67
8	(1 1 1 1 1 0 0)	185085	110.62
9	(1 1 1 1 1 1 0)	147094	66.13
10	(1 1 1 1 1 1 1)	52651	4.56

Fig. 14 shows the behavior of the functions *V(t)* and *W(t)* for some design strategies. These graphs correspond to a time interval when the majority of the design strategies are finished.

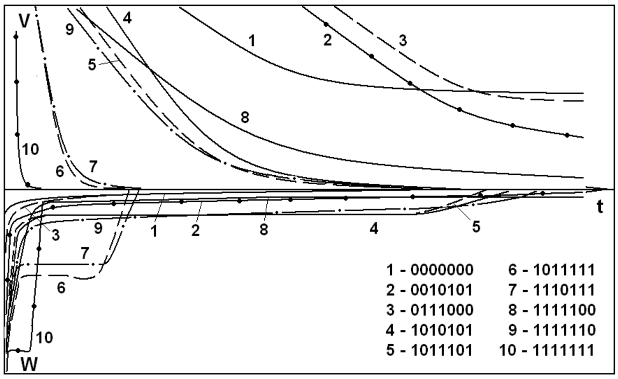


Fig. 14. Behavior of the functions *V(t)* and *W(t)* for some strategies during the design process for network in Fig. 13.

The strategies 6, 7, and 10 have a minimal relative computer time because the function *W(t)* for these strategies has a relatively large negative value during a long time of the design process in spite of the large value of Lyapunov function *V(t)* in initial time interval. On the contrary, the function *W(t)* has a relatively small value for the strategies 1, 2 and 3. That is why these strategies have a large computer design time. We can state that the large absolute value of the function *W(t)* on initial part of the design process provoke the less computer time.

We can state that the behavior of the Lyapunov function *V* and the relative time derivative *W* surely determine the design time. It means that it is possible be guided by means of these functions to predict the computer design time for any design strategy. We could analyzed the initial time interval of the functions *V(t)* and *W(t)* behavior for the different strategies and by this analysis we can predict the strategies that have a minimal computer design time.

5 Conclusion

The problem of the minimal-time design algorithm construction can be solved adequately on the basis of the control theory. The design process in this case is formulated as the controllable dynamic system. The Lyapunov function and its time derivative include the sufficient information to select more perspective design strategies from infinite set of the different design strategies that exist into the general design methodology. The special functions *W(t)* and *S(t)* have been proposed to predict the better design strategies with a minimal design time. There is a close relation between the computer time and the properties of the Lyapunov function of design process. These functions can be used as the principal tool to the time optimal design algorithm prediction. The successful solution of this problem permits to construct the minimal-time system design algorithm.

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