Study of Structure of Control Vector for the Problem of Analog Circuit Optimization

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Abstract: - The generalized methodology for electronic networks optimization was elaborated by means of the optimal control theory approach. A special control vector is defined to redistribute the compute expense between a network analysis and a parametric optimization. In this case the problem of the electronic system optimization is formulated as a classical problem of functional minimization of the optimal control theory. The optimization algorithm with a minimal computer time was defined as a controllable dynamic process with an optimal control vector. By this methodology the aim of the optimization process with a minimal computer time is presented as a transition process of some dynamic system that has the minimal transition time. The optimal position of the control vector switch points was determined as a principal characteristic of the algorithm with a minimal computer time for network optimization. The conception of the Lyapunov function of dynamic controllable system is used to analyze the principal characteristics of the process of network optimization. The special function that is a combination of Lyapunov function of the design process and its time derivative was proposed to predict the optimal control vector to construct the best algorithm for circuit optimization.

Key-Words: - Time-optimal design algorithm, circuit optimization, control theory formulation, Lyapunov function, switch points.

1 Introduction

The problem of the computer time reduction of a large system design is one of the essential problems of the total quality design improvement. Besides the traditionally used ideas of sparse matrix techniques and decomposition techniques defined decades ago [1-5] some another ways were proposed to reduce the total computer design time [6-7].

The other formulation of the circuit optimization problem was developed on heuristic level some decades ago [8-9]. This idea was based on the Kirchhoff laws ignoring for all the circuit or for the circuit part. The special cost function is minimized instead of the circuit equation solving. This idea was developed in practical aspect for the microwave circuit optimization [10] and for the synthesis of high-performance analog circuits [11] in extremely case, when the total system model was eliminated. The authors of the last papers affirm that the design time was reduced significantly. This last idea can be named as the modified traditional design strategy.

The generalized approach for the analog system design on the basis of the control theory formulation was elaborated in some previous works, for example [12]. This approach serves for the minimal-time design algorithm definition. On the other hand this approach gives the possibility to analyze with a great clearness the design process while moving along the trajectory curve into the design space. The main conception of this theory is the introduction of the special control functions, which, on the one hand generalize the design process and, on the other hand, they give the possibility to control the design process to achieve the optimum of the design cost function for the minimal computer time. This possibility appears because practically an infinite number of the different design strategies that exist within the bounds of the theory. The different design strategies have the different operation number and executed computer time. On the bounds of this conception, the traditional design strategy is only a one representative of the enormous set of different design strategies. The formulation of the intrinsic properties and special restrictions of the optimal design strategy is one of the first problems that need to be solved for the optimal algorithm construction.

2 Problem Formulation

The optimization process for any analog system can be defined in discrete form [12] as the problem of the generalized cost function $F(X,U)$ minimization by means of the system (1) with the constraints (2):

$$x_i^{s+1} = x_i^s + t_s \cdot f_i(X,U), \quad i = 1,2,...,N, \tag{1}$$

$$(1-u_j)g_j(X) = 0, \quad j = 1,2,\dots,M, \tag{2}$$

where $X \in R^N$, $X=(X',X'')$, $X' \in R^K$ is the vector of the independent variables and the vector $X'' \in R^M$ is the vector of dependent variables ($N=K+M$). All the functions $g_j(X)$ for all j presents the network model, s is the iterations number, t_s is the iteration parameter, $t_s \in R^1$, $H \equiv H(X,U)$ is the direction of the generalized cost function $F(X,U)$ decreasing, U is the vector of the special control functions $U=(u_1,u_2,...,u_m)$, where $u_j \in \Omega$; $\Omega = \{0;1\}$. The functions $f_i(X,U)$ for example for the gradient method are defined as:

$$f_i(X,U) = -\frac{\delta}{\delta x_i} F(X,U) \qquad i = 1,2,\dots,K \tag{3}$$

$$f_i(X,U) = -u_{i-K}\frac{\delta}{\delta x_i}F(X,U) + \frac{(1-u_{i-K})}{t_s}\{-x_i^s + \eta_i(X)\} \tag{3'}$$

$$i = K+1, K+2, \dots, N$$

where the operator $\frac{\delta}{\delta x_i}$ hear and below means $\frac{\delta}{\delta x_i}\varphi(X) = \frac{\partial\varphi(X)}{\partial x_i} + \sum_{p=K+1}^{K+M}\frac{\partial\varphi(X)}{\partial x_p}\frac{\partial x_p}{\partial x_i}$, x_i^s is equal to $x_i(t-dt)$; $\eta_i(X)$ is the implicit function ($x_i = \eta_i(X)$) that is determined by the system (2). The generalized cost function $F(X,U)$ can be defined for example as:

$$F(X,U) = C(X) + \psi(X,U) \tag{4}$$

where $C(X)$ is the non negative cost function of the design process, and $\psi(X,U)$ is the additional penalty function:

$$\psi(X,U) = \frac{1}{\varepsilon}\sum_{j=1}^{M} u_j \cdot g_j^2(X) \tag{5}$$

The continuous form of the problem definition is more adequate for the control theory application. This continuous form replaces Eq. (1) and can be defined by the next formula:

$$\frac{dx_i}{dt} = f_i(X,U), \quad i = 0,1,...,N. \tag{6}$$

This system together with equations (2), (3) and (4) composes the continuous form of the design process. The structural basis of different design strategies that correspond to the fixed control vector includes 2^M design strategies.

The control variables u_j have the time dependency in general case. The equation number j is removed from (2) and the dependent variable x_{K+j} is transformed to the independent when u_j=1. This independent parameter is defined by the formulas (6), (3). In this case there is no difference between formulas (3) and (3'). On the other hand, the Eq. (6) with the right part (3') is transformed to the identity $\frac{dx_i}{dt} = \frac{dx_i}{dt}$, when u_j=0, because $\eta_i(X) - x_i^s = x_i(t) - x_i(t-dt) = dx_i$. It means that at this time moment the parameter x_i is dependent one and the current value of this parameter can be obtained from the system (2) directly. This transformation of the vectors X' and X'' can be done at any time moment. The function $f_0(X,U)$ is determined as the necessary time for one step of the system (6) integration. This function depends on the concrete design strategy. The additional variable x_0 is determined as the total computer time T for the system design. In this case we determine the problem of the time-optimal system design as the classical problem of the functional minimization of control theory. In this context the aim of the optimal control is to result each function

$f_i(X,U)$ to zero for the final time T, to minimize the cost function and the total computer time x_0.

It is necessary to find the optimal behavior of the control functions u_j during the design process to minimize the total design computer time. The functions $f_i(X,U)$ are piecewise continued as the temporal functions and the structure of these functions can be found by approximate methods of the control theory [13-15].

This formulation of the design process permits the redistribution of the computer time expense between the solution of the problem (2) and the optimization procedure (1) for the function $F(X,U)$. The control vector U is the main tool for the redistribution process in this case. Practically an infinite number of the different design strategies are produced because the vector U depends on the optimization procedure current step. The traditional design strategy (TDS) is formulated in this case as a strategy with all functions u_j equal to 0. The problem of the optimal design strategy search is formulated now as the typical problem for the functional minimization of the control theory. The functional that needs to minimize is the total CPU time T of the design process. This functional depends directly on the operations number and on the design strategy that has been realized.

Now the process for analog network design is formulated as a dynamic controllable system. The minimal-time design process can be defined as the dynamic system with the minimal transition time In this case. So, we need to find the special conditions to minimize the transition time for this dynamic system.

3 Lyapunov Function

On the basis of the analysis in previous section we can conclude that the minimal-time algorithm has one or some switch points in control vector where the switching is realize among different design strategies. As shown in [16] it is necessary to switch the control vector from like modified traditional design strategy (MTDS) when all u_j equal to 1 to like traditional design strategy (TDS) with some adjusting. Some principal features of the time-optimal algorithm were determined previously. These are: 1) an additional acceleration effect that appeared under special circumstances [17]; 2) the start point special selection outside the separate hyper-surface to guarantee the acceleration effect, at least one negative component of the start value of the vector X is can be recommended for this; 3) an optimal structure of the control vector with the necessary switch points. The two first problems were discussed in [16-17]. The third problem is discussed in the present paper.

The main problem of the time-optimal algorithm construction is unknown optimal sequence of the switch points during the design process. We need to define a special criterion that permits to realize the optimal or quasi-optimal algorithm by means of the optimal switch points searching. A Lyapunov function of dynamic system serves as a very informative object for any system analysis in limits of the control theory. Traditionally this function is used for the stability analysis of dynamic systems. We propose to use a Lyapunov function of the design process for the optimal algorithm searching, particularly for the optimal switch points detect. The properties of the Lyapunov function give possibility to solve this problem.

There is a freedom of the Lyapunov function choice because of a non-unique form of this function. Let us define the Lyapunov function of the design process (1)-(5) by the following expression:

$$V(X) = \sum_i (x_i - a_i)^2 \tag{7}$$

where a_i is the stationary value of the coordinate x_i, in other words the set of all the coefficients a_i is the main objective of the design process. The function (7) satisfies all of the conditions of the standard Lyapunov function definition for the variables $y_i = x_i - a_i$.

We can use this form of the Lyapunov function if we already found the design solution someway. On the other hand, it is very important to control the stability of the design process during the optimization procedure. In this case we need to construct other form of the Lyapunov function that doesn't depend on the unknown stationary point. Let us define two new forms of the Lyapunov function by the next formulas:

$$V(X,U) = [F(X,U)]^r \tag{8}$$

$$V(X,U) = \sum_i \left(\frac{\partial F(X,U)}{\partial x_i} \right)^2 \tag{9}$$

where $F(X,U)$ is the generalized cost function of the design process. The formula (8) can be used when the general cost function is no negative and has zero value at the stationary point a. Other formula can be used always because all derivatives $\partial F / \partial x_i$ are equal to zero in the stationary point a.

We can define now the design process as a transition process for controllable dynamic system that can provide the stationary point (optimal point of the network optimization procedure) during some time. The problem of the time-optimal design algorithm construction can be formulated now as the problem of the transition process searching with the minimal transition time. There is a well-known idea [18-19] to minimize the time of the transition process by means of the special choice of the right hand part of the principal system of equations, in our case these are the functions $f_i(X,U)$. It is necessary to change the functions $f_i(X,U)$ by means of the control vector *U* selection to obtain the maximum speed of the Lyapunov function decreasing (the maximum absolute value of the Lyapunov function time derivative $\dot{V} = dV/dt$). Normally the time derivative of the Lyapunov function is non-positive for the stable processes. However, we can define now more informative function as a time derivative of Lyapunov function relatively the Lyapunov function: $W = \dot{V}/V$. In this case we can compare the different design strategies by means of the function *W(t)* behavior and we can search the optimal position for the control vector switch points.

4 Control Vector Structure

The optimal structure of the control vector *U* is the principal aim of the analysis of design process based on generalized methodology. This control vector's structure produces optimal or quasi optimal design process that minimizes the computer time. Functions *V(t)* and $\dot{V}(t)$ were the main objects of the analysis and its behavior has been analyzed during the design process. The behavior of the functions *V(t)*, $\dot{V}(t)$ and *W(t)* can define the total computer time for each design strategy [20].

The analysis of the design process for two-node passive nonlinear network in Fig. 1 is presented below.

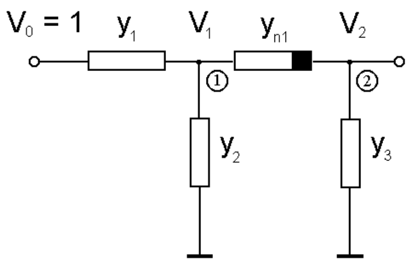

Fig. 1. Two-node nonlinear passive network.

The nonlinear element has the following dependency: $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$. The vector *X* includes five components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$. The model of this network (2) includes two equations (*M*=2) and the optimization procedure (5) includes five equations. This network is characterized by two dependent parameters and the control vector includes two control functions: $U=(u_1, u_2)$. Structural basis includes four different strategies with corresponding control vector: (00), (01), (10), and (11). Behavior of the functions *V(t)* and *W(t)* help us to determine the switch point optimal position of the control vector.

Taking into account the preliminary reasons about the optimal algorithm structure [17] we have been analyzed the strategy that consists of two parts. The first part is defined by the control vector (11) that corresponds to MTDS and the second part is defined by the control vector (00) that corresponds to TDS. So, the switching is realized between two strategies, (11) and (00).

The results of the network optimization corresponding total iteration number and computer time are presented in Table 1. The integration of the system (6) was realized by the constant integration step. The step for switch point increment is equal 20 to improve the identification of the difference between all curves.

Table 1. Iterations number and computer time for strategies with different switch points for network in Fig. 3.

N	Switch point	Iterations number	Total design time (sec)
1	147	8319	0.221
2	167	6501	0.172
3	187	3697	0.096
4	207	2860	0.073
5	227	3383	0.087
6	247	5429	0.142
7	267	6682	0.175

The behavior of the functions *V(t)* and *W(t)* during the design process after the switch point is shown in Fig. 2.

The analysis shows that the optimal switch point corresponds to the step 207 (graph 4 with dots in Fig. 2). The curves 1, 2, and 3 correspond to the switch point position before the optimal switch

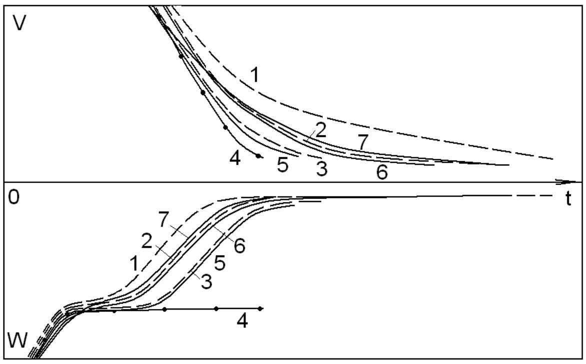

Fig. 2. Behavior of the functions *V(t)* and *W(t)* during the design process for seven different switch points (from 147 to 267) for network in Fig. 1.

point (curve 4), but the curves 5, 6, and 7 correspond to the switch point that lies after the optimal one. There is a decreasing of the computer time from curve 1 to curve 4. On the contrary, the computer time increases from curve 4 to curve 7. It means that curve 4 corresponds to the optimal position of the switch point.

The initial parts of *W(t)* dependencies of Fig. 2 are shown in Fig. 3 in large scale.

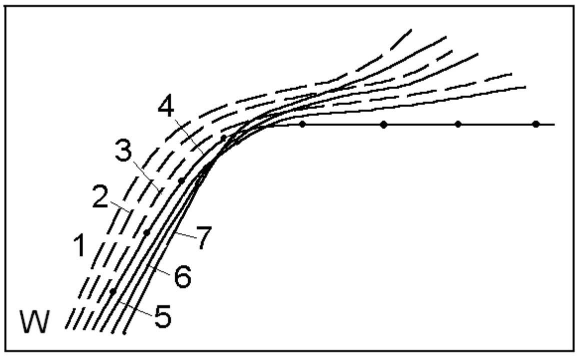

Fig. 3. Behavior of the functions *V(t)* and *W(t)* during the initial part of design process for network in Fig. 1.

We can see that the curves 1, 2, and 3, which correspond to the switch points before the optimal point (4) have not intersections. On the other hand, the curves 5, 6, and 7 that are based on the switch point after the optimal one have intersections and each this curve lies upper the curve 4 till some time point. It means that from this time moment the graph *W(t)* for the optimal switch point lies below all of other graph. So, from one hand the optimal switch point corresponds to a minimal computer time, from the other hand, this point corresponds to the graph of *W(t)* function that lies below all of other graphs. This property serves as a principal criterion for the optimal switch point selection.

The function *W(t)* that corresponds to the optimal switch point has a maximum absolute value leading off the 340th integration step. It means that from this integration step we can confidently predict the optimal switch point position that leads to the minimal computer design time.

The analysis of the design process for three-node passive nonlinear network in Fig. 4 is presented below.

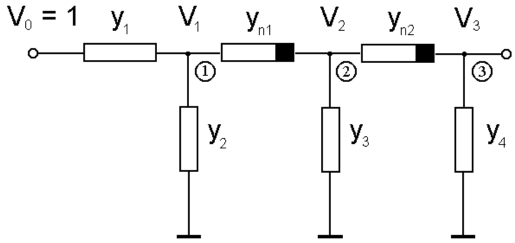

Fig. 4. Three-node nonlinear passive network.

The nonlinear elements are defined as: $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2$. The vector X includes seven components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5 = V_1$, $x_6 = V_2$, $x_7 = V_3$. The model of this network (2) includes three equations (M=3) and the optimization procedure (5) includes seven equations. This network is characterized by three dependent parameters and the control vector includes three control functions: $U = (u_1, u_2, u_3)$. Structural basis includes eight different strategies with corresponding control vector: (000), (001), (010), (011), (100), (101), (110), and (111). Behavior of the functions *V(t)* and *W(t)* help us to determine the switch point optimal position of the control vector.

Taking into account the above defined reasons about the optimal algorithm structure we have been analyzed the strategy that consists of two parts. The first part is defined by the control vector (111) that corresponds to MTDS and the second part is defined by the control vector (000) that corresponds to TDS. So, the switching is realized between two strategies, (111) and (000).

The optimal switch point was a principal objective of this analysis. The consecutive change of the switch point was realized for the integration step number from 2 to 20.

The behavior of the functions *V(t)* and *W(t)* during the design process after the switch point is shown in Fig. 5.

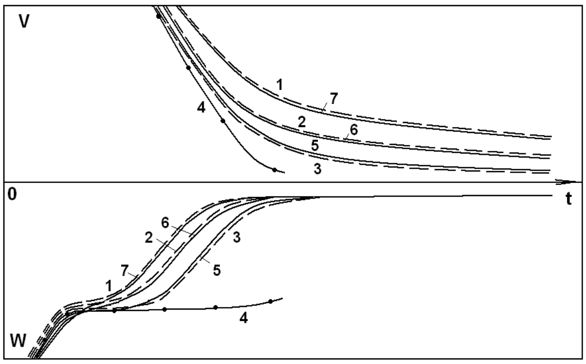

Fig. 5. Behavior of the functions *V(t)* and *W(t)* during the design process for seven different switch points (from 6 to 12) for network in Fig. 4.

As discussed above, the principal element of the minimal-time design algorithm is the optimal position of the control vector switch point. Fig. 5 shows the behavior of the functions *V(t)* and *W(t)* for seven different positions of the switch point. The corresponding total iteration number and computer time are presented in Table 2.

Table 2. Iterations number and computer time for strategies with different switch points for network in Fig. 4.

N	Switch point	Iterations number	Total design time (sec)
1	6	8409	0.659
2	7	6408	0.502
3	8	3141	0.246
4	9	1234	0.096
5	10	3310	0.259
6	11	5918	0.464
7	12	7404	0.581

The integration of the system (6) was realized by the constant integration step. The analysis shows that the optimal switch point corresponds to the step 9 (graph 4 with dots in Fig. 5). The curves 1, 2, and 3 correspond to the switch point position before the optimal switch point (curve 4), but the curves 5, 6, and 7 correspond to the switch point that lies after the optimal one. There is a decreasing of the computer time from curve 1 to curve 4. On the contrary, the computer time increases from curve 4 to curve 7. It means that curve 4 corresponds to the optimal position of the switch point.

The initial part of *W(t)* dependencies of Fig. 5 are shown in Fig. 6 in large scale.

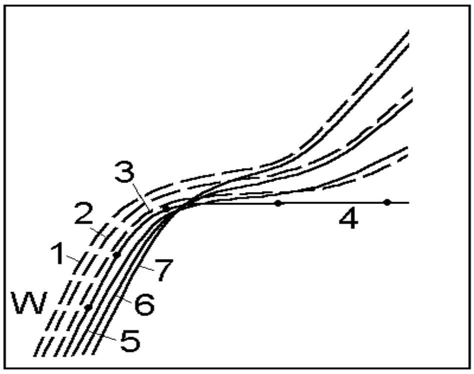

Fig. 6. Behavior of the functions *V(t)* and *W(t)* during the initial part of de sign process for network in Fig. 4.

We can see that the curves 1, 2, and 3, which correspond to the switch points before the optimal point (4) have not intersections. On the other hand, the curves 5, 6, and 7 that are based on the switch point after the optimal one have intersections and each this curve lies upper the curve 4 till some time point. It means that from this time moment the graph *W(t)* for the optimal switch point lies below all of other graph. So, from one hand the optimal switch point corresponds to a minimal computer time, from the other hand, this point corresponds to the graph of *W(t)* function that lies below all of other graphs. This property anew serves as a principal criterion for the optimal switch point selection. The function *W(t)* that corresponds to the optimal switch point has a maximum absolute value leading off the 15th integration step. It means that from this integration step we can confidently predict the optimal switch point position that leads to the minimal computer design time.

The next example corresponds to the fourth-node passive network that is shown in Fig. 7.

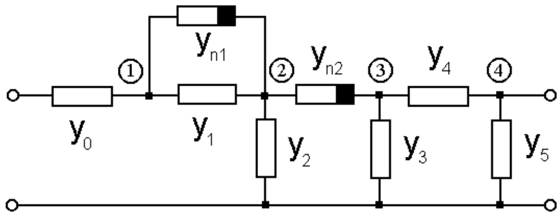

Fig. 7. Four-node nonlinear passive network.

The nonlinear elements have the same dependencies as in previous example. The vector X includes nine components: $x_1^2=y_1$, $x_2^2=y_2$, $x_3^2=y_3$, $x_4^2=y_4$, $x_5^2=y_5$ $x_6=V_1$, $x_7=V_2$, $x_8=V_3$, $x_9=V_4$.

The model of this network (2) includes four equations (M=4) and the optimization procedure (5) includes nine equations. This network is characterized by four dependent parameters and the control vector includes four control functions: $U=(u_1, u_2, u_3, u_4)$. Structural basis includes 16 different design strategies with corresponding control vector from (0000) to (1111). Behavior of the functions *V(t)* and *W(t)* help us to determine the optimal position of the switch point of the control vector.

We have been analyzed the strategy that consists of two parts. The first part is defined by the control vector (1111) that corresponds to MTDS and the second part is defined by the control vector (0000) that corresponds to TDS. So, the switching is realized between two strategies: (1111) and (0000). The consecutive change of the switch point was realized for integration step number from 2 to 40.

The behavior of the functions *V(t)* and *W(t)* during the design process after the switch point is shown in Fig. 8.

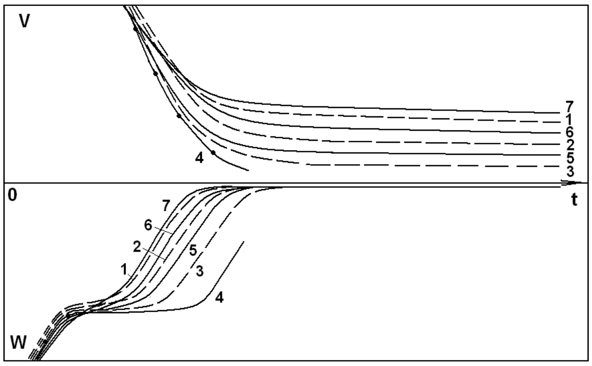

Fig. 8. Behavior of the functions *V(t)* and *W(t)* during the design process for seven different switch points (from 30 to 36) for network in Fig. 7.

As discussed above, the principal element of the minimal-time design algorithm is the optimal position of the control vector switch point. Fig. 8 shows the behavior of the functions *V(t)* and *W(t)* for seven different positions of the switch point. The corresponding total iteration number and computer time are presented in Table 3.

The integration of the system (6) was realized by the constant integration step. The analysis shows that the optimal switch point corresponds to the step 33 (graph 4 in Fig. 8). The curves 1, 2, and 3 correspond to the switch point position before the optimal switch point (curve 4), but the curves 5, 6, and 7 correspond to the switch point that lies after the optimal one.

Table 3. Iterations number and computer time for strategies with different switch points for network in Fig. 7.

N	Switch point	Iterations number	Total design time (sec)
1	30	12570	2.826
2	31	9677	2.174
3	32	4182	0.936
4	33	1020	0.223
5	34	8134	1.826
6	35	11612	2.609
7	36	13795	3.101

There is a decreasing of the computer time from curve 1 to curve 4. On the contrary, the computer time increases from curve 4 to curve 7. It means that curve 4 corresponds to the optimal position of the switch point.

So, the optimal switch point corresponds to the minimal computer time, from the other hand, this point corresponds to the graph of *W(t)* function that lies below of all the other graphs. This property selects the optimal switch point.

We can see that the function *W(t)* that corresponds to the optimal switch point has a maximum absolute value leading off the 54th integration step. It means that from this integration step we can predict the optimal switch point position that minimizes the computer design time.

Next example corresponds to the one-stage transistor amplifier in Fig. 9.

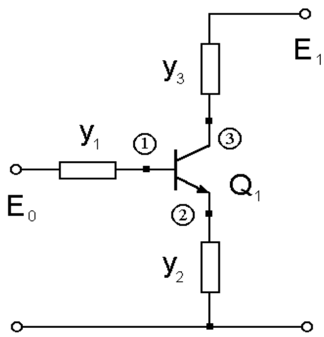

Fig. 9. One-stage transistor amplifier.

The vector X includes ten components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$, $x_6 = V_3$. The model of this network (2) includes three equations (M=3) and the optimization procedure (6) includes six equations. The total structural basis contains eight different design strategies. The control vector includes five control functions:

$U = (u_1, u_2, u_3)$. The Ebers-Moll static model of the transistor has been used [21].

Fig. 10 shows the behavior of the functions *V(t)* and *W(t)* during the design process with different switch points.

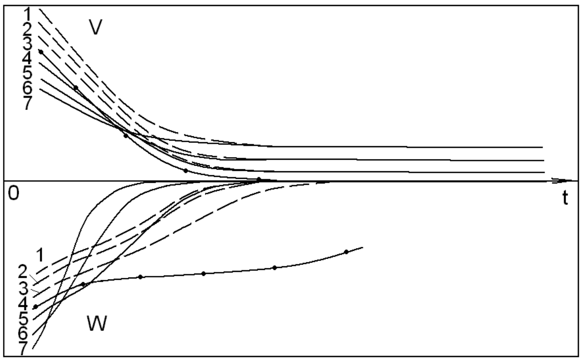

Fig. 10. Behavior of the functions *V(t)* and *W(t)* during the design process for seven different switch points (from 33 to 39) for network in Fig. 9.

The behavior of these functions helps us to determine the optimal position of the control vector switch point. We have been analyzed the strategy that consists of two parts. The first part is defined by the control vector (111) that corresponds to MTDS and the second part is defined by the control vector (000) that corresponds to TDS. The optimal switch point was an aim of the analysis. The consecutive change of the switch point was realized for the integration step number from 2 to 50. The behavior of the functions *V(t)* and *W(t)* for the switch points from 33 to 39 are shown in this figure and the data, which correspond to these graphs, are presented in Table 4. The analysis shows that the optimal switch point corresponds to the step 36 (graph with dots). The computer design time has a minimal value for this step.

Table 4. Iterations number and computer time for strategies with different switch points for network in Fig. 9.

N	Switch point	Iterations number	Total design time (sec)
1	33	2433	0.404
2	34	2180	0.361
3	35	1748	0.289
4	36	61	0.01
5	37	1705	0.281
6	38	2111	0.349
7	39	2349	0.389

We can see that the function *W(t)* has a maximum absolute value for the optimal switch step (number 4) leading off the 55th integration step. It means that from this integration step we can confidently predict the optimal switch point position that leads to the minimal computer design time.

The next example corresponds to the two-stage transistor amplifier in Fig. 11.

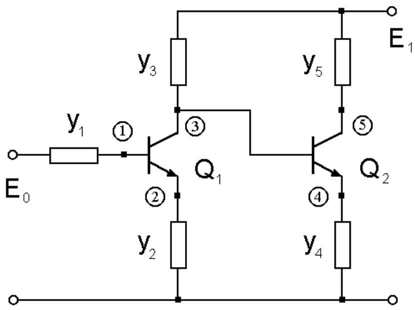

Fig. 11. Two-stage transistor amplifier.

The vector *X* includes ten components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, $x_6 = V_1$, $x_7 = V_2$, $x_8 = V_3$, $x_9 = V_4$, $x_{10} = V_5$. The model of this network (2) includes five equations (M=5) and the optimization procedure (6) includes ten equations. The total structural basis contains 32 different design strategies. The control vector includes five control functions: $U = (u_1, u_2, u_3, u_4, u_5)$. Fig. 12 shows the behavior of the functions *V(t)* and *W(t)* for some design strategies with different switch points including the optimal one.

The data, which correspond to these graphs, are presented in Table 5. The integration of the system (6) was realized by the optimal variable integration step.

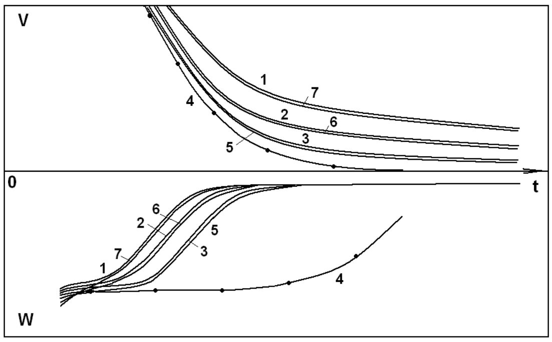

Fig. 12. Behavior of the functions *V(t)* and *W(t)* during the design process for seven different switch points (from 7 to 13) for network in Fig. 11.

Table 5. Iterations number and computer time for strategies with different switch points for network in Fig. 11.

N	Switch point 1	Switch point 2	Iterations number	Total design time (sec)
1	7	8	4900	9.912
2	8	9	4486	9.113
3	9	10	3785	7.691
4	10	11	1354	2.742
5	11	12	3618	7.341
6	12	13	4424	8.981
7	13	14	4882	9.893

As for previous example, the design of two-transistor cell amplifier has been proposed as a combination of MTDS and TDS. In this case the quasi-optimal control vector includes two switch points. We changed the control vector from (11111) to (00000) and from (00000) to (11111). The consecutive change of the switch point was realized for the integration step's number from 2 to 20.The behavior of the functions *V(t)* and *W(t)* for the optimal switch steps and some steps near the optimal confidently detect the optimal position of the switch points.

We observe a specific behavior of the function *W(t)* near the optimal switch point's position. Before the optimal switch point the function *W(t)* graphs are "parallel". Function *W(t)* has the maximum negative value for the optimal switch points. The graphs of the function *W(t)* that correspond to the optimal switch point's position (number 4) and before the optimal position (1, 2 and 3) have not intersection. After the optimal points the graphs of the function *W(t)* intersect the graphs that correspond to the optimal switch point and before the optimal one. It means that we can detect the optimal position of the switch points during the initial design interval.

So, the structure of the optimal control vector i.e. the structure of the time optimal design strategy can be defined by means of the analysis of the relative time derivative of the Lyapunov function during the initial time interval of the design process.

The analysis of the design process for three-stage transistor amplifier in Fig. 13 is presented below. The vector X includes ten components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, $x_6^2 = y_6$, $x_7^2 = y_7$, $x_8 = V_1$, $x_9 = V_2$, $x_{10} = V_3$, $x_{11} = V_4$, $x_{12} = V_5$, $x_{13} = V_6$, $x_{14} = V_7$.

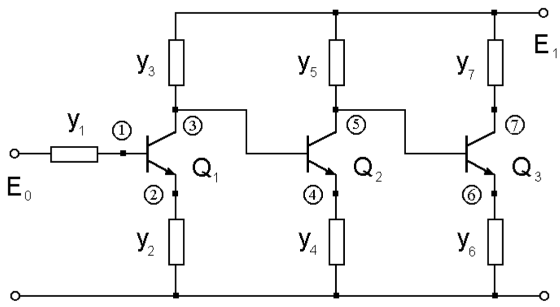

Fig. 13. Three-stage transistor amplifier.

The model of this network (2) includes seven equations (*M*=7) and the optimization procedure (6) includes 14 equations. The total structural basis contains 128 different design strategies and the control vector includes seven control functions.

As shown in previous paper [20] the quasi optimal structure of the control vector can be constructed on basis of two strategies. The first strategy can be selected from the subset of strategies that have the structure like MTDS. The other strategy can be selected from the subset of strategies that have the structure like TDS. We selected two strategies with two control vector (1111111) and (0000000) and two possible switch points. The first switch point corresponds the changing from the strategy with control vector (1111111) to strategy (0000000) and the other switch point corresponds to the opposite changing from control vector (0000000) to (1111111). The first switch point corresponds to *n*–th step of optimization procedure and the second switch point corresponds to *n*+6 step.

The searching of the optimal position of the control vector switch points was realized on basis of the analysis of the functions *V(t)* and $\dot{V}(t)$. It would be convenient to analyze the properties of these functions applying the coordinates V - $\dot{V}$. In this case the time of design, which corresponds to the current point of the optimization procedure, serves like a parameter. The behavior of the function $\dot{V}(t)$ is presented in Fig. 14 for different switch points.

Fig. 14a corresponds to the total optimization process and Fig 14b corresponds to the small rectangle presented in Fig. 14a.

Diagrams showed in these figures correspond to the strategies that have seven consecutive values of switch points beginning 10 till 16 for the first switch point and beginning 16 till 22 for the second one. Corresponding results are presented in Table 6. Strategy 1 has switch points 10 and 16, strategy 2 has switch points 11 and 17, etc.

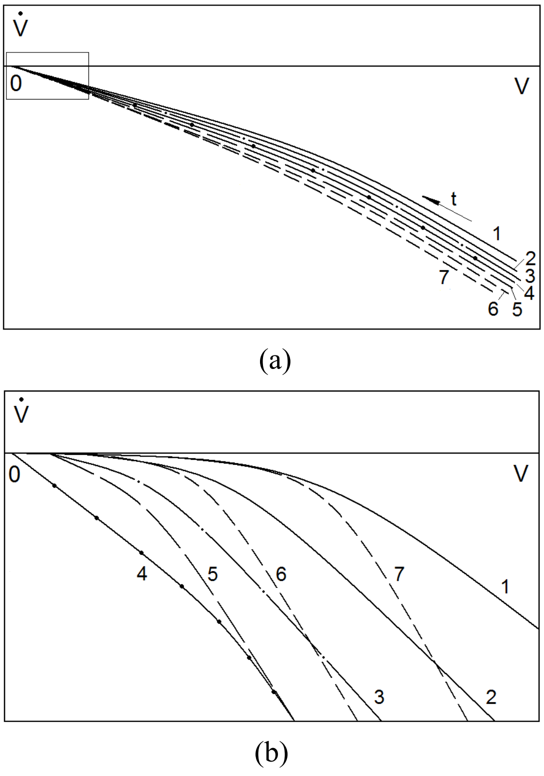

Fig. 14. Dependence of time derivative of Lyapunov function for different switch points.

Table 6. Data of some strategies with different switch points.

N	Switch point 1	Switch point 2	Iterations number	Total design time (sec)
1	10	16	8187	154.31
2	11	17	7432	140.04
3	12	18	6125	115.36
4	13	19	2087	39.14
5	14	20	10259	193.33
6	15	21	11610	218.81
7	16	22	12372	233.16

We can see that the strategy 4 with switch points 13 and 19 has a minimal computer time, so it is an optimal one. Diagram with dots corresponds to this strategy. Diagrams by solid lines correspond to strategies before the optimal switch point position and diagrams by dash lines correspond to the strategies after the optimal switch point position.

The behavior of the functions V - $\dot{V}$ for the optimal switch point position and some switch points near the optimal one can confidently detect the optimal position of the switch point. We observe a specific behavior of the derivative function near the optimal switch point position. Before the optimal switch point the graphs of the derivative function are quasi "parallel". The graphs of this function that correspond to the optimal switch point's position (number 4) and before the optimal position (1, 2 and 3) have not intersection. On the contrary, after the optimal points the graphs of this function intersect the graphs that correspond to the optimal switch point and before the optimal one.

The qualitative changing in behavior of the function $\dot{V}$ graphs before and after the optimal switch point serves as the special indicator to detect the optimal switch point position. It means that we can detect the optimal position of the switch points during the initial design interval with a confidence.

It is interesting to analyze the step of the first intersection between different graphs of the function $\dot{V}$. Corresponding data are presented in Table 7.

Table 7. Data of some strategies with different switch points.

Number of strategy	1	2	3	4	5	6
5	1352	1350	1319	412		
6	1274	1090	417	315	266	
7	826	421	302	255	229	214

The first line of the table corresponds to numbers of graphs that were intersected. The first column corresponds to numbers of graphs that intersect other graphs. The principal information is the step number that corresponds to the point of intersection. It is clear that before the optimal switch point there are no intersections. The graph 5 that is the first graph after the optimal switch point intersects the optimal graph 4 in 412-th step number, but the total step number of the optimal graph is equal 2087. It means that we can confidently identify the optimal graph, which lies below all others, after 412 steps of the optimization procedure. These steps occupy less than 20% of the total design time of the optimal design strategy. On the other hand the graph number 6 intersect the optimal graph 4 in 315-th step of the optimization procedure. Intersection between 7-th and 6-th graphs is observed in 214-th step. The presence at list one of similar intersection is an evidence of the fact that the current switch point lies after the optimal position. It is a sufficient sign of the optimal switch point identification.

The analysis of the optimization process is presented below for transistor amplifier showed in Fig. 15. In this case we can define ten independent variables $y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}$ (K=10) and eleven dependent variables $V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9, V_{10}, V_{11}$, ($M$=11) for the traditional design strategy. The optimization algorithm includes the system of 21 equations and the circuit model includes 11 nonlinear equations. The structural basis of different design strategies includes 2048 strategies and control vector includes 11 components. The quasi optimal strategy can be composed as a combination of MTDS and TDS with two switch points. The first switch point corresponds to the *n*-th step of optimization procedure and control vector changing from (11111111111) to (00000000000), and the second switch point corresponds to the n+2 step and changing from control vector (00000000000) to (11111111111).

The optimal distribution of switch points has been analyzed on the behavior of the Lyapunov function *V(t)* and its time derivative $\dot{V}(t)$.

The behavior of the derivative function is presented in Fig. 16 for different switch points. All the graphs are beginning from the point, which lies in 40-th step of the optimization procedure.

The trajectories of this figure correspond to the strategies that have the switch points from 5 to 65 with the step increment 5. The numerical results that correspond to these trajectories are shown in Table8.

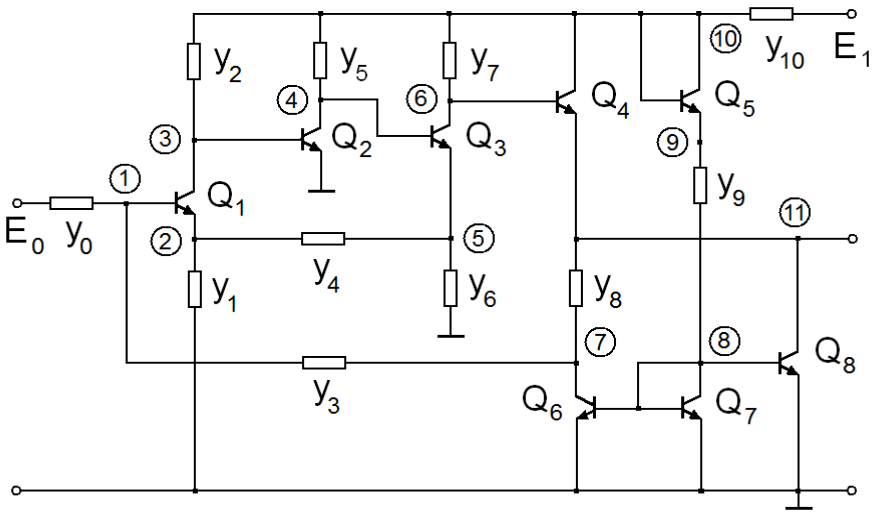

Fig. 15. Transistor amplifier.

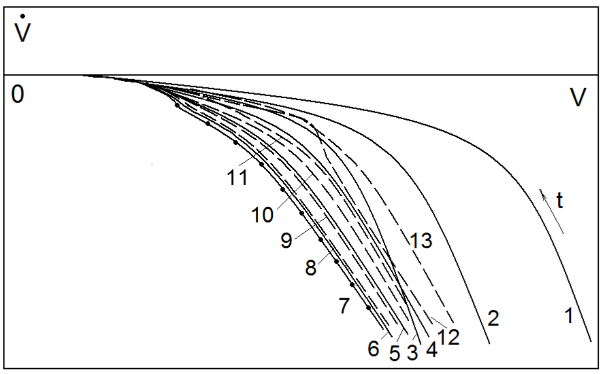

Fig. 16. Time derivative of Lyapunov function.

The strategy that corresponds to the trajectory 1 has the first switch point on 5-th step of optimization procedure and the second switch point on 7-th step. The strategy 2 has two switch points in 10 and 12 steps and so on. The analysis of data of Table I give us a conclusion that strategy 7 is an optimal one and has a minimal computer time 7.68 sec. The switch points 35 and 37 are optimal in this case. The optimal trajectory 7 with dots in graph of Fig. 16 corresponds to these switch points.

All trajectories have been drawn beginning from 40-th step after the first switch point. In this case we can identify the optimal strategy immediately by means of the function $\dot{V}(t)$ analysis.

Table 8. Strategies with different switch points.

N	Switch point 1	Switch point 2	Iterations number	Total design time (sec)
1	5	7	19360	19.424
2	10	12	12652	12.691
3	15	17	11713	11.754
4	20	22	11100	11.130
5	25	27	9391	9.424
6	30	32	8049	8.071
7	35	37	7655	7.682
8	40	42	8189	8.213
9	45	47	8930	8.959
10	50	52	10252	10.277
11	55	57	10423	10.451
12	60	62	10745	10.782
13	65	67	11047	11.078

The trajectory that corresponds on the optimal strategy lies below of all other trajectories. It means that we can identify the optimal strategy beginning 50-th step of the optimization procedure. In this case the necessary computer time for determine the optimal switch point is less than 1% of the optimal strategy computer time. So, the computer time waste needed to obtain the optimal design strategy is neglected. Summarized all obtained results we can conclude that the behavior of the time derivative of Lyapunov function of design process gives us the possibility to identify the optimal switch points of the control vector, i.e. the optimal or quasi optimal control vector structure. This analysis is preliminary but it shows a principal possibility to obtain an optimal structure of the control vector by means of study of the initial part of the design process.

5 Conclusion

The problem of the minimal-time design algorithm construction can be solved adequately on the basis of the control theory. The design process in this case is formulated as the controllable dynamic system. The Lyapunov function of the design process and its time derivative include the sufficient information to select more perspective design strategies from the infinite set of the different design strategies that exist into the generalized design methodology. The special function *W(t)* was proposed to predict the structure of the time optimal design strategy. This function can be used as the principal tool to construct an optimal structure of the control vector.

Acknowledgment

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