Generalized Thévenin/ Helmholtz and Norton/ Mayer Theorems of Electric Circuits With Variable Resistances

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Abstract: - The generalized equivalent circuits, which develop the known theorems, are formulated. It appears that the load straight line at various values of a changeable element (resistor) of an active two-pole passes into a bunch of these lines. The bunch centre coordinates do not depend on this changeable element. It is proposed to use as the parameters of the generalized equivalent generator such a load current and voltage, which proved the current across this element equal to zero. The application of projective coordinates instead of resistance values allows obtaining suitable formulas of the recalculation of the load current, to define the scales for the load and variable element.

Key-Words: - equivalent circuit, active two-pole, load straight line, projective geometry, geometric circuit theory.

1 Introduction

In the theory of the electric circuits, in case of variable parameter of elements, one of the analysis problems is the establishment of the dependence of the regime parameter changes on the respective change of the element parameter. In practice, it can be DC power supply systems with a variable load. To simplify the calculation of such networks, Thévenin/ Helmholtz and Norton/ Mayer theorems are used [1, 2- 4]. According to these theorems, the fixed part of a circuit, concerning the terminals of the dedicated load, is replaced by an equivalent circuit or equivalent generator. The open circuit voltage, internal resistance or short circuit current is the parameters of this equivalent generator. Considering importance of ideas of the equivalent generator, the attention is given to the respective theorems in education [5, 6]. Also, these theorems attract the attention of researchers [7, 8, 9].

The parameters of the equivalent generator can be used as scales for normalized values of the load parameters or regimes. Such a definition of regimes (hereinafter referred as relative regimes) allows comparing or setting the regimes of the different systems.

However, this known equivalent generator does not completely disclose the property of a circuit. For example, power supply systems with the basic (priority) load and the variable auxiliary (buffer) load or voltage regulator. In this case, the change of such an element leads to change of the open circuit voltage and short circuit current, as the parameters of the equivalent generator. Thus, the problem of calculation of this circuit and finding of the equivalent generator parameters arises again.

In a number of previous papers of the author, the generalized equivalent generator, which develops Thévenin/ Helmholtz and Norton/ Mayer theorems, is proposed. It appears that the load straight line at various values of a changeable element passes into a bunch of these lines. Since the bunch centre coordinates do not depend on this changeable element, they can be accepted as the parameters of the generalized equivalent generator [10, 11, 12]. Also, the approach based on projective geometry for interpretation of changes (kinematics) of regimes is developed [13]. It allows revealing the invariant properties of a circuit, i.e. such expressions, which turn out identical to the load and element changes. Such invariant expressions allow obtaining convenient formulas of recalculation of the load current.

The methodically simpler and reasonable statement of the basic obtained results is offered further.

2 Equivalent Generator of an Active Two-Pole with Variable Elements

Let us consider an electric circuit with a conductivity Y_{L1} of the basic load and a conductivity Y_{L2} of the auxiliary load (variable element) in Fig.1.

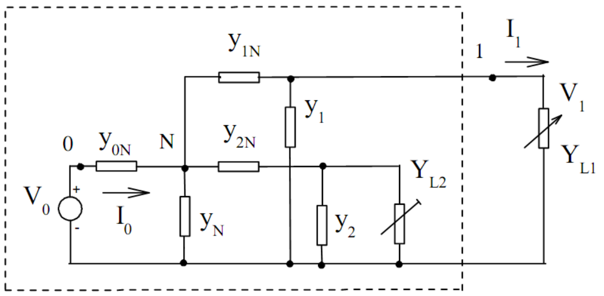

Fig.1 Active two-pole with a load Y_{L1} and a variable element Y_{L2}

This circuit can be considered as an active two-port A network relatively to the specified loads in Fig.2.

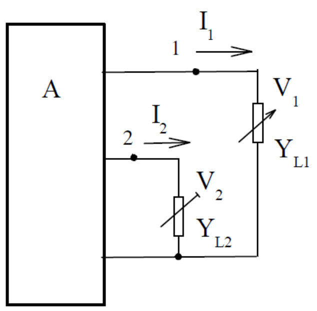

Fig.2 Active two-port A network with the specified loads

Taking into account the specified directions of currents, this network is described by the following system of the equations

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -Y_{11} & Y_{12} \\ Y_{12} & -Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} I_1^{SC,SC} \\ I_2^{SC,SC} \end{bmatrix}. \quad (1)$$

where the Y parameters are

$$Y_{11} = y_1 + y_{1N} - \frac{y_{1N}^2}{y_\Sigma}, \quad y_\Sigma = y_{0N} + y_N + y_{1N} + y_{2N},$$

$$Y_{12} = y_{2N}\frac{y_{1N}}{y_\Sigma}, \quad Y_{22} = y_2 + y_{2N} - \frac{y_{2N}^2}{y_\Sigma}.$$

The short circuit SC current of all the loads are

$$I_1^{SC,SC} = Y_{10}V_0 = y_{0N}\frac{y_{1N}}{y_\Sigma}V_0,$$

$$I_2^{SC,SC} = Y_{20}V_0 = y_{0N}\frac{y_{2N}}{y_\Sigma}V_0.$$

2.1 Disadvantages of the known equivalent generator

Taking into account the current

$$I_2 = Y_{L2}V_2 \quad (2)$$

and system of equation (1), we obtain the expression of a load straight line

$$I_1 = -\left[Y_{11} - \frac{Y_{12}^2}{Y_{L2} + Y_{22}}\right] \cdot V_1 + \left[Y_{10} + \frac{Y_{12}Y_{20}}{Y_{L2} + Y_{22}}\right] \cdot V_0 \quad . \quad (3)$$

The operating point with variable coordinate (V_1, I_1) moves on the load straight line with the parameter Y_{L2} at the expense of change of the load conductivity Y_{L1} as shown in Fig.3.

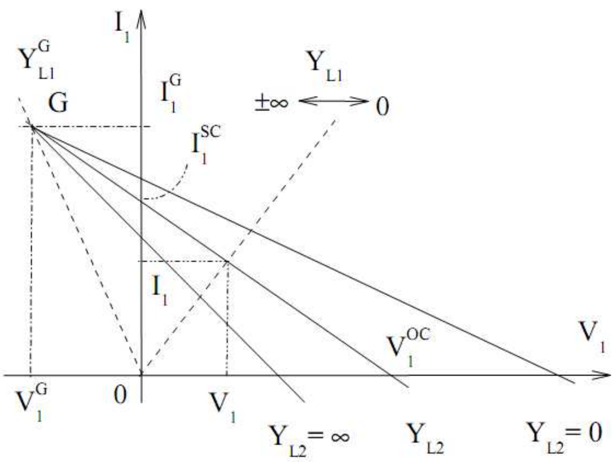

Fig.3. Family of load straight lines with the parameters Y_{L2} and load Y_{L1}

The voltage $V_1 = 0$ in the short circuit regime. In this case, the short circuit current or coordinate of the intersection point of load straight line (3) with the axis I_1 is

$$I_1^{SC} = \left[Y_{10} + \frac{Y_{12}Y_{20}}{Y_{L2} + Y_{22}}\right] \cdot V_0. \quad (4)$$

Similarly, the current $I_1 = 0$ in the open circuit regime.
Then, the open circuit voltage is

$$V_1^{OC} = \frac{I_1^{SC}}{Y_i}, \qquad (5)$$

where the value

$$Y_i = Y_{11} - \frac{Y_{12}^2}{Y_{L2} + Y_{22}} = \frac{Y_{L2}Y_{11} + \Delta_Y}{Y_{L2} + Y_{22}} \qquad (6)$$

is the internal conductivity of circuit relatively to the load Y_{L1}; the Δ_Y is the determinate of the matrix Y parameters.
Taking into account the entered parameters, equation (3) becomes as

$$I_1 = -Y_i V_1 + I_1^{SC}. \qquad (7)$$

This expression we present as

$$I_1 - I_1^{SC} = -Y_i V_1. \qquad (8)$$

Thus, we obtain the equation of the straight line passing through the point I_1^{SC}. In turn, the internal conductivity Y_i defines a slope angle of this line. So, the values I_1^{SC}, Y_i are the parameters of Norton/ Mayer equivalent generator.
By analogy to (8) and taking into account (5), we obtain the equation of the straight line passing through the point V_1^{OC}

$$I_1 = (V_1^{OC} - V_1)Y_i. \qquad (9)$$

So, the values V_1^{OC}, Y_i are the parameters of known Thévenin/ Helmholtz equivalent generator. We note that the parameters I_1^{SC}, V_1^{OC} depend on the conductivity Y_{L2} of a changeable element.

2.2 Generalized Norton/ Mayer equivalent generator

Let us study features of load straight line (3). This expression (3) represents a bunch of straight lines with the parameter Y_{L2}. To find the coordinates V_1^G, I_1^G of the bunch centre G of these lines, it is convenient to use the extreme values of parameters, i.e. $Y_{L2} = 0$, $Y_{L2} = \infty$.
These lines are shown in Fig.3. In this case, expression (3) gives the following system of equations

$$\begin{cases} I_1(0) = -\left[Y_{11} - \dfrac{Y_{12}^2}{Y_{22}}\right] \cdot V_1 + \\ \qquad + \left[Y_{10} + \dfrac{Y_{12}Y_{20}}{Y_{22}}\right] \cdot V_0 \\ I_1(\infty) = -Y_{11}V_1 + Y_{10}V_0 \quad . \end{cases} \qquad (10)$$

For the point of intersection we have that

$$I_1^G = I_1(0) = I_1(\infty).$$

The solving of system (10) gives the values of voltage

$$V_1^G = -\frac{Y_{20}}{Y_{12}} V_0 = -\frac{y_{0N}}{y_{1N}} V_0 \qquad (11)$$

and current

$$I_1^G = \left[Y_{10} + \frac{Y_{11}Y_{20}}{Y_{12}}\right] \cdot V_0 = y_{0N}\left[1 + \frac{y_1}{y_{1N}}\right] \cdot V_0. \qquad (12)$$

The obtained values V_1^G, I_1^G, and internal conductivity Y_i allow to present equations (3) or (8) in another form

$$I_1 - I_1^G = -Y_i (V_1^G + V_1). \qquad (13)$$

Thus, we obtain the equation of the straight line, passing through the point I_1^G, V_1^G. The internal conductivity Y_i defines a slope angle of this line. So, the values I_1^G, V_1^G and Y_i are the parameters of the generalized Norton/ Mayer equivalent generator in Fig.4.

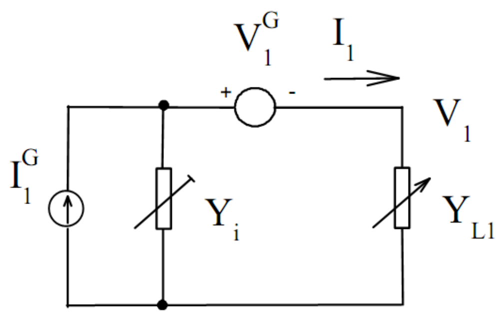

Fig.4. Generalized Norton/ Mayer equivalent generator

We note that
besides the basic energy source of one kind (a current source I_1^G) there is an additional energy source of another kind (a voltage source V_1^G) that it is possible to consider as a corresponding theorem.

It is natural, when the value $V_1^G = 0$, we obtain the known Norton/ Mayer equivalent generator. In this case $I_1^G = I_1^{SC}$.

Let us note that physically the centre G of the bunch corresponds to such a voltage V_1^G and current I_1^G of the load $Y_{L1} = Y_{L1}^G$, when the current of the element Y_{L2} is equal to zero. According to this condition, from (1) it is also possible to find values (11), (12) of the parameters V_1^G, I_1^G. Then, the corresponding load conductivity

$$Y_{L1}^G = \frac{I_1^G}{V_1^G} = -\frac{Y_{11}Y_{20} + Y_{10}Y_{21}}{Y_{20}} = -(y_{1N} + y_1). \quad (14)$$

The load is a power source because of a negative value of the conductivity.

2.3 Generalized Thévenin/ Helmholtz equivalent generator

In the above case, the centre G of the bunch is in the second quadrant of coordinate system and so $V_1^G < 0, I_1^G > 0$. It is natural to consider a case that the centre G is in the fourth quadrant as shown in Fig.5.

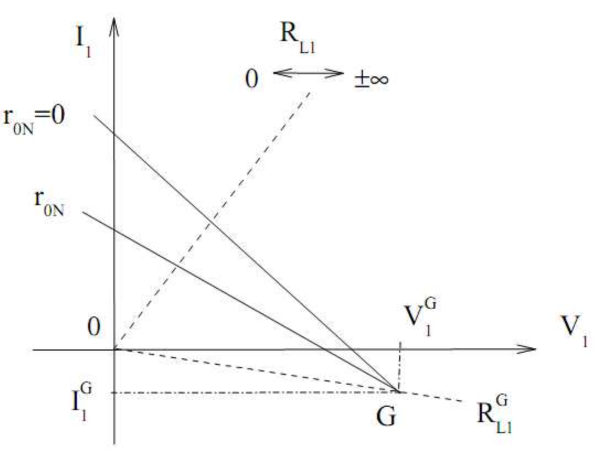

Fig.5. Centre of a bunch is in the fourth quadrant

To do this, we consider an electric circuit with the basic load R_{L1} and variable element (voltage regulator) r_{0N} in Fig.6.

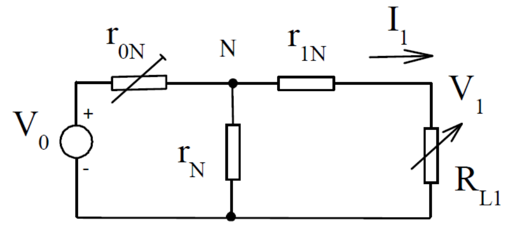

Fig.6. Circuit with a variable element r_{0N}

Similarly, there is a bunch of the straight lines with the parameter r_{0N}. Let us define the centre of this bunch. At once it is visible that the current across the resistance r_{0N} will be equal to zero if the voltage $V_N = V_0$. Then, the load voltage and current are

$$V_1^G = \frac{r_{1N} + r_N}{r_N} V_0 > V_0 , \quad (15)$$

$$I_1^G = -\frac{V_0}{r_N}. \quad (16)$$

The load resistance

$$R_L^G = \frac{V_1^G}{I_1^G} = -(r_{1N} + r_N). \quad (17)$$

In turn, the internal resistance

$$R_i = r_{1N} + \frac{r_{0N} \cdot r_N}{r_{0N} + r_N}. \quad (18)$$

Thus, the equation of the straight line passing through the point I_1^G, V_1^G has the form

$$I_1 + I_1^G = -\frac{V_1 - V_1^G}{R_i}. \quad (19)$$

So, the values I_1^G, V_1^G and R_i are the parameters of the generalized Thévenin/ Helmholtz equivalent generator in Fig.7.

We note that
besides the basic energy source of one kind (a voltage source V_1^G) there is an additional energy source of another kind (a current source I_1^G) that it is possible to consider as the corresponding theorem.

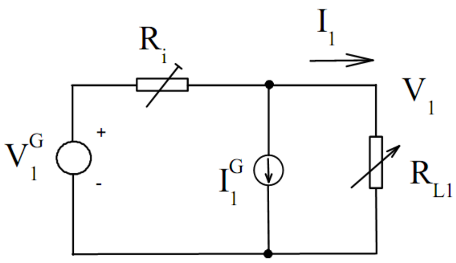

Fig.7 Generalized Thévenin/ Helmholtz equivalent generator

It is natural when the value $I_1^G = 0$, we obtain the known Thévenin/ Helmholtz equivalent generator. In this case $V_1^G = V_1^{OC}$.

3 Analysis of Operating Regimes of the Generalized Equivalent Generator

The centre G position (the second or fourth quadrant) is defined by the kind of an active two-pole as an energy source. If the active two-pole shows more properties of a current source, the case of Fig. 3 takes place. If it shows more properties of a voltage source, we have the case of Fig. 7.

Let us demonstrate how the internal conductivity Y_i and respectively conductivity Y_{L2} influence on the kind or type of the generalized equivalent generator in Fig.4. The corresponding family of the load straight lines is shown in Fig.8.

We use further the inverse expression to (6)

$$Y_{L2} = \frac{Y_{22}Y_i - \Delta_Y}{Y_{11} - Y_i}. \qquad (20)$$

The conductivities have the following characteristic value:

$$\text{a)} \quad Y_i = 0,\; Y_{L2}^I = -\frac{\Delta_Y}{Y_{11}}, \qquad (21)$$

which defines the generalized equivalent generator as an ideal current source;

$$\text{b)} \quad Y_i = \infty,\; Y_{L2}^V = -Y_{22}, \qquad (22)$$

which defines the generalized equivalent generator as an ideal voltage source;

$$\text{c)} \quad Y_i^0 = -Y_{L1}^G = \frac{Y_{11}Y_{20} + Y_{10}Y_{21}}{Y_{20}},$$

$$Y_{L2}^0 = -Y_{22} + \frac{Y_{20}Y_{12}}{Y_{10}} = -(y_{2N} + y_2), \qquad (23)$$

which corresponds to the beam $G0$ and defines a "zero- order" source, when the current and voltage of the load are always equal to zero for all its values.

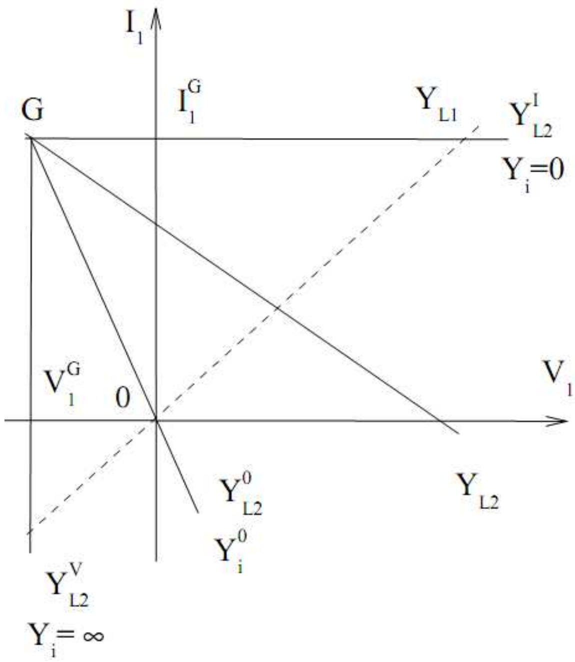

Fig. 8 Family of the load straight lines with characteristic values of Y_i and Y_{L2}

The Fig.9 presents this case of generalized equivalent generator that demonstrates the "zero-order" generator. The load voltage $V_1 = 0$ because $V_i^0 = -V_1^G$.

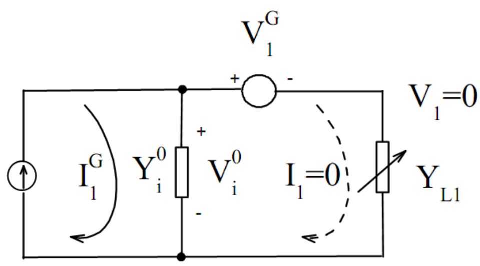

Fig.9 "Zero- order" generator

So, the variable element can have these three specified characteristic values. These characteristic values are defined at a qualitative level. This brings up the problem of determination in the relative or normalized form of conductance value Y_{L2} regarding of these characteristic values. In this case, it is possibly to define the kind of an active two-pole as an energy source. Therefore, as though obvious values $Y_{L2}=0$, $Y_{L2}=\infty$ are not characteristic ones concerning load.

Let us view possible load characteristic values. The traditional values, as $Y_{L1}=0$, $Y_{L1}=\infty$ and too Y_{L1}^{G} will be characteristic values according to Fig.8. The physical sense of these values is clear.

Let us show how the internal resistance R_i and respectively the changeable element r_{0N} influence on the type of the generalized equivalent generator in Fig.7. The corresponding family of the load straight lines is shown in Fig.10.
We use further the inverse expression to (18)

$$r_{0N}=\frac{r_N\cdot(R_i-r_{1N})}{r_N-(R_i-r_{1N})}. \qquad (24)$$

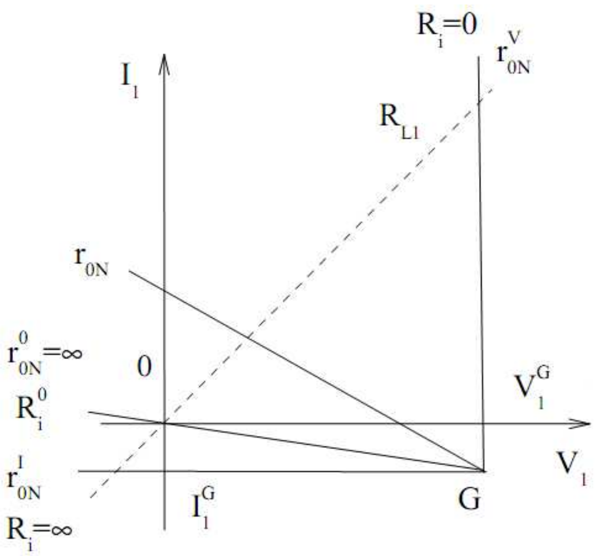

Fig.10 Family of the load straight lines with characteristic values R_i and r_{0N}

The resistances have the following characteristic value:

$$\text{a)} \qquad R_i=0,\ r_{0N}^V=-\frac{r_N\cdot r_{1N}}{r_N+r_{1N}}, \qquad (25)$$

which defines the generalized equivalent generator as an ideal voltage source;

$$\text{b)} \qquad R_i=\infty,\ r_{0N}^I=-r_N, \qquad (26)$$

which defines the generalized equivalent generator as an ideal current source;

$$\text{c)} \qquad R_i^0=-R_{L1}^G=r_{1N}+r_N,\ r_{0N}^0=\infty, \qquad (27)$$

which corresponds to the beam $G0$ and defines the "zero- order" source. The Fig.11 presents the generalized equivalent generator that demonstrates the "zero- order" generator. The load voltage $V_1=0$ because $V_i^0=-V_1^G$.

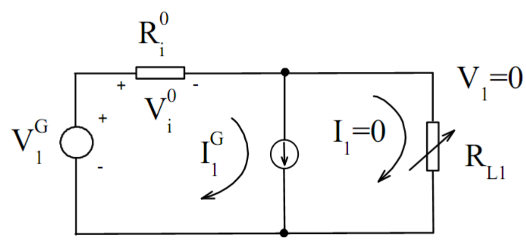

Fig.11. "Zero- order" generator

So, the variable element and load can have these three specified characteristic values.

4 Invariant Characteristics of Operating Regime Changes

Let us consider a common change of the load and variable element of a circuit in Fig.1. The corresponding load straight lines are shown in Fig.12.

Let the initial value of variable element be Y_{L2}^1 and subsequent value be Y_{L2}^2. Similarly, the initial value of load equal Y_{L1}^1 and subsequent one is Y_{L1}^2. Let us consider the straight line of the initial load Y_{L1}^1. The three straight lines (with characteristic values $Y_{L2}^{I}, Y_{L2}^{0}, Y_{L2}^{V}$ of element Y_{L2} as a parameter of these lines) and the two lines with parameters Y_{L2}^{1}, Y_{L2}^{1} intersect this line Y_{L1}^{1}. The points $A_1, 0, B_1, C_1, F_1$ are points of this intersection. In turn, the point $A_2, 0, B_2, C_2, F_2$ are points of intersection of the line Y_{L1}^{2}.

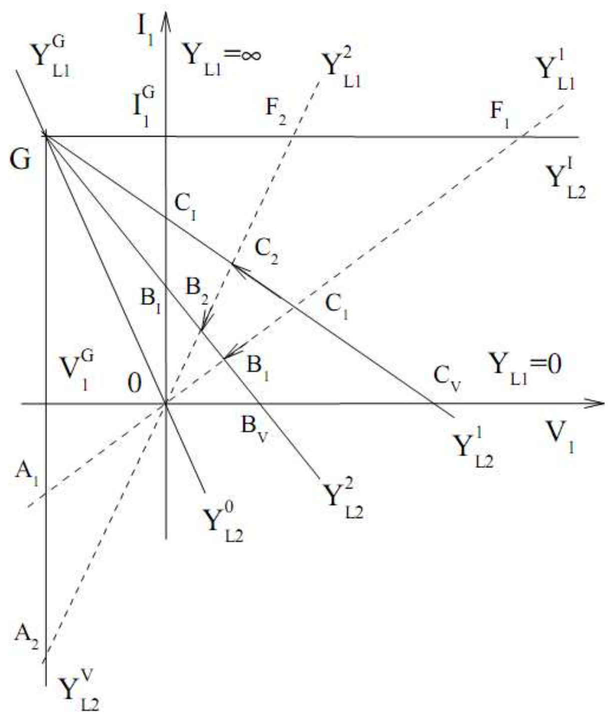

Fig.12 Common change of the load and variable element

Therefore, a projective map (conformity) of points of one line Y_{L1}^{1} into points of other line Y_{L1}^{2} takes place. This conformity is set by a projection centre G.

Similarly, we consider the straight line Y_{L2}^{1}. The three straight lines (with the characteristic values of load Y_{L1} and two lines with parameters Y_{L1}^{1}, Y_{L1}^{2}) intersect this line Y_{L2}^{1}. The points C_V, C_1, C_2, C_I, G are points of this intersection. In turn, the point B_V, B_1, B_2, B_I, G are points of intersection of the line Y_{L2}^{2}. Therefore, the projective map (conformity) of points of one line Y_{L2}^{1} into points of other line Y_{L2}^{2} takes place. This conformity is set by the projection centre 0. The presented conformities of points of straight lines present transformations of projective geometry.

As shown in a number of papers, it is convenient to use projective geometry for the analysis of circuits with variable elements [14,15,16]. The projective transformation is also set by three pairs of respective points. As pairs of these points, it is convenient to use the points corresponding to the characteristic values of a load and variable element. The projective transformations preserve a cross ratio of four points, or otherwise, the cross ratio is an invariant of the projective transformations. Further, we will show application of such invariants.

4.1 Definition of relative operating regime at load change

The cross ratio m_{L1}^{1} of four points (three of these are the characteristic ones C_V, C_I, G of the line Y_{L2}^{1}, and the fourth C_1 is a point of the initial regime Y_{L1}^{1}) has the view

$$m_{L1}^{1} = (C_V \; C_1 \; C_I \; G) = \\ = \frac{C_1 - C_V}{C_1 - G} \div \frac{C_I - C_V}{C_I - G} \quad . \tag{28}$$

The same value of the cross ratio will be for the points B_V, B_1, B_I, G of the line Y_{L2}^{2}

$$m_{L1}^{1} = (B_V \; B_1 \; B_I \; G) = \\ = \frac{B_1 - B_V}{B_1 - G} \div \frac{B_I - B_V}{B_I - G} \quad . \tag{29}$$

For finding the value of this cross ratio, it is necessary to define coordinates of these points. To do this, we map the line Y_{L2}^{1} into the axis of current. There is also the projective transformation, which is set by an infinitely remote centre. Then, the cross ratio is expressed by the components of current

$$m_{L1}^{1} = (0 \; I_1^{C1} \; I_1^{CI} \; I_1^{G}) = \\ = \frac{I_1^{C1} - 0}{I_1^{C1} - I_1^{G}} \div \frac{I_1^{CI} - 0}{I_1^{CI} - I_1^{G}} \quad . \tag{30}$$

The corresponding values of this cross ratio are shown in Fig.13. The cross ratio in geometry underlies the definition of a “distance” between the points C_1, C_I concerning the extreme or base points C_V, G. In turn, the point C_I is a scale or unit point.

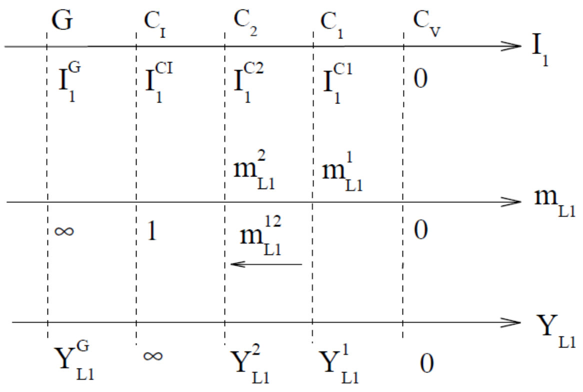

Fig.13 Mutual corresponds of the load current, conductivity, and cross ratio

Similarly, the cross ratio of the subsequent regime points

$$m_{L1}^{2}=(0\ I_1^{C2}\ I_1^{CI}\ I_1^{G})=\\=\frac{I_1^{C2}-0}{I_1^{C2}-I_1^{G}}\div\frac{I_1^{CI}-0}{I_1^{CI}-I_1^{G}}\quad. \tag{31}$$

The view or structure of expressions (30) and (31) shows that it is possible to use the division of these cross ratios, i.e.

$$m_{L1}^{12}=m_{L1}^{1}\div m_{L1}^{2}=\frac{I_1^{C1}-0}{I_1^{C1}-I_1^{G}}\div\frac{I_1^{C2}-0}{I_1^{C2}-I_1^{G}}=\\=(0\ I_1^{C1}\ I_1^{C2}\ I_1^{G})=\frac{\frac{I_1^{C1}}{I_1^{G}}-0}{\frac{I_1^{C1}}{I_1^{G}}-1}\div\frac{\frac{I_1^{C2}}{I_1^{G}}-0}{\frac{I_1^{C2}}{I_1^{G}}-1}. \tag{32}$$

This cross ratio is the "distance" between the points of the initial and subsequent regimes of the line Y_{L2}^{1}. The same "distance" will be between the points of the initial and subsequent regimes of the line Y_{L2}^{2}

$$m_{L1}^{12}=(0\ I_1^{B1}\ I_1^{B2}\ I_1^{G})\,. \tag{33}$$

Physically, it is evidently because the initial and subsequent regime is set by the change of the common load $Y_{L1}^{1}\rightarrow Y_{L1}^{2}$. The equality of cross ratios (28), (29) is also explained.

Then, it is possible to express cross ratio (30) or (32) by load conductivities. Let us present equation (13) of the generalized equivalent generator by the following view

$$I_1=(I_1^{G}-Y_i V_1^{G})\frac{Y_{L1}}{Y_{L1}+Y_i}\,. \tag{34}$$

This fractionally linear expression is a projective transformation, which maps the values of the load conductivity into the values of the load current. The corresponding points of these values are shown in Fig.13. Therefore, it is possible, by formalized method, to express, at once, cross ratio (30) by the load conductivities

$$m_{L1}^{1}=(0\ I_1^{C1}\ I_1^{CI}\ I_1^{G})=(0\ Y_{L1}^{1}\ \infty\ Y_{L1}^{G})=\\=\frac{Y_{L1}^{1}-0}{Y_{L1}^{1}-Y_{L1}^{G}}\div\frac{\infty-0}{\infty-Y_{L1}^{G}}=\frac{Y_{L1}^{1}-0}{Y_{L1}^{1}-Y_{L1}^{G}}\quad. \tag{35}$$

So, we consider cross ratio (35) as the projective coordinate of the initial or running regime points I_1^{C1}, Y_{L1}^{1}. This coordinate is expressed by invariant (identical) manner by various regime parameters.

Too, it is possible, by formalized method, to express cross ratio (32) by load conductivities

$$m_{L1}^{12}=(0\ I_1^{C1}\ I_1^{C2}\ I_1^{G})=(0\ Y_{L1}^{1}\ Y_{L1}^{2}\ Y_{L1}^{G})=\\=\frac{Y_{L1}^{1}-0}{Y_{L1}^{1}-Y_{L1}^{G}}\div\frac{Y_{L1}^{2}-0}{Y_{L1}^{2}-Y_{L1}^{G}}=\frac{\frac{Y_{L1}^{1}}{Y_{L1}^{G}}-0}{\frac{Y_{L1}^{1}}{Y_{L1}^{G}}-1}\div\frac{\frac{Y_{L1}^{2}}{Y_{L1}^{G}}-0}{\frac{Y_{L1}^{2}}{Y_{L1}^{G}}-1}. \tag{36}$$

So, we consider the cross ratio of type (32), (36) as the change of running regime. This change is expressed by invariant manner through various regime parameters. Therefore,
***usually used regime changes by increments (as formal)* are eliminated.**

In turn, the values I_1^{G}, Y_{L1}^{G} are the scales for normalizing of values of current and conductivity. Then, expressions (32), (36) present the relative regimes. It permits to compare or set the regime of the different circuits with various parameters.

Let us note the properties of a cross ratio. If to interchange the components I_1^{C1}, I_1^{C2} of expression (32), then we get

$$m_{L1}^{12} = 1/m_{L1}^{21}. \quad (37)$$

Also, the group property takes place

$$m_{L1}^{3} = m_{L1}^{32} \cdot m_{L1}^{2} = m_{L1}^{32} \cdot m_{L1}^{21} \cdot m_{L1}^{1} = m_{L1}^{31} \cdot m_{L1}^{1}. \quad (38)$$

Let us obtain the subsequent value of current from expression (32); we have

$$I_1^{C2} = \frac{I_1^G I_1^{C1}}{(1-m_{L1}^{12})I_1^{C1} + m_{L1}^{12} I_1^G}. \quad (39)$$

The obtained transformation with the parameter m_{L1}^{12} allows realizing the direct recalculation of the current at load change. This expression is especially convenient in case of group or set of load changes on account of performance of group property (38).

4.2 Definition of relative operating regime at change of element Y_{L2}.

The cross ratio m_{L2}^1 of the four point (three of these are the characteristic ones $0, A_1, F_1$ of line Y_{L1}^1, and the fourth point C_1 of initial regime Y_{L2}^1) has the view

$$m_{L2}^1 = (0\ C_1\ A_1\ F_1) = \frac{C_1 - 0}{C_1 - F_1} \div \frac{A_1 - 0}{A_1 - F_1}. \quad (40)$$

The points $0, F_1$ are chosen as base ones. That will be explained later.

The same value of the cross ratio will be for the points $0, C_2, A_2, F_2$ of the line Y_{L1}^2

$$m_{L2}^1 = (0\ C_2\ A_2\ F_2) = \frac{C_2 - 0}{C_2 - F_2} \div \frac{A_2 - 0}{A_2 - F_2}. \quad (41)$$

Cross ratio (40) is expressed by the components of current

$$m_{L2}^1 = (0\ I_1^{C1}\ I_1^{A1}\ I_1^G) = \frac{I_1^{C1} - 0}{I_1^{C1} - I_1^G} \div \frac{I_1^{A1} - 0}{I_1^{A1} - I_1^G}. \quad (42)$$

So, the cross ratio is the "distance" between the points C_1, A_1 concerning the base points. In turn, the point C_I is a unit point.

Similarly, the cross ratio of the points of subsequent regime

$$m_{L2}^2 = (0\ I_1^{B1}\ I_1^{A1}\ I_1^G) = \frac{I_1^{B1} - 0}{I_1^{B1} - I_1^G} \div \frac{I_1^{A1} - 0}{I_1^{A1} - I_1^G}. \quad (43)$$

The "distance" between the points of initial and subsequent regimes of line Y_{L1}^1 has the view

$$\begin{aligned} m_{L2}^{12} &= m_{L2}^1 \div m_{L2}^2 = \\ &= \frac{I_1^{C1} - 0}{I_1^{C1} - I_1^G} \div \frac{I_1^{B1} - 0}{I_1^{B1} - I_1^G} = (0\ I_1^{C1}\ I_1^{B1}\ I_1^G). \end{aligned} \quad (44)$$

The same "distance" is between the points of initial and subsequent regimes of line Y_{L1}^2

$$\begin{aligned} m_{L2}^{12} &= (0\ I_1^{C2}\ I_1^{B2}\ I_1^G) = \\ &= \frac{I_1^{C2} - 0}{I_1^{C2} - I_1^G} \div \frac{I_1^{B2} - 0}{I_1^{B2} - I_1^G}. \end{aligned} \quad (45)$$

Let us express cross ratio (42) or (44) by the load conductivities Y_{L2}. Expression (34) $I_1(Y_i) = I_1(Y_{L2})$ for the given load is the projective transformation, which maps the points Y_i, Y_{L2} into the points of current. Therefore, it is possible, by formalized method, to express, at once, cross ratio (42), 944) by the conductivities Y_{L2}, Y_i

$$\begin{aligned} m_{L2}^1 &= (0\ I_1^{C1}\ I_1^{A1}\ I_1^G) = (Y_i^0\ Y_i^1\ \infty\ 0) = \\ &= (Y_{L2}^0\ Y_{L2}^1\ Y_{L2}^V\ Y_{L2}^I) = \\ &= \frac{Y_{L2}^1 - Y_{L2}^0}{Y_{L2}^1 - Y_{L2}^I} \div \frac{Y_{L2}^V - Y_{L2}^0}{Y_{L2}^V - Y_{L2}^I} = \\ &= \frac{Y_i^1 - Y_i^0}{Y_i^1 - 0} \div \frac{\infty - Y_i^0}{\infty - Y_{L2}^I} = \frac{Y_i^1 - Y_i^0}{Y_i^1}. \end{aligned} \quad (46)$$

$$\begin{aligned} m_{L2}^{12} &= (0\ I_1^{C1}\ I_1^{B1}\ I_1^G) = (Y_i^0\ Y_i^1\ Y_i^2\ 0) = \\ &= (Y_{L2}^0\ Y_{L2}^1\ Y_{L2}^2\ Y_{L2}^I) = \\ &= \frac{Y_{L2}^1 - Y_{L2}^0}{Y_{L2}^1 - Y_{L2}^I} \div \frac{Y_{L2}^2 - Y_{L2}^0}{Y_{L2}^2 - Y_{L2}^I} = \\ &= \frac{Y_i^1 - Y_i^0}{Y_i^1 - 0} \div \frac{Y_i^2 - Y_i^0}{Y_i^2 - Y_{L2}^I}. \end{aligned} \quad (47)$$

Also, the group property takes place

$$m_{L2}^{3}=m_{L2}^{32}\cdot m_{L2}^{2}= \\ =m_{L2}^{32}\cdot m_{L2}^{21}\cdot m_{L2}^{1}=m_{L2}^{31}\cdot m_{L2}^{1}. \quad (48)$$

Let us obtain the subsequent value of current from expression (44). Then

$$I_1^{B1}=\frac{I_1^G I_1^{C1}}{(1-m_{L2}^{12})I_1^{C1}+m_{L2}^{12}I_1^G}. \quad (49)$$

The obtained transformation with the parameter m_{L2}^{12} allows realizing the direct recalculation of current.

4.3 Definition of relative operating regime at common change of load Y_{L1} and element Y_{L2}

Let the common or composite change of regime be given as $C_1\to C_2\to B_2$. Then, the view of expressions (45) and (32) shows that it is possible to use the multiplication of these cross ratios as a compound change of regime

$$m^{12}=m_{L2}^{12}\cdot m_{L1}^{12}= \\ =\frac{I_1^{C1}-0}{I_1^{C1}-I_1^G}\div\frac{I_1^{B2}-0}{I_1^{B2}-I_1^G}=(0\ \ I_1^{C1}\ I_1^{B2}\ I_1^G). \quad (50)$$

In this resultant expression the intermediate components are reduced at the expense of the choice of identical basic points. Therefore, we obtain the resultant value of current for the point B_2

$$I_1^{B2}=\frac{I_1^G I_1^{C1}}{(1-m^{12})I_1^{C1}+m^{12}I_1^G}. \quad (51)$$

5 Example

Let the elements of a circuit in Fig.1 be given as follows

$V_0=5$, $y_{0N}=1.25$, $y_N=0.5$, $y_{1N}=1.25$, $y_{2N}=5$, $y_1=0.25$, $y_2=0.1$.

The value dimensions are not indicated.

The system of equation (1)

$$\begin{bmatrix}I_1\\I_2\end{bmatrix}=\begin{bmatrix}-1.3046 & 0.7812\\ 0.7812 & -1.975\end{bmatrix}\cdot\begin{bmatrix}V_1\\V_2\end{bmatrix}+\begin{bmatrix}0.9765\\3.9062\end{bmatrix}.$$

Let us consider the conductivity $Y_{L2}^1=0.5$.

The parameters of the initial regime, point C_1,

$$Y_{L1}^1=0.5,\quad I_1^{C1}=0.7091.$$

The parameters of the subsequent regime, point C_2,

$$Y_{L1}^2=1,\quad I_1^{C2}=1.074.$$

Parameters (4), (5) of the known equivalent generator, points C_I, C_V,

$$I_1^{CI}=2.2095,\ V_1^{CV}=2.088.$$

Internal conductivity (6) of the circuit

$$Y_i=\frac{1.3046\cdot Y_{L2}+1.9664}{Y_{L2}+1.975}=1.058. \quad (52)$$

Parameters (11), (12) of the Norton/ Mayer equivalent generator, point G,

$$V_1^G=-5,\ I_1^G=7.5.$$

Equation (13) of Norton/Mayer equivalent generator

$$I_1-7.5=-Y_i(5+V_1).$$

Value (14) of the load conductivity, point G,

$$Y_{L1}^G=-1.5.$$

Let us now consider the conductivity $Y_{L2}^2=2.5$.

The parameters of the initial regime, point B_1,

$$Y_{L1}^1=0.5,\quad I_1^{B1}=0.4971.$$

The parameters of the subsequent regime, point B_2,

$$Y_{L1}^2=1,\quad I_1^{B2}=0.7649.$$

The parameters of the points B_I, B_V

$$I_1^{BI}=1.658,\ V_1^{BV}=1.42.$$

Internal conductivity (6) of the circuit

$$Y_i=1.1682.$$

Next, we are finding the characteristic values of internal conductivity Y_i and variable element Y_{L2}. Expression (20)

$$Y_{L2} = \frac{1.975 \cdot Y_i - 1.9664}{1.3046 - Y_i}.$$

Value (21) of the ideal current source, $Y_i = 0$,

$$Y_{L2}^{I} = -1.5071.$$

Value (22) of the ideal voltage source, $Y_i = \infty$,

$$Y_{L2}^{V} = -1.975.$$

Values (23) of the “zero-order” source

$$Y_i^0 = 1.5,\ Y_{L2}^0 = -5.1.$$

5.1 Operating regime at load change.

Cross ratio (30) of the initial regime, point C_1,

$$m_{L1}^1 = (0\ \ I_1^{C1}\ I_1^{CI}\ I_1^G\ ) = \\ = \frac{0.7091 - 0}{0.7091 - 7.5} \div \frac{2.209 - 0}{2.209 - 7.5} = 0.25.$$

Let us check the cross ratio value of the point B_1

$$m_{L1}^1 = (0\ \ I_1^{B1}\ I_1^{BI}\ I_1^G\ ) = \\ = \frac{0.4971 - 0}{0.4971 - 7.5} \div \frac{1.658 - 0}{1.658 - 7.5} = 0.25.$$

We are checking cross ratio value (35) by the load conductivity

$$m_{L1}^1 == \frac{Y_{L1}^1 - 0}{Y_{L1}^1 - Y_{L1}^G} = \frac{0.5 - 0}{0.5 + 1.5} = 0.25.$$

Cross ratio (31) of the subsequent regime, point C_2,

$$m_{L1}^2 = (0\ \ I_1^{C2}\ I_1^{CI}\ I_1^G\ ) = \\ = \frac{1.074 - 0}{1.074 - 7.5} \div \frac{2.209 - 0}{2.209 - 7.5} = 0.4.$$

Let us check this value by the load conductivity

$$m_{L1}^2 == \frac{Y_{L1}^2 - 0}{Y_{L1}^2 - Y_{L1}^G} = \frac{1 - 0}{1 + 1.5} = 0.4.$$

Regime change (32) and (36)

$$m_{L1}^{12} = m_{L1}^1 \div m_{L1}^2 = 0.25 \div 0.4 = 0.625.$$

The corresponded points of values are shown in Fig.14.

The subsequent value of current (39)

$$I_1^{C2} = \frac{7.5 \cdot 0.7091}{(1 - 0.625) \cdot 0.7091 + 0.625 \cdot 7.5} = 1.074.$$

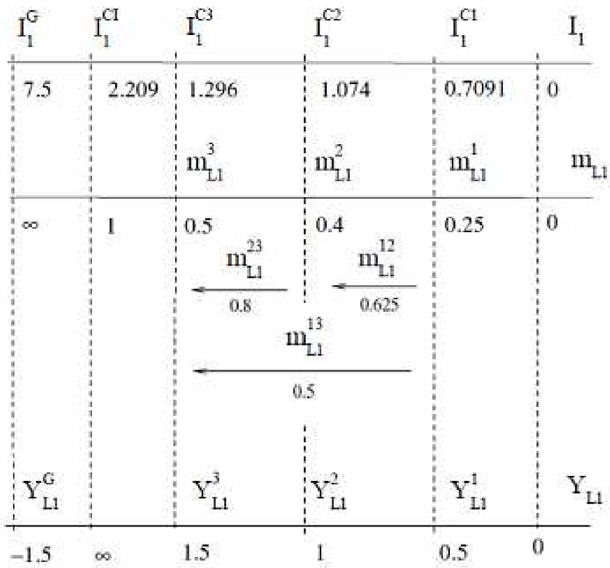

Fig.14 Example of mutual corresponds of regime parameters

Let the load once again be changed, $Y_{L1}^3 = 1.5$. Then, regime change (36) or (38) in regard to the load $Y_{L1}^2 = 1$

$$m_{L1}^{23} = m_{L1}^2 \div m_{L1}^3 = 0.4 \div 0.5 = 0.8.$$

Corresponding current value (39)

$$I_1^{C3} = \frac{7.5 \cdot 1.074}{(1 - 0.8) \cdot 1.074 + 0.8 \cdot 7.5} = 1.296.$$

The common regime change relatively to the load $Y_{L1}^1 = 0.5$

$$m_{L1}^{13} = m_{L1}^1 \div m_{L1}^3 = 0.25 \div 0.5 = 0.5.$$

We have obtained the same value (39) of the current

$$I_1^{C3} = \frac{7.5 \cdot 0.7091}{(1 - 0.5) \cdot 0.7091 + 0.5 \cdot 7.5} = 1.296.$$

5.2 Operating regime at change of element Y_{L2}

"Distance" (44) between the initial point C_1 and the subsequent point B_1

$$m_{L2}^{12} = \frac{I_1^{C1} - 0}{I_1^{C1} - I_1^G} \div \frac{I_1^{B1} - 0}{I_1^{B1} - I_1^G} =$$

$$= \frac{0.7091 - 0}{0.7091 - 7.5} \div \frac{0.4971 - 0}{0.4971 - 7.5} = 1.471$$

Let us check the same value (45) of points C_2, B_2

$$m_{L2}^{12} = \frac{1.074 - 0}{1.074 - 7.5} \div \frac{0.7649 - 0}{0.7649 - 7.5} =$$

$$= 0.16713 \div 0.11356 = 1.471 \quad ,$$

and the same value (47) of conductivities Y_{L2}^1, Y_{L2}^2

$$m_{L2}^{12} = \frac{Y_{L2}^1 - Y_{L2}^0}{Y_{L2}^1 - Y_{L2}^I} \div \frac{Y_{L2}^2 - Y_{L2}^0}{Y_{L2}^2 - Y_{L2}^I} =$$

$$= \frac{0.5 + 5.1}{0.5 + 1.5071} \div \frac{2.5 + 5.1}{2.5 + 1.5071} = 1.471 .$$

Subsequence current value (49)

$$I_1^{B1} = \frac{7.5 \cdot 0.7091}{(1 - 1.471) \cdot 0.7091 + 1.471 \cdot 7.5} = 0.4971 .$$

5.3 Common change of load Y_{L1} and Y_{L2}

Common regime change (50)

$$m^{12} = 1.471 \cdot 0.625 = 0.1044 \div 0.1135 = 0.919 .$$

The resultant current value, point B_2

$$I_1^{B2} = \frac{7.5 \cdot 0.7091}{(1 - 0.9196) \cdot 0.7091 + 0.919 \cdot 7.5} = 0.764 .$$

6 Development of the Obtained Results

The presented results are generalized for active two-port [17] and multiport [18]. The application of projective coordinates gives capability to obtain formulas of recalculation of load currents of active multiport for various cases [19, 20, 21].

7 Conclusion

-The generalized equivalent generator of active two-pole with the variable parameters simplifies of circuit analysis, gives more profound idea about the interrelation of operating regimes and parameters of elements, can be useful in the education purposes.

- The application of the projective coordinates instead of conductivities or resistance allows obtaining the suitable formulas of the recalculation of load current, to define the scales for load and variable element.

-The obtained results can be applied in particular to AC linear circuits, to circuits with dependent sources of voltage and current.

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