Magnitude Approximation of IIR Digital Filter using Greedy Search Method

RANJIT KAUR[1], DAMANPREET SINGH[2]
Department of Electronics & Communication[1], Department of Computer Science & Enginnering[2]
Punjabi University, Patiala[1]
Sant Longowal Institute of Engineering & Technology, Longowal[2]
INDIA
ranjit24_ucoe@pbi.ac.in[1], damanpreetsingh@sliet.ac.in[2]

Abstract: - The paper presents a greedy search method based on binary successive approximation-evolutionary search (BSA-ES) strategy to design stable infinite impulse response (IIR) digital filter using L_1 optimality criterion. The stability constraints are well taken care of during the design procedure. The filter designed based on L_1-approximation error possesses flat pass-bands and stop-bands to that of the least square design. A comparison has been made with other design techniques, demonstrating that BSA-ES obtains better results for designing digital IIR filters than the existing genetic algorithm (GA) based methods.

Key-Words: - Digital infinite impulse response (IIR) filters, Greedy algorithms, Stability, Magnitude response.

1 Introduction

The optimal design problem of digital infinite impulse response (IIR) filters has attracted much attention during past decades. The IIR filter design problem has been tackled using various optimization techniques such as p-error, weighted least square and ripple magnitudes (tolerances) of both pass-band and stop-band [1]-[3]. Due to the success of L_p-norm minimization, the designs based on L_p-normed minimization has been successfully applied in various applications such as: finite impulse response (FIR) filter design [5]-[6] and IIR filter design [3]-[4]. For p=1, this generalized L_p criterion reduces to the conventional L_1 approximation.

The design of IIR filters is problematic as: i) the error surface of IIR filters is usually nonlinear and multimodal, conventional gradient-based design methods may easily get stuck in the local minima of error surface; ii) IIR filters can be unstable and that is why the stability constraints must be included into IIR filter design problems. Hence, it is necessary to use an efficient optimization algorithm, robust to the local minima problem and possibly globally convergent.

Recently several optimization algorithms based on modern heuristics optimization algorithms [7]-[20] have been proposed for the design of digital IIR filters such as particle swarm optimization [13], simulated annealing [14], tabu search [15], ant colony optimization [16], artificial immune algorithm [17], hybrid taguchi genetic algorithm (HTGA) [18], immune algorithm (TIA) [19], hierarchical genetic algorithm (HGA) [20] and many more.

The intent of this paper is to apply a greedy search method based on binary successive approximation evolutionary search (BSA-ES) to design stable IIR digital filter. The values of the filter coefficients are optimized with BSA-ES approach to achieve L_1-norm approximation error criterion in terms of magnitude response. The credibility of.the proposed method has been demonstrated in [21] to solve the economic-emission load dispatch problem by searching the generation pattern of committed units. The problem formulation and design methodology is detailed below.

2 Problem Formulation

The transfer function of IIR filter can be represented by cascading first and second order sections to avoid the coefficient quantization problem which causes instability. In cascade realization coefficient range is limited. The structure of cascading type digital IIR filter is [8]:

$$H(\omega,x) = A\left(\prod_{i=1}^{M}\frac{1+a_{1i}e^{-j\omega}}{1+b_{1i}e^{-j\omega}}\right) \times \left(\prod_{k=1}^{N}\frac{1+c_{1k}e^{-j\omega}+c_{2k}e^{-2j\omega}}{1+d_{1k}e^{-j\omega}+d_{2k}e^{-2j\omega}}\right) \tag{1}$$

where

$x = [a_{11}, b_{11}, ..., a_{1M}, b_{1M}, c_{11}, c_{21}, d_{11}, d_{21}, ..., c_{1N}, c_{2N}, d_{1N}, d_{2N}, A]^T$. The Vector x denotes the filter coefficients of dimension $V \times 1$ with $V = 2M + 4N + 1$ and A is the gain of the filter. Digital filter design problem involves the determination of a set of filter coefficients which meet performance specifications such as pass-band width and corresponding gain, width of the stop-band and attenuation, band edge frequencies, and tolerable peak ripple in the pass band and stop-band. The magnitude response is specified at K equally spaced discrete frequency points in pass-band and stop-band. The L_p-norm approximation error for the magnitude response is defined as [4].

$$e(x) = \left\{ \sum_{i=0}^{K} \left| H_d(\omega_i) - \left| H(\omega_i, x) \right| \right|^p \right\}^{1/p} \quad (2)$$

In IIR filter design problem fixed grid approach is used [4]. For $p=1$, the magnitude response error denotes the L_1-norm error and is defined as given below:

$$e(x) = \sum_{i=0}^{K} \left| H_d(\omega_i) - \left| H(\omega_i, x) \right| \right| \quad (3)$$

Desired magnitude response $H_d(\omega_i)$ of IIR filter is given as:

$$H_d(\omega_i) = \begin{cases} 1, & for \ \ \omega_i \in passband \\ 0, & for \ \ \omega_i \in stopband \end{cases} \quad (4)$$

The design of causal recursive filters requires the inclusion of stability constraints. Therefore, the stability constraints given by (5a) to (5e) which are obtained by using the Jury method [22] on the coefficients of the digital IIR filter in (1) are included in the optimization process.

Mathematically, IIR filter problem is formulated as below:

$$\text{Minimize} \, F = e\ (x) \quad (5)$$

Subject to: the stability constraints:

$$1 + b_{1i} \geq 0 \ \ (i = 1, 2, ..., M) \quad (5a)$$

$$1 - b_{1i} \geq 0 \ \ (i = 1, 2, ..., M) \quad (5b)$$

$$1 - d_{2k} \geq 0 \ (k = 1, 2, ..., N) \quad (5c)$$

$$1 + d_{1k} + d_{2k} \geq 0 \ (k = 1, 2, ..., N) \quad (5d)$$

$$1 - d_{1k} + d_{2k} \geq 0 \ (k = 1, 2, ..., N) \quad (5e)$$

The stability constraints given by (5a) to (5e) have been forced to satisfy by updating the coefficients with random variation.

3 Solution Methodology

A direct search methodology is applied to examine trial solutions, sequentially. The process of departing from a given point to the next improved point is called a *move*. A move is termed a *success* if objective improves; otherwise, it is a *failure*. The direct search technique makes two types of move. First move is an exploratory move designed to acquire knowledge concerning the behavior of the function. This move is performed in the neighborhood of the current point systematically to find the best point around the current point. Second move is pattern move [23].

3.1 Exploratory Move

In exploratory move, the current point is perturbed in positive and negative directions along each variable one at a time and the best point is recorded. The current point is changed to the best point at the end of each variable perturbation. If the point found at the end of all variable perturbations is different than the original point, the exploratory move is a success; otherwise the exploratory move is a failure. In any case, the best point is considered to be the outcome of the exploratory move.

3.2 Evolutionary Search Method

Evolutionary method is proposed to search the optimal value of filter coefficients. In this method, 2^V feasible solutions are generated for V number of filter coefficients. A (V) dimensional hypercube of side Δ is formed around the point. x_i^C represents filter coefficients from the current point in the hyperspace. The better feasible solution is obtained from objective function of the problem. Another hypercube is formed around the better point, to continue the iterative process. All the corners of the hypercube represented in binary *(V)* bits equivalent code, generated around the current set of filter coefficients, are explored for the desired solution simultaneously. Table I shows the pattern of filter coefficients for 3-filter coefficients where 3 bits code is considered to represent the corners of the 3-dimensional hypercube (Fig. 1).

Serial numbers of hypercube corners in decimal are converted into their binary equivalent code. The deviation from the current centre point is obtained by replacing 0's with -Δ and 1's with +Δ in code associated with hypercube corners. As the number of filter coefficients increases, the number of hypercube corners increases exponentially. The process of exploring the better solution from all corners of the hypercube becomes time consuming, which needs some efficient search technique that should explore all the corners of the hypercube with minimum number of function evaluations and comparisons.

Table 1 Filter Coefficient Vector at Hypercube Corners

Hyper cube Corners	Possible combinations of 3-bits $C_2 C_1 C_0$	Distance of hypercube corners from centre point x_1^C, x_2^C, x_3^C	Pattern of filter coefficients at the hypercube corners		
0	0 0 0	$-\Delta_1-\Delta_2-\Delta_3$	$x_1^c-\Delta_1$	$x_2^c-\Delta_2$	$x_3^c-\Delta_3$
1	0 0 1	$-\Delta_1-\Delta_2+\Delta_3$	$x_1^c-\Delta_1$	$x_2^c-\Delta_2$	$x_3^c+\Delta_3$
2	0 1 0	$-\Delta_1+\Delta_2-\Delta_3$	$x_1^c-\Delta_1$	$x_2^c+\Delta_2$	$x_3^c-\Delta_3$
3	0 1 1	$-\Delta_1+\Delta_2+\Delta_3$	$x_1^c-\Delta_1$	$x_2^c+\Delta_2$	$x_3^c+\Delta_3$
4	1 0 0	$+\Delta_1-\Delta_2-\Delta_3$	$x_1^c+\Delta_1$	$x_2^c-\Delta_2$	$x_3^c-\Delta_3$
5	1 0 1	$+\Delta_1-\Delta_2+\Delta_3$	$x_1^c+\Delta_1$	$x_2^c-\Delta_2$	$x_3^c+\Delta_3$
6	1 1 0	$+\Delta_1+\Delta_2-\Delta_3$	$x_1^c+\Delta_1$	$x_2^c+\Delta_2$	$x_3^c-\Delta_3$
7	1 1 1	$+\Delta_1+\Delta_2+\Delta_2$	$x_1^c+\Delta_1$	$x_2^c+\Delta_2$	$x_3^c+\Delta_3$

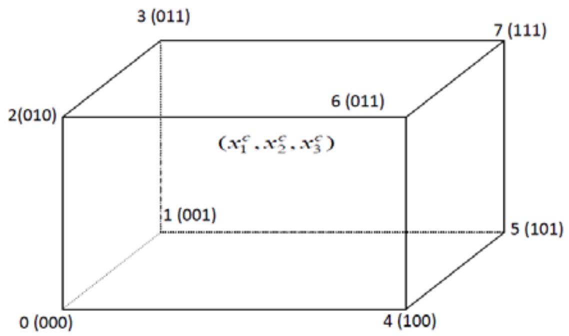

Fig.1. Three dimensional hypercube representing corners in decimal

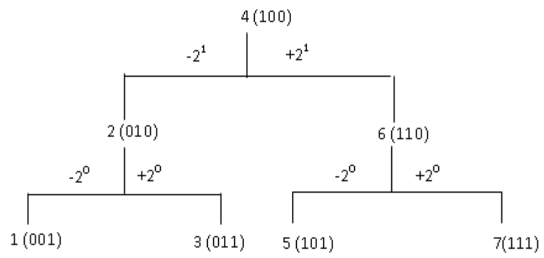

Fig. 2. Binary successive approximation for 3-bits code.

3.3 Binary Successive Approximation (BSA) Strategy

To reduce the computational burden, search is performed on filter coefficients pattern exploiting evolutionary optimization and BSA strategy to search the optimal solution. BSA strategy to search the filter coefficients is elaborated in Fig. 2, where the solution procedure moves towards the optimal solution by comparing two solutions at a time represented by two corners of hypercube.

The search process is started by initializing vector decision variable x_i^{ck}, giving objective F^c. To perform the BSA strategy by the iterative process C_i^k is initially selected as follows:

$$C_i^k = \begin{cases} 1 \ ; \text{for } (i=1) \\ 0 \ ; \text{for } (i=2, 3, \cdots, V) \end{cases} \quad (6)$$

Two corners, with reference to selected corner, are generated for the comparison as follows:

$$C_{1i}^k = \begin{cases} 1 \ ; \text{for } \ i+1 \\ C_j^k \ ; \ \text{for } (j=1, 2,.., i, \ (i+2), ..., V) \end{cases} \quad (7)$$

$$C_{2i}^k = \begin{cases} 0 & ;\text{for } i \\ C_{1j}^k & ;\text{for } (j=1, 2,...,(i\text{-}1), \\ & (i+1),....,V) \end{cases} \quad (8)$$

In reference to these two corners, vectors of filter coefficients are generated as depicted in Table I. Mathematically, it is represented in generalized form:

$$x_{mi}^k = x_i^{ck} + \Delta_{mi}^k \; ; (\; m=1,\; 2) \quad (i=1, 2,\cdots,V) \quad (9)$$

where

$$\Delta_{mi}^k = \begin{cases} +\Delta_i & \text{if } C_{mi}^k = 1 \\ -\Delta_i & \text{if } C_{mi}^k = 0 \end{cases} \quad (10)$$

$$(m=1,\; 2)\;\; (\; i=1, 2,\cdots,V)$$

Initial value of increment to filter coefficients is decided by

$$\Delta_i = \left(x_i^{\max} - x_i^{\min}\right)/\delta \quad (11)$$

Compute x_{mi}^k using (9) and then objective functions at x_{1i}^k and x_{2i}^k are evaluated as follows:

$$F_m^k = f\left(x_{mi}^k\right) \quad (m=1,\; 2) \quad (12)$$

Minimum of the two points is selected to be compared with the rest of the corners, generated subsequently by (7) and (8).

$$F^k = \min\left\{F_1^k, F_2^k\right\}; \quad (13)$$

The selected corner for the generation of next two corners is

$$C_i^k = \begin{cases} C_{1i}^k & \text{if } F_1^k < F_2^k \\ C_{2i}^k & \text{if } F_2^k < F_1^k \end{cases} \quad (i=1, 2,\cdots,V) \quad (14)$$

Table 2 Comparison of Number of Function Evaluations

Number of filter coefficients (V)	Number of corners of hypercube (2^V)	Number of comparisons by BSA method ($2 \times V$))
3	8	6
6	64	12
8	256	16
10	1,024	20
13	8,192	26
17	1,31,072	34
42	4,398,046,511,104	84

The process is repeated till all the corners of hypercube are explored by BSA method and overall minimum is selected to find the new center point for the next iteration. So, this procedure ends when last element of C_i^k vector contains 1 or the last branch of the binary successive approximation tree is reached which ensures that all the corners are explored. In this method the number of comparisons is reduced by a large amount. This is elaborated in Table 2 for different number of filter coefficients.

3.4 Pattern Move

The pattern move is designed to utilize the information acquired in the exploratory move, and accomplish the minimization of the function by moving in the direction of the established "pattern". A new point is found by jumping from the current best point x_i^k along a direction connecting the previous best point x_i^{k-1} and is executed as given below.

$$x_i^{k+1} = x_i^k + \eta\left(x_i^k - x_i^{k-1}\right) \quad (\; i=1, 2,\cdots,V). \quad (15)$$

4 Design Examples and Comparisons

For designing digital IIR filter 200 equally spaced points are set within the frequency domain $[0,\pi]$ and for the purpose of comparison, the lowest order of the digital IIR filter is set exactly the same as that given in [20] for the LP, HP, BP, and BS filters. Therefore, in this paper, the order of the digital IIR filter is a fixed number not a variable in the optimization process. The objective of designing the digital IIR filters is to minimize the objective function given by (5) with the stability constraints stated by (5a) to (5e) under the prescribed design conditions given in Table 3.

The examples of [19] and [20] are considered to test and compare the performance of the proposed BSA-ES approach. From the evaluated results with the proposed method presented in Table 4 and depicted in Fig. 3, it can be observed that, for the LP, HP, BP, and BS filters, the proposed BSA-ES approach gives the smaller L_1-norm approximation errors and the better magnitude performances in both pass-band and stop-band than the genetic algorithm based method given in [19] and [20].

The pole zero diagrams for LP, HP, BP and BS filters are presented in Fig. 4. It can be observed that the designed filters follow the stability constraints imposed in the design procedure as all the poles lie inside the unit circle. The poles magnitude and angles in radian are given by (0.8590, ± 0.6275), (0.6710, 0) for LP, (0.8597, ±2.4981), (0.6529, 3.1416) for HP, (0.8719, ± 1.1890), (0.8716, ±1.9470), (0.7212, ±1.5683) for BP and (0.7186,

±0.9509), (0.7103, ±2.1762) for BS filters. The first number in the parentheses is the magnitude of pole and the second number is the angle in radians. The stability of filter is not influenced by the zeros lying outside the unit circle. The designed IIR filter models obtained by the proposed BSA-ES approach are given below.

Table 3 Prescribed Design Conditions on LP, HP, BP and BS Filters.

Filter type	Pass-band	Stop-band	Maximum Value of $\lvert H(\omega, x)\rvert$
Low-Pass	$0 \le \omega \le 0.2\pi$	$0.3\pi \le \omega \le \pi$	1
High-Pass	$0.8\pi \le \omega \le \pi$	$0 \le \omega \le 0.7\pi$	1
Band-Pass	$0.4\pi \le \omega \le 0.6\pi$	$0 \le \omega \le 0.25\pi$ $0.75 \le \omega \le \pi$	1
Band-Stop	$0 \le \omega \le 0.25\pi$ $0.75 \le \omega \le \pi$	$0.4\pi \le \omega \le 0.6\pi$	1

$$H_{LP}(z) = 0.032089 \frac{(z+1.031726)(z^2 - 0.247472z + 1.020750)}{(z-0.671004)(z^2 - 1.390778z + 0.737950)} \tag{16}$$

$$H_{HP}(z) = 0.041083 \frac{(z-1.174660)(z^2 + 0.513211z + 0.941516)}{(z+0.652986)(z^2 + 1.375604z + 0.739122)} \tag{17}$$

$$H_{BP}(z) = 0.033198 \left(\frac{(z^2 - 0.028335\,z - 0.821419)(z^2 - 0.000465z - 1.031771)(z^2 + 0.001073z - 0.855457)}{(z^2 - 0.003541\,z + 0.520164)(z^2 - 0.649751\,z + 0.760342)(z^2 + 0.640542z + 0.759858)} \right) \tag{18}$$

$$H_{BS}(z) = 0.412049 \frac{(z^2 + 0.362508z + 0.994287)(z^2 - 0.402986z + 1.005084)}{(z^2 + 0.808577z + 0.504602)(z^2 - 0.834901z + 0.516474)} \tag{19}$$

Table 4 Design Results for LP, HP, BP and BS Digital IIR Filter

	L_1-norm error	Pass-band performance (Ripple magnitude)	Stop-band performance (Ripple magnitude)
LP Filter			
BSA-ES Approach	3.6690	$0.9205 \le \lvert H(e^{j\omega})\rvert \le 1.017$ (0.0973)	$\lvert H(e^{j\omega})\rvert \le 0.1577$ (0.1577)
TIA Approach[19]	3.8157	$0.8914 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.1086)	$\lvert H(e^{j\omega})\rvert \le 0.1638$ (0.1638)
HGA Approach[20]	4.3395	$0.8870 \le \lvert H(e^{j\omega})\rvert \le 1.009$ (0.1139)	$\lvert H(e^{j\omega})\rvert \le 0.1802$ (0.1802)
HP Filter			
BSA-ES Approach	4.0657	$0.9448 \le \lvert H(e^{j\omega})\rvert \le 1.013$ (0.0682)	$\lvert H(e^{j\omega})\rvert \le 0.1501$ (0.1501)
TIA Approach[19]	4.1819	$0.9229 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0771)	$\lvert H(e^{j\omega})\rvert \le 0.1424$ (0.1424)
HGA Approach[20]	14.5078	$0.9224 \le \lvert H(e^{j\omega})\rvert \le 1.003$ (0.0779)	$\lvert H(e^{j\omega})\rvert \le 0.1819$ (0.1819)

BP Filter			
BSA-ES Approach	1.3910	$0.9909 \le \lvert H(e^{j\omega})\rvert \le 1.005$ (0.0149)	$\lvert H(e^{j\omega})\rvert \le 0.0616$ (0.0616)
TIA Approach[19]	1.5204	$0.9681 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0319)	$\lvert H(e^{j\omega})\rvert \le 0.0679$ (0.0679)
HGA Approach[20]	5.2165	$0.8956 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.1044)	$\lvert H(e^{j\omega})\rvert \le 0.1772$ (0.1772)
BS Filter			
BSA-ES Approach	3.1233	$0.9348 \le \lvert H(e^{j\omega})\rvert \le 1.007$ (0.0726)	$\lvert H(e^{j\omega})\rvert \le 0.1326$ (0.1326)
TIA Approach[19]	3.4750	$0.9259 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0741)	$\lvert H(e^{j\omega})\rvert \le 0.1178$ (0.1278)
HGA Approach[20]	6.6072	$0.8920 \le \lvert H(e^{j\omega})\rvert \le 1.000$ 0.1080)	$\lvert H(e^{j\omega})\rvert \le 0.1726$ (0.1726)

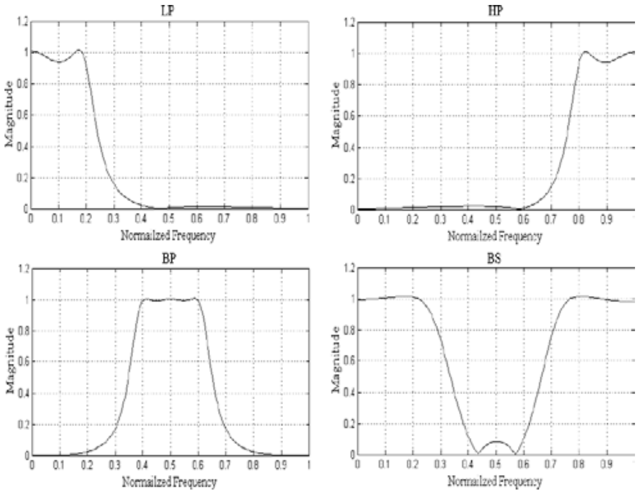

Fig. 3. Magnitude response of LP, HP, BP and BS filter respectively

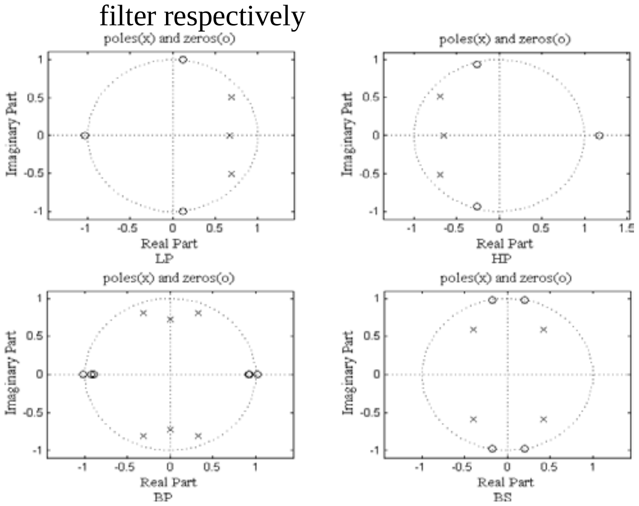

Fig. 4. Pole-zero plots for LP, HP, BP and BS filter respectively

5 Conclusion

In this paper, a novel BSA-ES search methodology has been applied for the design of stable digital IIR filters based on L_1-norm approximation error. On the basis of results obtained for the design of digital IIR filter, it is concluded that the proposed BSA-ES method is better method as compared to the existing GA based methods and it satisfies prescribed amplitude specifications consistently. The main advantage of the method is that there is no need to compute the derivative of the functions and solution is obtained with relative small known number of comparison and function evaluations. In order to overcome the limitation of the interactive method, it is proposed to search the optimal pattern with the help of BSA. The method is equally applicable to solve multi-objective optimization problems and falls in the category of interactive solution procedure.

References:

[1] W.-S. Lu, S. Pei, and C. Tseng, "A weighted least-squares method for the design of stable 1-D and 2-D IIR digital filters", *IEEE Trans. Signal Process.*, vol. 46, no. 1, pp. 1–10, Jan. 1998.

[2] C. Tseng and S. Lee, Minimax design of stable IIR digital filter with prescribed magnitude and phase responses", *IEEE Trans. Circuits Syst. I, Reg. Papers,* vol. 49, no. 4, pp. 547–551, Apr. 2002.

[3] C. Tseng, "Design of stable IIR digital filter based on least p-power error criterion," *IEEE Trans. Circuits Syst.—I*, vol. 51, no. 9, pp. 1879–1888, Sep. 2004.

[4] A. G Deczky, "Synthesis of recursive filters using the minimum p-error criterion", *IEEE Trans. Audio Electroacoust.*, vol. 20, no.4, pp. 257–263, 1972.

[5] C. S. Burrus, J. A. Barreto, and I. W. Selesnick, “Iterative weighted least-squares design of FIR filters”, *IEEE Trans. Signal Processing*, vol.42, pp. 2926–2936, Nov. 1994.

[6] S. C. Pei and C. C. Tseng, “Least mean p-power error criterion for adaptive FIR filter”, *IEEE J. Selected Areas Commun.*, vol. 12, pp.1540–1547, Dec. 1994.

[7] S.P.Harris and E.C Ifeachor, “Automatic design of frequency sampling filters by hybrid genetic algorithm techniques”, *IEEE Trans. Signal Process.*, vol.46, no. 12, pp. 3304–3314,1998.

[8] G. Vanuytsel, P. Boets, L.V. Biesen and S. Temmerman, “Efficient hybrid optimization of fixed-point cascaded IIR filter coefficients”, *in Proc. IEEE Int. Conf. Instrum. Meas. Technol.*, Anchorage, AK, pp.793–797, 2002.

[9] G.X. Zhang, W.D. Jin and F. Jin, “Multi-criterion satisfactory optimization method for designing IIR digital filters”, *in Proc. Int. Conf. Commun. Technol.*, Beijing, China, pp.1484–1490, 2003.

[10] Y. Liu, S.Q. Shao, H. Zhao, X.F. Liao and J.B. Yu, “An application of genetic algorithms with guiding strategy in the design of digital filters”, *in Proc. Int. Conf. Commun., Circuits Syst.*, Chengdu, China, pp.1141–1145, 2004.

[11] Nikos E. Mastorakis, Ioannis F. Gonos and M. N. S. Swamy, “Design of two-dimensional recursive filters using genetic algorithms”, *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol.50, no. 5, pp. 634-639, 2003.

[12] M. Nilsson, M. Dahl and I. Claesson, “Digital filter design of IIR filters using real valued genetic algorithm”, *WSEAS Trans. Circuit Syst*., vol.3, no. 1, pp. 29–34, 2004.

[13] J. Sun, W.B. Xu and B. Feng, “A global search strategy of quantum behaved particle swarm optimization”, in. *Proc. IEEE Conf. Cybernetics Intell. Syst.*, Singapore, pp. 111–116, 2004.

[14] S. Chen, R. H. Istepanian and B. L. Luk, “Digital IIR filter design using adaptive simulated annealing”, *J. Digital Signal Process*., vol. 11, no. 3, pp. 241–251, 2001.

[15] A. Kalinli and N. Karaboga, “A new method for adaptive IIR filter design based on tabu search algorithm”, *Int. J. Electron. Commun. (AEÜ)*, vol. 59, no. 2, pp. 111–117, 2005.

[16] N. Karaboga, A. Kalinli and D. Karaboga, “Designing IIR filters using ant colony optimisation algorithm”, *J. Engg. Appl. Artificial Intell*., vol. 17, no. 3, pp. 301–309, 2004.

[17] A. Kalinli and N. Karaboga, “Artificial immune algorithm for IIR filter design,” *J. Eng. Appl. Artif. Intell.*, vol. 18, no. 5, pp. 919–929, Dec. 2005.

[18] J.-T. Tsai, J.-H. Chou, and T.-K. Liu, “Optimal design of digital IIR filters by using hybrid Taguchi genetic algorithm”, *IEEE Trans. Ind. Electron.*, vol. 53, no. 3, pp. 867–879, Jun. 2006.

[19] J.-T. Tsai and J.-H. Chou, “Optimal design of digital IIR filters by using an improved immune algorithm”, *IEEE Trans. Signal Process.*, vol. 54, no. 12, pp. 4582–4596, Dec. 2006.

[20] K. S. Tang, K. F. Man, S. Kwong, and Z. F. Liu, “Design and optimization of IIR filter structure using hierarchical genetic algorithms”, *IEEE Trans. Ind. Electron.*, vol. 45, no. 3, pp. 481–487, Jun. 1998.

[21] J.S. Dhillon, J.S. Dhillon and D.P. Kothari, “Economic-emission load dispatch using binary successive approximation-based evolutionary search”, *IET Gener. Transm. Distrib*., vol. 3, no. 1, pp. 1–16, 2009.

[22] I. Jury, *Theory and Application of the Z-Transform Method*. New York: Wiley, 1964.

[23] D.P. Kothari and J.S. Dhillon, *Power system optimization,* PHI Pvt. Ltd., New Delhi, 2011.