Quadrature Hybrid Miniaturization on Single-Layer Substrate

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Abstract: - This paper presents an exact design technique to obtain formulas for compact quadrature hybrid coupler. We minimize the physical size based on even-and odd mode analysis. The proposed structure is relatively simple as the coupler can be fabricated on a single-layer printed circuit board. After finishing the design, full prototype of proposed coupler is fabricated and tested to validate the design. The obtained results reveal that we can reduce the coupler size up to 60% from the conventional one while maintaining its characteristics such as return loss, insertion loss and phase difference between through and coupled ports.

Key-Words: - even-odd mode; quadrature hybrid coupler; miniaturization; 3G and LTE

1 Introduction

So far, evolution on mobile wireless communication systems has been greatly developed rapidly. The developments have been made to adapt the systems from analog to digital transmissions, and/or for operating in wider frequency range. Standard technology of each generation and related application have been adopted and thus further developed. Until now, beamforming technique has gained lots of attention from researchers around the world as it is able to improve the performance of wireless communication systems. The key to success for beam formation is beamforming network. The key element of famous beamforming network e.g. Butler matrix is a quadrature hybrid [1]-[5]. According to this, miniaturization of quadrature hybrid is an attractive topic nowadays as this can result in compact beamforming network. From literatures, a slot-coupled multi-section quadrature hybrid for UWB applications has been proposed [6]. However, this has to be fabricated on multi-layer Printed-Circuit Board (PCB). Also, the authors of [7] have proposed a compact wideband branch-line hybrid coupler. As a result, the size of hybrid coupler can be reduced to 44.8% comparative to the conventional one. Furthermore, the works presented in [8-9] have presented the size reduction of quadrature hybrid up to 60% and 49%, respectively. Also a particular interest to the designs of compact and wideband quadrature hybrid coupler is presented in [10] based on the odd/even mode analysis. However, most of the works related to miniaturization of quadrature hybrid have proposed a complex structure such as fabricating on multilayer PCB or somehow its layout is too small to be fabricated. Therefore, this paper proposes a miniaturization of quadrature hybrid in which the proposed structure is relatively simple as it can be fabricated on single layer PCB. Some design examples for 900-3000 GHz band are demonstrated. The performance of proposed coupler comparing to conventional one is shown in term of computer simulation and experiment.

The remainder of paper is organized as follows. Section II presents the brief concept of quadrature hybrid coupler. Section III describes the design for miniaturization of quadrature hybrid proposed in this paper. In this section, we also present a detailed analysis based on even- and odd mode excitations by reducing the physical size of the quarter-wave transmission line using formulas relating to the characteristics impedances of all branch lines. After presenting its structure, some simulation results are shown to indicate its performance. The performance comparison obtained from the proposed and conventional ones is presented in section IV. Section V presents the full prototype of proposed quadrature hybrid coupler followed by the experimental results for confirming the true performance of the proposed concept. Finally, Section VI concludes the paper.

2 Quadrature Hybrid Coupler

Fig. 1 shows geometry of a quadrature (90°) hybrid coupler which is a four-terminal device, otherwise known as quadrature coupler or branch-line coupler. The phase difference between signal outputs, ports 2 and 3 as indicated in Fig. 1, is 90° [11]. As we can see in the figure, its dimension is given by quarter-wavelength at the center frequency. The coupler is usually fabricated on a simple PCB. So far, a conventional quadrature hybrid coupler works well for single frequency or within a limited frequency band.

3 Analysis of Proposed Quadrature Hybrid Coupler

In Fig. 2, the modification is made by adding four stubs in the coupler. The reason is that adding some stubs makes change to overall impedance of the coupler. To find an analytic solution, the structure is decomposed into the superposition of odd/even mode excitation [9-10], as shown Fig. 3. The proposed quadrature hybrid coupler consists of nine quarterwavelength lines (top circuit), as shown in Fig. 3 (a). Since the coupler is symmetric, the normal mode theory can be employed to analyze the characteristic impedance of the quarter-wavelength lines. The diagram shown in Fig. 3 (a) can be decomposed into the superposition of even and odd -mode excitations as shown in Fig. 3 (b) and (c), respectively. Since the circuit has a linear behavior, the actual response (the scattered wave) can be obtained from the sum of responses to the even and odd mode excitations. For a symmetry or anti-symmetry of the excitation, the four-terminal network can be decomposed into a set of two decoupled twoport networks, as shown in Fig. 3. We present a detailed analysis based on even- and odd mode as follows.











Fig.3 Schematic diagrams of proposed coupler in (a) normal mode (b) odd mode (c) even mode.

3.1 Even and Odd mode analysis

Analyzing the top circuit for odd mode excitation as shown in Fig. 3(b), we find that transmission line is a short-circuited stub. To analyze the circuit behavior, we represent them by ABCD matrices as follows.

- The ABCD matrix of a lossless transmission line section of characteristic Z_a and Z_b in which the electrical lengths βl_a and βl_b are represented by $(2\pi/\lambda)l_a$ and $(2\pi/\lambda)l_b$ respectively.

- The admittance of shunt short-circuited stub, Z_{sh}, is characterized by $Y = -jZ_{sh}\cot\beta l_{sh}$. Then, we find that $Z_{csh} = Z_c + 2Z_{sh}$ and electrical length $\beta l_{csh} = (2\pi/\lambda)l_{csh}$.

- The admittance of the shunt short-circuited stub Z_{sv} is characterized by $Y = -jZ_{sv}\cot\beta l_{sv}$ and electrical length is $\beta l_{sv} = (2\pi/\lambda)l_{sv}$

Analyzing the top circuit for even mode excitation as shown Fig. 3(c), we find that transmission line behaves like an open-circuited stub. Similarly to odd mode analysis, we obtain the admittance of the shunt open-circuited stubs as $Y = j \tan \beta l$

At the center frequency, the scattering parameters in term of even-and odd-mode reflection and transmission coefficients for twoport networks of Figs. 3(b) and (c) are determined as represented by S-parameters: S_{11} , S_{21} , S_{31} , and S_{41} for the entire coupler which can be formulated as follows:

$$S_{11} = \frac{1}{2} \Gamma_e + \frac{1}{2} \Gamma_o$$
 (1-a)

$$S_{21} = \frac{1}{2}T_e + \frac{1}{2}T_o$$
 (1-b)

$$S_{31} = \frac{1}{2} T_e - \frac{1}{2} T_o$$
 (1-c)

$$S_{41} = \frac{1}{2} \Gamma_e - \frac{1}{2} \Gamma_o$$
 (1-d)

$$[\mathbf{S}] = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} & \mathbf{S}_{13} & \mathbf{S}_{14} \\ \mathbf{S}_{21} & \mathbf{S}_{22} & \mathbf{S}_{23} & \mathbf{S}_{24} \\ \mathbf{S}_{31} & \mathbf{S}_{32} & \mathbf{S}_{33} & \mathbf{S}_{34} \\ \mathbf{S}_{41} & \mathbf{S}_{42} & \mathbf{S}_{43} & \mathbf{S}_{44} \end{bmatrix}$$
(2)

where $\Gamma_{e,o}$ and $T_{e,o}$ are even and odd -mode reflection and transmission coefficients for the two-port networks of the one presented in Fig 3. Then, we convert ABCD parameters to S – parameters as follows.

$$\Gamma_{0} = \frac{A_{0}+B_{0}/Zo-C_{0}Zo-D_{0}}{A_{0}+B_{0}/Zo+C_{0}Zo+D_{0}}$$
(3)

$$\Gamma_{e} = \frac{A_{e}+B_{e}/Zo-C_{e}Zo-D_{e}}{A_{e}+B_{e}/Zo+C_{e}Zo+D_{e}}$$
(4)

$$T_{0} = \frac{2}{A_{0}+B_{0}/Zo+C_{0}Zo+D_{0}}$$
(4)

$$T_{e} = \frac{2}{A_{e}+B_{e}/Zo+C_{e}Zo+D_{e}}$$

3.2 Formulation

This formulation is accomplished by first choosing the design parameters of the conventional quadrature hybrid to give a reasonable approximation having the optimum compensation. In this paper, we minimize its physical size based on even-and odd mode analysis. One way to reduce the size of conventional quadrature hybrid is minimizing length (l) and width (w) influencing its characteristic impedance. The (2) to (7) are utilized to find the optimum length and width using own developed Matlab programming. In the programming, some parameters are given to be a criteria for optimization such as return loss, isolation loss and phase between through and coupled ports.

Next, one example of the proposed concept is demonstrated. The aim of proposed design is to minimize the area in a branch-line hybrid coupler shown in Fig. 3. The design parameters for miniaturization are the lengths (l_a , l_b , l_c , l_{sh} , l_{sv}) and the widths (w_a , w_b , w_c , w_{sv} , w_{sh}) of the transmission-lines shown in Fig. 3.

$$S11 = S22 = S33 = S44 = \frac{1}{2}\Gamma_{e} + \frac{1}{2}\Gamma_{o}$$
$$= \frac{1}{4} \left[\frac{1}{A_{e}} \left(\frac{B_{e}/Zo - C_{e}Zo}{B_{e}/Zo + C_{e}Zo} \right) \right] + \frac{1}{4} \left[\frac{1}{A_{o}} \left(\frac{B_{o}/Zo - C_{o}Zo}{B_{o}/Zo + C_{o}Zo} \right) \right]$$
...(5-a)

$$S21 = S12 = S43 = S34 = \frac{1}{2}Te + \frac{1}{2}To$$
$$= \frac{1}{2} \left[\frac{1}{A_e} \left(\frac{1}{B_e/Zo + C_eZo} \right) \right] + \frac{1}{4} \left[\frac{1}{A_o} \left(\frac{1}{B_o/Zo + C_oZo} \right) \right]$$
...(5-b)

$$S31 = S13 = S42 = S24 = \frac{1}{2}T_{e} - \frac{1}{2}T_{o}$$
$$= \frac{1}{2} \left[\frac{1}{A_{e}} \left(\frac{1}{B_{e}/Zo + C_{e}Zo} \right) \right] - \frac{1}{4} \left[\frac{1}{A_{o}} \left(\frac{1}{B_{o}/Zo + C_{o}Zo} \right) \right]$$
...(5-c)

$$S41 = S14 = S32 = S23 = \frac{1}{2}\Gamma_{e} - \frac{1}{2}\Gamma_{o}$$
$$= \frac{1}{4} \left[\frac{1}{A_{e}} \left(\frac{B_{e}/Zo - C_{e}Zo}{B_{e}/Zo + C_{e}Zo} \right) \right] - \frac{1}{4} \left[\frac{1}{A_{o}} \left(\frac{B_{o}/Zo - C_{o}Zo}{B_{o}/Zo + C_{o}Zo} \right) \right]$$
...(5-d)

Then, we obtain ABCD parameters for odd mode excitation as follows:

$$\begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix}$$

= [E01]×[E02]×[E03]×[E04]×[E05]×[E06]×[E07]
...(6)

when

$$Eo1 = \begin{bmatrix} \cos(M_a) & jZasin(M_a) \\ j\frac{1}{Za}sin(M_a) & \cos(M_a) \end{bmatrix}$$
$$Eo2 = \begin{bmatrix} 1 & 0 \\ -j\frac{Z_c + Z_{sh}}{Z_c Z_{sh}} \cot\frac{2\pi}{\lambda}\frac{l_{csh}}{2} & 1 \end{bmatrix}$$
$$Eo3 = \begin{bmatrix} \cos(M_b) & jZbsin(M_b) \\ j\frac{1}{Zb}sin(M_b) & \cos(M_b) \end{bmatrix}$$

$$Eo4 = \begin{bmatrix} 1 & 0 \\ j\frac{1}{Zsv}\tan(M_a) & 1 \end{bmatrix}$$
$$Eo5 = \begin{bmatrix} \cos(M_b) & jZbsin(M_b) \\ j\frac{1}{Zb}sin(M_b) & \cos(M_b) \end{bmatrix}$$
$$Eo6 = \begin{bmatrix} 1 & 0 \\ -j\frac{Z_c + Z_{sh}}{Z_c Z_{sh}} \cot\frac{2\pi}{\lambda}\frac{l_{csh}}{2} & 1 \end{bmatrix}$$
$$Eo7 = \begin{bmatrix} \cos(M_a) & jZasin(M_a) \\ j\frac{1}{Za}sin(M_a) & \cos(M_a) \end{bmatrix}$$

Also, we obtain ABCD parameters for even mode excitation as follows:

$$\begin{bmatrix} A_e & B_e \\ C_e & D_e \end{bmatrix}$$

= [Ee1]×[Ee2]×[Ee3]×[Ee4]×[Ee5]×[Ee6]×[Ee7]
...(7)

when

$$\begin{aligned} & \text{Eel} = \begin{bmatrix} \cos(M_a) & j\text{Zasin}(M_a) \\ j\frac{1}{\text{Za}}\sin(M_a) & \cos(M_a) \end{bmatrix} \\ & \text{Ee2} = \begin{bmatrix} 1 & 0 \\ j\frac{1}{\text{Zc}}\tan(M_c) & 1 \end{bmatrix} \\ & \text{Ee3} = \begin{bmatrix} \cos(M_b) & j\text{Zbsin}(M_b) \\ j\frac{1}{\text{Zb}}\sin(M_b) & \cos(M_b) \end{bmatrix} \\ & \text{Ee4} = \begin{bmatrix} 1 & 0 \\ j\frac{1}{\text{Zsv}}\tan(M_a) & 1 \end{bmatrix} \\ & \text{Ee5} = \begin{bmatrix} \cos(M_b) & j\text{Zbsin}(M_b) \\ j\frac{1}{\text{Zb}}\sin(M_b) & \cos(M_b) \end{bmatrix} \\ & \text{Ee6} = \begin{bmatrix} 1 & 0 \\ j\frac{1}{\text{Zb}}\sin(M_b) & \cos(M_b) \end{bmatrix} \end{aligned}$$

$$Ee7 = \begin{bmatrix} \cos(M_a) & jZasin(M_a) \\ j\frac{1}{Za}sin(M_a) & \cos(M_a) \end{bmatrix}$$

where

$$M_{a} = \frac{2\pi}{\lambda} l_{a} \qquad M_{b} = \frac{2\pi}{\lambda} \frac{l_{b}}{2} \qquad M_{c} = \frac{\pi}{\lambda} l_{c}$$
$$\lambda = \frac{V_{p}}{f}$$
$$v_{p} = \frac{3 \times 10^{8}}{\sqrt{\varepsilon_{e}}}$$
$$\varepsilon_{e} = \frac{\varepsilon_{r} + 1}{2} + \frac{\varepsilon_{r} - 1}{2} \frac{1}{\sqrt{1 + 12\frac{d}{W_{csh}}}}$$

The miniaturization starts with some characteristic of Conventional(Con) quadrature hybrid coupler having its characteristic of 50 Ω as follows referring to Fig. 3:

- $w_{con,a}$ = width of conventional coupler at Z_a of 50 Ω
- $w_{con,b}$ = width of conventional coupler at Z_b of $50/\sqrt{2} \Omega$
- $w_{con,c}$ = width of conventional coupler at Z_c of 50 Ω
- $l_{con,a}$, $l_{con,b}$, $l_{con,c}$ = line-length at $\lambda/4$

when some reductions of other widths and lengths are given during frequencies 0.9-2 GHz as follows:

 $w_a = w_{con,a}/2$ $w_b = w_{con,b}/2$ $w_c = w_{con,c}-2.2$ mm $l_a = l_{con,a}/3$ $l_b = l_{con,b}-2.2$ mm $l_c = l_{con,c}-2.2$ mm

After calculating parameters w_a , w_b , w_c , l_a , l_b and l_c , we limit of frequency response for l_{sh} and w_{sh} as follows. Please note that the following formulas are obtained from simulation when the frequency is varied from 0.9 to 3 GHz.

$$l_{sh} = -10f + 23$$
; 0.9 GHz $\leq fc \leq 1.4$ GHz

$$l_{sh} = -5f + 16 \quad ; \quad 1.4 \text{ GHz} < fc < 2.1 \text{ GHz}$$

$$w_{sh} = 3.4 \quad ; \quad 0.9 \text{ GHz} \le fc < 1.7 \text{ GHz}$$

$$w_{sh} = 3.2 \quad ; \quad fc = 1.7 \text{ GHz}$$

$$w_{sh} = 3 \quad ; \quad 1.7 \text{ GHz} < fc < 2.1 \text{ GHz}$$

$$w_{sv} = 5.6 \quad ; \quad 0.9 \text{ GHz} \le fc \le 1.2 \text{ GHz}$$

$$w_{sv} = 5.2 \quad ; \quad 1.2 \text{ GHz} < fc \le 1.5 \text{ GHz}$$

$$w_{sv} = 5 \quad ; \quad 1.5 \text{ GHz} < fc \le 1.8 \text{ GHz}$$

$$w_{sv} = -3f + 10 \quad ; \quad 1.8 \text{ GHz} < fc < 2.1 \text{ GHz}$$

and so on for operating frequencies from 2.1-2.7 GHz:

$$w_{a} = w_{con,a}/2$$

$$w_{b} = w_{con,b}/2$$

$$w_{c} = w_{con,c} - 2.2 \text{ mm}$$

$$l_{a} = l_{con,a}/2.95$$

$$l_{b} = l_{con,b} - 2 \text{ mm}$$

$$l_{c} = l_{con,c} - 2 \text{ mm}$$

$$l_{sh} = 5.6 \qquad ; fc = 2.1 \text{ GHz}$$

$$l_{sh} = -3f + 11.8 \quad ; 2.1 \text{ GHz} < fc < 2.8 \text{ GHz}$$

$$w_{sh} = 3.1 \qquad ; 2.1 \text{ GHz} \le fc < 2.3 \text{ GHz}$$

$$w_{sh} = 3 \qquad ; 2.3 \text{ GHz} \le fc \le 2.5 \text{ GHz}$$

$$w_{sv} = 4.7 \qquad ; fc = 2.1 \text{ GHz}$$

$$w_{sv} = 4.3 \qquad ; 2.1 \text{ GHz} < fc < 2.4 \text{ GHz}$$

$w_{sv} = 4$; 2.4 GHz \leq fc $<$ 2.6 GHz

$$w_{sv} = 5$$
; 2.6 GHz $\leq fc < 2.8$ GHz

then, for frequencies from 2.8-3 GHz:

$$w_{a} = w_{con,a}/2$$

$$w_{b} = w_{con,b}/2$$

$$w_{c} = w_{con,c} - 2.2 \text{ mm}$$

$$l_{a} = l_{con,a}/2.95$$

$$l_{b} = l_{con,b} - 2 \text{ mm}$$

$$l_{c} = l_{con,c} - 2 \text{ mm}$$

$$l_{sh} = -2.5\text{f} + 11 \quad ; 2.8 \text{ GHz} \le fc \le 3 \text{ GHz}$$

$$w_{sh} = 3 \qquad ; 2.8 \text{ GHz} \le fc \le 3 \text{ GHz}$$

$$w_{sv} = 4 \qquad ; 2.8 \text{ GHz} \le fc \le 3 \text{ GHz}$$

The final values of w_a , w_b , w_c , l_a , l_b , l_c , l_{sh} , and w_{sh} are obtained from the optimization function in own developed programming.

4 Simulation Results

To see the performance of proposed design, the return and insertion losses are evaluated using CST microwave studio. Figs. 4 and 5 show the simulated S-parameters of the proposed quadrature hybrid coupler. The different center frequencies (f_c) from 0.9 to 3 GHz are chosen to validate the proposed design. As we can see in both figures, we can have the best return loss which is much lower than -10 dB for all center frequencies. Then, one response example at 1.8 GHz is chosen to reveal full performance of the new design as shown in Fig. 6. This plot shows S-parameters for the proposed all and conventional couplers. As we can see, they have



Fig.4 Simulated S11 (magnitude) of proposed at 0.9-2 GHz.



Fig.5 Simulated S11 (magnitude) of proposed coupler at 2.1-3 GHz.



Fig.6 Simulated S-parameters (magnitude) of Conventional(Con) vs Proposed(Pro) couplers at 1.8 GHz.

a good agreement. In addition, the proposed coupler provides phase difference of 90° between through and coupled ports at 1.8 GHz similar to the conventional one as can be seen in Fig. 7.

To confirm the performance of proposed design, another example at 2.8 GHz is shown in Fig. 8. In this figure, the ones obtained from proposed and conventional couplers are compared. As expected, we can have a good agreement between them. Also, in term of phase difference between through and coupled ports, we obtain 90° at the designated frequency, 2.8 GHz as seen in Fig. 9.

Next, the full prototype of proposed quadrature hybrid coupler is fabricated to reflect its true performance comparative with the conventional one.

5 Experimental Results

Fig. 10 shows photograph of quadrature hybrid coupler designed at 1.8 GHz. Please note the utilized PCB has its dielectric constant of 4.8 and thickness of 1.66 mm. In this figure, the overall size and dimension of proposed and conventional ones are compared. The area of conventional coupler (top) is $67.32 \times 30.20 \text{ mm}^2$ when area of proposed coupler (bottom) is $33.02 \times 23.01 \text{ mm}^2$. According to this, we can reduce the size of conventional coupler up to 63%. Fig. 11 shows measured S-parameters of proposed and conventional ones. As we can see, they provide similar behaviour during the designated frequency, 1.8 GHz. Also for phase difference between through and coupled ports, the proposed and conventional ones have a good agreement as can be seen in Fig. 12 at 1.8 GHz.

One more prototype at 2.8 GHz is constructed and tested as its photograph is shown in Fig. 13. The size of conventional one is 43.29×22.13 mm² while the obtained size of the coupler employing the proposed design is 22.96×16.87 mm². This means that we can reduce the size of



Fig.7. Simulated phase difference between through and coupled ports of conventional and proposed couplers at 1.8 GHz.



Fig.8. Simulated S-parameters (magnitude) of Conventional(Con) and Proposed(Pro) couplers at 2.8 GHz.



Fig.9. Simulated phase difference between through and coupled ports of conventional and proposed couplers at 2.8 GHz.



Fig.10. Size comparison between conventional (top) and proposed (bottom) quadrature hybrid coupler at 1.8 GHz.



Fig.11. Measured S-Parameters (magnitude) of Conventional(Con) vs Proposed(Pro) couplers at 1.8 GHz.



Fig.12. Measured phase difference between through and coupled ports of conventional and proposed couplers at 1.8 GHz.



Fig.13. Size comparison between conventional (top) and proposed (bottom) quadrature hybrid coupler at 2.8 GHz.



Fig.14. Measured S-Parameters (magnitude) of Conventional(Con) vs Proposed(Pro) couplers at 2.8 GHz.



Fig.15. Simulated phase difference between through and coupled ports of Conventional(Con) vs Proposed(Pro) couplers at 2.8 GHz.

proposed and conventional ones is compared for both magnitude (Fig. 14) and phase (Fig. 15). As we can see in both figures, they have a good agreement for return loss, insertion loss and also phase difference between through and coupled ports even the new design is 40% smaller in size.

6 Conclusion

This paper has proposed a design procedure for compact quadrature hybrid coupler. The proposed design is based on oddeven mode analysis. The structure of the proposed coupler is relatively simple as it can be fabricated on a single-layer PCB. Some design examples are chosen at 1.8 and 2.8 GHz. A number of simulations have been run to show the performance of proposed couplers at various operating frequencies. The obtained simulation have shown that results the propose miniaturization can maintain the coupler's characteristic in term of S-parameters as the conventional one. To validate the proposed concept, some prototype of couplers were constructed and tested at 1.8 and 2.8 GHz. The obtained experimental results have revealed that we can reduce the coupler size up to 60% while maintaining its characteristic as the ones obtained from conventional coupler.

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