Second-Degree Polynomial Model of Laser Generation for a CuBr Laser

NIKOLAY DENEV[1], ILIYCHO ILIEV[2] and SNEZHANA GOCHEVA-ILIEVA[3]

[1] Department of Applied Physics,
Technical University of Sofia
8 Climent Ohridski Blvd, 1000 Sofia
BULGARIA
ndenevtph@dir.bg

[2] Department of Physics,
Technical University of Sofia, branch Plovdiv
25 Tzanko Djusstabanov Str., 4000 Plovdiv
BULGARIA
iliev55@abv.bg

[3] Department of Applied Mathematics and Modeling, Faculty of Mathematics and Informatics
University of Plovdiv 'Paisii Hilendarski'
24 Tzar Assen Str., 4000 Plovdiv
BULGARIA
snow@uni-plovdiv.bg

Abstract: - The subject of investigation is a copper bromide vapor laser, generating laser emissions in the visible spectrum (510.6 nm and 578.2 nm). A statistical model has been developed based on experiment data. The goal is to analyze the state of existing lasers and to predict the behavior of new laser sources. A second-degree polynomial model has been developed to determine output laser generation in relation to 10 independent input laser characteristics. The model describes 96.7% of examined experiment data. An adequacy diagnosis has been performed on the obtained model. The model is applied to predict the output power of the laser source in relation to new data of the input characteristics.

Key-Words: - Copper bromide laser (CuBr), Laser generation, Second order model, Statistical Prediction model, Multiple linear regression, Factor analysis, Cluster analysis.

1 Introduction

Copper and copper compound vapor lasers continue to be the subject of active scientific research. This type of lasers are the most powerful sources in the visible spectrum (λ = 510.6 and 578.2 nm). Their main advantages are the high quality of the laser beam and the ability to focus it onto a single spot of the order of a few microns. These lasers have a wide range of applications in various fields of medicine and medical research, in industry for the microprocessing of materials: drilling, cutting, labeling, and etching. They are widely used for scientific research - for isotopic separation of various chemical elements, in chemistry and physics, for the pumping of other types of lasers [1, 2]. For this reason, they are subject to particular interest and rank among the 12 most commercialized lasers in the world. They are manufactured in many countries throughout the world - Russia, USA, China, England, Bulgaria, etc.

The particular subject of consideration in this study is one of the subtypes of copper halide lasers - copper bromide (CuBr) vapor lasers generating in the visible spectrum. The use of copper halides gives significant advantages to this type of lasers when compared to pure copper lasers. The first of these advantages is that the operating temperature of the active laser tube is reduced by around 1000 ^{o}C as compared to pure copper lasers working at temperatures of about 1500 ^{o}C. This allows the use of cheaper material for the manufacture of the laser tube - usually quartz. Another advantage is that CuBr vapor lasers are cooled by air and do not require additional water- or another type of cooling. This makes operating CuBr vapor laser much easier. Simple insulation is used for the active laser volume - mineral- or glass-fiber wool. Another result of the lower operating temperature is the significantly reduced time for initial heating of the charge and time to operational readiness of the device in comparison to pure metal vapor lasers. Furthermore, it has been experimentally established that maintaining an optimal mode of operation at lower temperatures and the use of suitable design materials leads to a significantly longer service life of metal halide vapor lasers compared to that of pure metal vapor ones.

The copper vapor laser is considered to have been thoroughly studied but the work on increasing laser output power continues to be topical because this would broaden its range of applications and commercial significance [1, 2].

In recent years on the base of the big amount of available experiment data on copper bromide laser different statistical models were developed. The aim of these studies is to derive the essential information about the direct relations between the input laser characteristics as geometrical design, applied electrical power, pressure of the neutral gas and others on the output laser characteristics as laser generation and laser efficiency. The main part of the obtained empirical models is presented in [3-7]. More details are given below in section 4.

In this study a new polynomial type regression model, based on all possible terms up to second degree is obtained. The model describes 96.7% of all examined data.

The model is developed by using the IBM SPSS statistical software [8].

2 Subject of study

The subject of investigation is a copper bromide vapor laser which is an original Bulgarian design developed at the Laboratory of Metal Vapor Lasers at the Georgi Nadjakov Institute of Solid State Physics of the Bulgarian Academy of Sciences, Sofia (see [1, 9-10]). A conceptual schematic of the laser source is given in Fig. 1.

Neon is used as a buffer gas. In order to improve efficiency, small quantities of hydrogen are added [1]. Unlike the high-temperature pure copper vapor laser, the copper bromide vapor laser is a low-temperature one, with an active zone temperature of 500 °C. The laser tube is made out of quartz glass without high-temperature ceramics as a result of which it is significantly cheaper and easier to manufacture. The discharge is heated by electric current (self-heating). It produces light impulses tens of nanoseconds long. Its main advantages are: short initial heating period, stable laser generation, relatively long service life, high values of output power and laser efficiency.

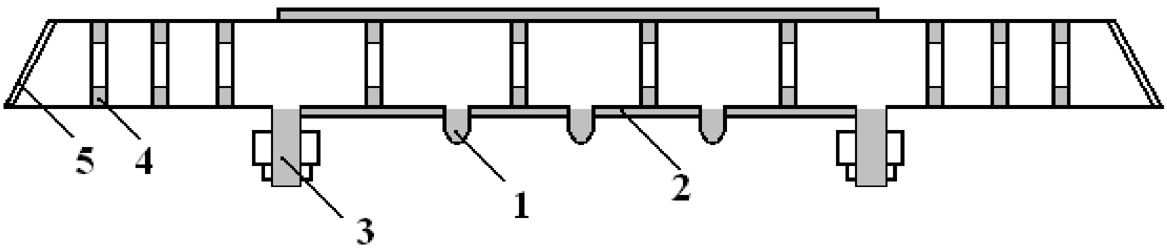

Fig. 1. Construction of CuBr laser tube: 1- reservoirs with copper bromide, 2- heat insulation of the active volume, 3- copper electrodes, 4- inner diaphragms, 5- tube windows.

3 Experimental data

We examine experimental data for various CuBr lasers, published in [1, 11] and included therein literature. By their geometry, the CuBr lasers under consideration are usually classified into three basic groups: small-bore lasers of inside diameter D<20 mm, medium-bore lasers of diameter D=20 to 40 mm and large-bore lasers of D>40 mm.

The total number of data investigated in this paper includes n=387 experiments.

The following 10 input laser characteristics (independent variables, predictors) are used: the inner diameter of the laser tube, D (mm); the inner diameter of the diaphragms, dr (mm); the active-volume length (distance between the electrodes), L (cm); the supplied electric power, Pin (kW); the electric power per unit length (with allowance for 50% loss), $PL = Pin / L$ (W cm^{-1}); the electric pulse repetition frequency, PRF (kHz); the buffer gas (neon) pressure, PNe (Torr); the additional gas (hydrogen) pressure, $PH2$ (Torr); the equivalent capacitance of the capacitor battery, C (nF); and the temperature of the reservoir filled with CuBr, Tr (°C).

The average output laser power (laser generation) $Pout$ (W) will be considered as the main dependent variable.

4 Analysis of the known parametric regression models of a CuBr laser

The initial approach requires finding a linear regression relationship of the type:

$$\begin{aligned}\hat{P}out &= a_0 + a_1 D + a_2 dr + a_3 L + a_4 Pin \\ &+ a_5 PH2 + +a_6 PL + a_7 PRF \\ &+ a_8 PNe + a_9 C + a_{10} Tr\end{aligned} \quad (1)$$

where $a_i,\ i = 0,1,...,10$ are the regression coefficients (parameters) to be determined.

The factor and regression analysis [3] performed showed that there is a high degree of correlation between the predictors and that an equation of type (1) is not statistically significant. For this reason, the factor analysis procedure was applied that grouped the independent variables in 3 factors. The factors included only 6 of the 10 predictors and are distributed as follows: $F1$ (D, dr, Pin, L), $F2$ (PL), and $F3$ ($PH2$). In accordance with the properties of factor analysis there is a high degree of correlation between the variables in each factor. Between the variables, grouped in different factors, there is no such correlation or it is very weak [12, 13].

In this way, using factors as new independent variables a regression equation was obtained of the type:

$$\hat{P}out = b_0 + b_1 F_1 + b_2 F_2 + b_3 F_3 .$$

This method is called principle component regression [3, 13]. The factor variables are linearly independent and through regression analysis the following linear relationship was established [6]:

$$\begin{aligned}\hat{P}out &= 40.598 + 29.717 F_1 + 4.155 F_2 \\ &+ 12.941 F_3\end{aligned} \quad (2)$$

Based on the analysis of the results from [6] and using the stepwise regression method with factor variables, polynomial models of second and third degree were also obtained in the following form:

$$\begin{aligned}\hat{P}out &= 38.283 + 28.090 F_1 + 4.866 F_2 \\ &+ 11.992 F_3 + 2.336 F_1^2\end{aligned} \quad (3)$$

And

$$\begin{aligned}\hat{P}out &= 38.270 + 39.511 F_1 + 8.243 F_2 F_3^2 \\ &+ 8.718 F_3^3 + 6.884 F_3^2 - 2.294 F_1^3 \\ &+ 3.053 F_2^2 F_3 - 0.976 F_2^3\end{aligned} \quad (4)$$

Table 1 shows that by increasing the order of nonlinearity, the accuracy of the results is improved. This is in accordance with the common polynomial theory of complex systems [14].

It must be noted, that a disadvantage of the applied principle regression models (2)-(4) is that the actual physical variables are "hidden" behind the factor variables. Also the prediction of laser power requires the application of some specialized software and its skilled use. This makes the approach difficult for wider use.

For this reason, the issue of developing nonlinear and polynomial parametric models containing explicitly the 10 independent variables is topical. For the first time such a second-degree polynomial model using the data described above in section 3 was developed in [7]. The resulting regression equation is in the form:

Table 1. Comparison of the statistical summary of polynomial regression models.

Polynomial regression model	R^2	Std. error of the Estimate
Linear model -eq. (2)	0.946	7.92540
Second order model eq. (3)	0.950	7.63382
Third order model eq. (4)	0.957	5.81272

$$\begin{aligned}\hat{P}out = &-188.821 + 2.728dr + 38.349Pin \\ &+10.014PL + 126,250C + 11.018Pin.C \\ &-0.025Pin.PRF - 7.577C.PL - 2.517C.dr \\ &+9.766C.PH2 + 1.643Pin.PL\end{aligned} \quad (5)$$

The obtained second-degree regression model (5) is partial because out of all possible combinations only 6 of second degree were used: {*Pin.C*}, {*Pin.PRF*}, {*C.PL*}, {*C.dr*}, {*C.PH*2} и {*Pin.PL*}.

The following statistical indexes were obtained: the coefficient of determination of the resulting model (5) is $R^2 = 0.973$ and the standard error of the estimate is 5.3136, which is better than all currently known parametric models (see Table 1).

In this paper we improve model (5) by developing a complete polynomial type model with terms up to the second degree.

5 Complete second-degree polynomial model of laser generation

5.1 Model construction

The relationship is to be found in general form, which means that for *Pout* it can be of the type:

$$\begin{aligned}\hat{P}out = &b_0 + b_1D + b_2dr + b_3L + b_4Pin + b_5PL + b_6P_{H2} + b_7Prf + b_8Pne + b_9C + b_{10}Tr + b_{11}D.dr \\ &+b_{12}D.L + b_{13}D.Pin + b_{14}D.PL + b_{15}D.P_{H2} + b_{16}D.PRF + b_{17}D.Pne + b_{18}D.C + b_{19}D.Tr + b_{20}dr.L \\ &+b_{21}dr.Pin + b_{22}dr.PL + b_{23}dr.P_{H2} + b_{24}dr.PRF + b_{25}dr.Pne + b_{26}dr.C + b_{27}dr.Tr + b_{28}L.Pin \\ &+b_{29}L.PL + b_{30}L.P_{H2} + b_{31}L.PRF + b_{32}L.Pne + b_{33}L.C + b_{34}L.Tr + b_{35}Pin.PL + b_{36}Pin.P_{H2} \\ &+b_{37}Pin.PRF + b_{38}Pin.Pne + b_{39}Pin.C + b_{40}Pin.Tr + b_{41}PL.P_{H2} + b_{42}PL.PRF + b_{43}PL.Pne \\ &+b_{44}PL.C + b_{45}PL.Tr + b_{46}P_{H2}.PRF + b_{47}P_{H2}.Pne + b_{48}P_{H2}.C + b_{49}P_{H2}.Tr + b_{50}PRF.Pne \\ &+b_{51}PRF.C + b_{52}PRF.TR + b_{53}Pne.C + b_{54}Pne.Tr + b_{55}C.Tr + b_{56}D^2 + b_{57}dr^2 + b_{58}L^2 \\ &+b_{59}Pin^2 + b_{60}PL^2 + b_{61}P_{H2}^2 + b_{62}PRF^2 + b_{63}Pne^2 + b_{64}C^2 + b_{65}Tr^2\end{aligned} \quad (6)$$

Equation (6) includes all 10 variables as first degree terms and all possible combinations of 10 second degree terms with repetition. A total of 66 unknown regression coefficients need to be defined: $b_i,\ i = 0,1,...,65$. Of all unknown coefficients, only those which are statistically significant must be selected, in particular those with a significance Sig.<0.05 at usual level of significance 0.05. To this end a stepwise linear regression method was applied with the 65 predictors, given in (6). The stepwise procedure has the advantage of monitoring the level of significance of unknown coefficients. Each coefficient for which the condition Sig.< 0.05 is not fulfilled is removed from the equation and no longer participates in the regression analysis.

Some of the results are presented in Table 2.

It shows that in this case 12 steps were needed to calculate all statistically significant coefficients and to remove all the rest. The obtained regression coefficients of the model are statistically significant at level 0.001.

The ANOVA showed that the overall model (7) is significant with Sig.=0.000. The goodness of fit indices are as follows: the correlation coefficient is *R*=0.983 and the coefficient of determination is R^2=0.967; Std. Error of the Estimate = 6.56416.

Table 2 shows that save for the constant (b_0), of the 65 predictors in (6) only 10 statistically significant ones remain: *dr.Pin*, *Pin.C, Pin*, *Pin2, L.PRF*, *PRF.C*, *D.PL*, *dr.C*, *D.PH*2, and *C*. Of these only two are linear: *Pin* and *C*, all the rest are nonlinear. The formal approach employed once again confirms that the processes in the active laser volume exhibit strong nonlinear characteristics. The following unstandardized model equation is obtained for *Pout*:

$$\begin{aligned}\hat{P}out = &-18.308 + 0.686dr.Pin - 7.567Pin.C \\ &+27.569Pin - 4.362Pin^2 - 0.006L\,RF \\ &+0.274PRF.C - 0.030D.PL - 0.612dr.C \\ &+0.259D.P_{H2} + 18.093C\end{aligned} \quad (7)$$

Model equation (7) contains 8 of the 10 independent variables: *dr*, *D*, *L*, *Pin*, *PL*, *PRF*, *PH*2, and *C*. It does not include *PNe* and *Tr*.

This means those two quantities influence laser generation insignificantly and in future experimental studies their predetermined optimum values need to be used.

Table 2. Unstandardized (B) and standardized (Beta) coefficients obtained through stepwise regression procedure in SPSS.

Model		Unstandardized Coefficients		Standardized Coefficients			95.0% Confidence Interval for B	
		B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound
12	(Constant)	-18.308	5.291		-3.460	.001	-28.713	-7.904
	dr_Pin	0.686	0.086	1.793	7.947	.000	0.516	0.855
	Pin_C	-7.567	2.013	-0.480	-3.759	.000	-11.526	-3.609
	Pin	27.569	5.322	0.984	5.181	.000	17.105	38.033
	*Pin*_2	-4.362	0.716	-0.874	-6.089	.000	-5.770	-2.953
	L_PRF	-0.006	0.001	-0.295	-5.346	.000	-0.009	-0.004
	PRF_C	0.274	0.057	0.215	4.771	.000	0.161	0.387
	D_PL	-0.030	0.006	-0.097	-5.411	.000	-0.042	-0.019
	dr_C	-0.612	0.135	-0.523	-4.516	.000	-0.878	-0.345
	*D_PH*2	0.259	0.065	0.102	4.005	.000	0.132	0.385
	C	18.093	1.434	0.312	12.621	.000	15.275	20.912

a. Dependent Variable: *Pout*

The results from Table 2, column Beta, allows to write the so called standardized equation

$$\begin{aligned}\hat{\overline{Pout}} = {} & 1.793\overline{dr\,Pin} + 0.984\overline{Pin} - 0.874\overline{Pin}^2 \\ & - 0.523\overline{dr}.\overline{C} - 0.480\overline{PinC} \\ & + 0.312\overline{C} - 0.295\overline{LPRF} + 0.215\overline{PRFC} \\ & + 0.102\overline{DPH2} - 0.097\overline{DPL}\end{aligned} \tag{8}$$

In equation (8) the terms are included in descending order of the absolute value of their coefficients. In this equation the coefficients indicate the relative influence on *Pout* by the predictors they stand before. The application of equation (8) is discussed below.

5.2 Application of the model

The derived second-degree model will be applied for prediction of the values of *Pout* both for existing and future experiment.

5.2.1 Prediction of the examined experiments

Using model equation (7) it is possible to calculate the value of *Pout* for any experiment.

Fig. 2 compares the results of using equation (7) for prediction *Pout* for all respective experiment data. It is apparent that the confidence interval for 95% contains 96.7% of the experiment data.

As a more detailed example in Table 3 some experiment data cases are compared, including *Pout* (column 9) and calculated predicted values *PoutPre*(column 10) as per equation (7).

Column 11 shows the relative error in percent for each row. The average relative error is 7.08%. The results once again show that the nonlinearity in the processes of the laser volume is strong and there is ample ground for the development of nonlinear models.

5.2.2 Prediction of future experiment

The next step is to predict the new values of *Pout* which have not been obtained by experiments. Equation (7) can be used to predict and develop new laser sources with higher output power.

Some of the results are given in Table 4. In order to determine the direction change for the values of 8 significant quantities, we need to analyze equation (7). Table 3 shows that improving output power requires an increase of quantities D, dr, L, Pin PH2 and a decrease of PL, PRF, and C. This means that these three quantities need to be reduced so as to increase Pout. The change of their values is shown in columns 5, 7, and 8, Table 4. The calculated value of laser output power is given in column 9.

The change of laser characteristics *C* and *PRF* within 10% can lead to a change (in case of an increase) of *Pout* up to 20%. This once again shows the strong nonlinear relationship between *Pout* and on the input laser operating conditions. Such conclusions can only be drawn by developing nonlinear models.

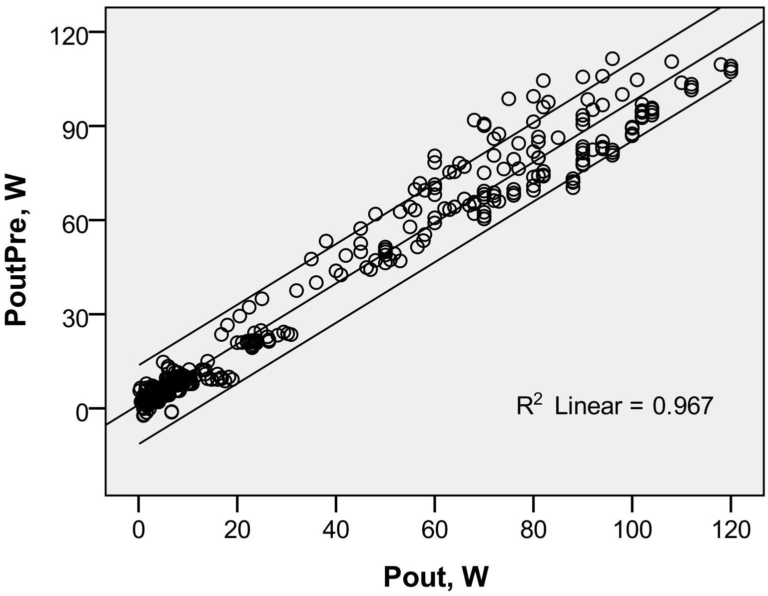

Fig. 2. Comparison between experiment values of laser output power *Pout* and those calculated using second-degree regression model (7), denoted by *PoutPre*.

Table 3. Examples of comparison of the experiment values of laser output power *Pout* with *PrePout*, obtained using equation (7).

D, mm	*dr*, mm	*L*, cm	*Pin*, kW	*PL*, W/cm	*PH*2, Torr	*PRF*, KHz	*C*, pF	*Pout*, W	*PoutPre*, W	*δ*, %
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
58	58	200	4	10.00	0.6	13	1.3	104	95.77	7.92
58	58	200	5	12.50	0.6	15	1.3	108	110.52	2.34
58	58	200	4.5	11.25	0.6	16	1.3	110	103.77	5.66
58	58	200	4.5	11.25	0.6	17.5	1.3	112	103.44	7.64
58	58	200	5	12.50	0.6	17.5	1.3	118	109.60	7.12
58	58	200	5	12.50	0.6	17.5	1.3	120	108.20	9.83
58	58	200	5	12.50	0.6	16.5	1.3	120	109.13	9.06

Table 4. Values predicted for a "hypothetical experiment" using model equation (7).

D, mm	*dr*, mm	*L*, cm	*Pin*, kW	*PL*, W/cm	*PH*2, Torr	*PRF*, KHz	*C*, pF	*PoutPre*, W
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
58	58	205	5.1	12.43902	0.6	16.0	1.30	111.73
60	58	210	5.2	12.38095	0.6	15.5	1.28	113.67
62	60	215	5.3	12.32558	0.65	15.0	1.25	122.68
65	62	220	5.4	12.27273	0.65	14.8	1.20	131.91
70	68	225	5.4	12.00000	0.7	14.0	1.18	151.80
72	70	230	5.35	11.63043	0.7	13.5	1.15	159.19

5.2.3 Identification of the most important dependences and physical interpretation

The input electrical power *Pin* is of the highest importance in equation (7). It participates a total of 4 times, on its own and as a second degree, as well as in combination with two other independent variables. This means that *Pin* has the strongest influence on laser generation. Two of the items are positive and the other two - negative. This can also be concluded from equation (8) where *Pin* participates in the first three addends with the highest relative coefficients.

In order to qualitatively consider the influence of *Pin* on *Pout*, we have assigned the following fixed values to the other independent variables: *D*=60 mm, *dr*=58 mm, *L*=210 cm, *PH*2= 0.6 Torr, *PRF*=16 kHz, *C*=1.3 pF. The resulting relationship is given in Fig. 3.

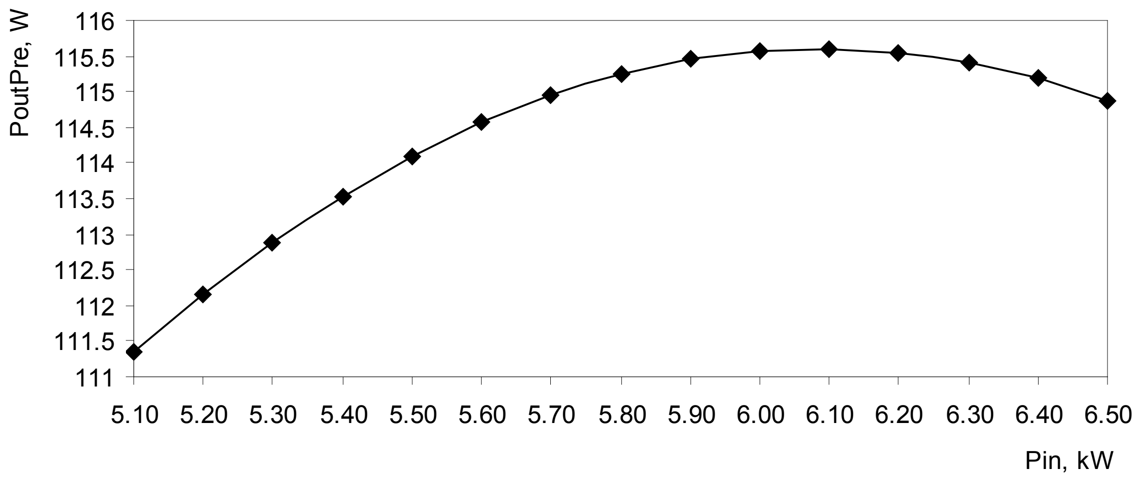

Fig. 3. Dependence of *Pout* on *Pin* at the following fixed values *D*=60mm, *dr*=58 mm, *L*=210 cm, *PH*2= 0.6 Torr, *PRF*=16 kHz , *C*=1.3 pF.

As expected, the relationship is once again nonlinear with a well-defined optimum value of *Pin*. The resulting graphic clearly reflects the physical processes occurring in the laser tube when the supplied electric power is increased. Initially its influence on the processes of laser generation is positive. This is due to the increased energy of the electrons and the population of the upper laser level. After a certain critical value the negative processes take over due to the overheating of the laser tube - thermo-ionizing instability of the gas discharge, thermo-chemical deterioration of the active compound, thermal population of the lower laser level. All this leads to decreased laser generation.

5.3 Diagnosis of the model

Any statistical model is adequate if the basic assumptions for its validity are met. In the case of multiple linear regression it is important to check the main theoretical requirements of the method [12, 13]. The main of these is the total behavior of the relationship, which can be linear or closed to linear with respect to the predictors. This is relevant by observing Fig. 2. The second rule is the multivariate normal distribution of the data. In our case this is difficult to check because of the very high dimension of the data cloud (65 predictors), and we assume this point is fulfilled.

The adequacy diagnostic of the regression model also requires the consideration of the distribution of the residuals which are calculated as differences between the experimental and predicted values of the dependent variable. The theoretical assumption for the regressions is that these are random quantities with normal distribution.

The histogram in Fig. 4 shows that the distribution of the standardized residuals for model (7), (8) is very close to normal with the average value close to zero, and a standard deviation of 0.987, which is nearly 1. The validation of this basic assumption shows that the model is adequate and significant factors are missing [3, 13].

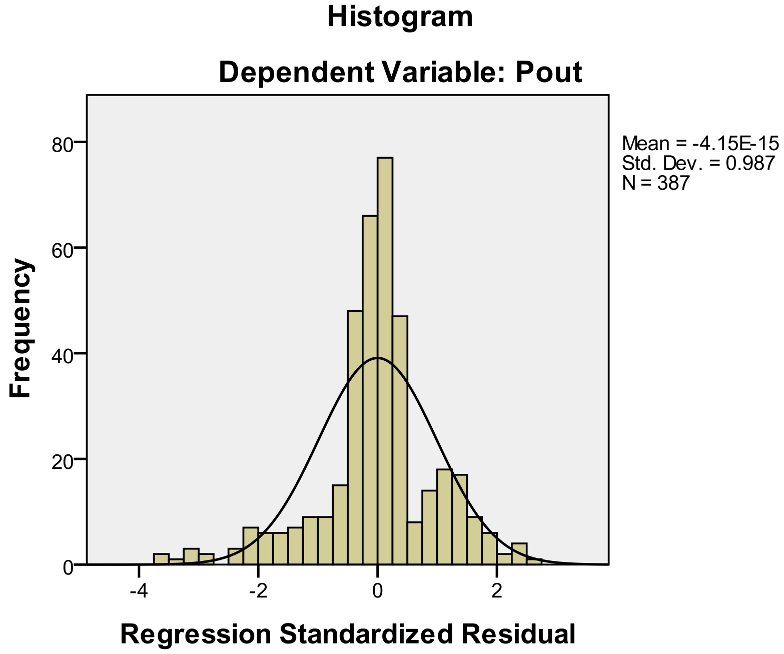

Fig. 4. Histogram of the standardized residuals of regression model (7) of *Pout*.

6 Classification analysis of variables

The derived regression model (7) indicates that out of a total of 65 predictors in the general equation (6) only 10 are statistically significant. The objective of this paragraph is to establish the degree of influence each of these quantities has on *Pout*. A partial answer to this question is given by the standardized equation (8). The value of the coefficients shows the relative participation of first and second degree items. We will use the procedures of classification analysis to investigate this influence in more detail. The goal of classification analysis is to classify the variables according to specific target groups [12].

6.1 Classification by factor analysis

Factor analysis is a statistical procedure which is used to reduce the number of variables describing the object of investigation to fewer independent ones called factors. Generally, one factor groups together variables which correlate strongly. The variables from different factors exhibit a week correlation between each other. We can distinguish 2 main objectives of factor analysis:

- defining the relationships between variables (classification of variables)
- reducing the number of variables needed to describe the data.

In order to perform factor analysis and to generate the factors we have used the method of Principal Component Analysis (PCA). The rotation of factors was carried out by Promax method [13, 3].

By means of SPSS software and by following the general procedure of factor analysis [3] the ten significant predictors in model (7) have been grouped into 4 factors (components). It was found that this number of factors describes 96.783% of the total set of experiment data, as it is seen in Table 5.

Table 5. Total variance explained for 4 factors.

Component	Initial Eigenvalues		
	Total	% of Variance	Cumulative %
1	6.737	61.250	61.250
2	1.622	14.747	75.996
3	1.505	13.684	89.680
4	0.781	7.103	96.783

The distribution of variables and their partial importance into factors is given in Table 6. The laser generation *Pout* has been added to these. As shown, 6 of the variables(*D_PH*2, *dr_Pin*, *Pin*, *Pin_*2, *dr_C* and *Pin_C*) together with *Pout* are included in the first factor. Therefore, these variables demonstrate a strong mutual correlation and exert the strongest influence on *Pout*. The remaining 4 variables (*PRF_C*, *L_PRF*, *C*, and *D_PL*) are distributed in the second, third, and fourth factor and respectively have a weaker influence on *Pout*.

Table 6. Grouping variables from equation (7) by factor with their corresponding factor loadings.[a]

Variable	Factor 1	2	3	4
Pout	1.022			
D_PH2	1.018			
dr_Pin	0.988			
Pin	0.958			
Pin_2	0.893			
dr_C	0.767			
Pin_C	0.727			
PRF_C		1.000		
L_PRF		0.797		
C			1.036	
D_PL				0.966

[a]Extraction Method: Principal Component Analysis. Rotation Method: Promax with Kaiser Normalization.

6.2 Classification by cluster analysis

Cluster analysis is a multidimensional statistical procedure which involves the division or grouping of a given set of objects into similar groups (clusters or classes). All objects assigned to a given group must be similar according to a preset criterion. Elements from different groups must be different from one another. Also, the number of groups may be unknown beforehand, and there may be no known internal structure for each group [12].

From many existing cluster analysis methods we have applied the hierarchical agglomerative method, which is appropriate for n<500. The analysis was carried out by standardized variable, assuming each variable as a point in n-dimensional vector space, n=387. In the beginning each point is considered as a single cluster. Then at each step the two more similar clusters are grouped in a new cluster and so on.

In order to quantitatively assess the concept of similarity some kind of metrics (or distance) is introduced. The distance between two points is measured. The similarity or difference between clusters is determined by the distance between them. Two objects (clusters) are identical or similar when the distance between them is zero. The greater the distance, the less they are similar or identical.

The most common distances used in cluster analysis are the usual Euclidean distance, squared Euclidean distance, Chebishev distance, etc. [3, 12]. The squared Euclidean distance between the vectors in n-dimensional vector space was used in our analysis.

Another basic moment is the cluster method of combining two clusters in a new one. This depends on the chosen formula, defined a distance between two clusters, containing one or more points. In this analysis the average linkage between groups method was used [3, 12].

The following Table 7 shows the grouping of 10 predictors from (7) and *Pout* in 2-5 clusters.

Table 7. Cluster membership of model predictors and *Pout* in 2 to 5 clusters, obtained by Average linkage method and squared Euclidean distance.

Variable	5 Clusters	4 Clusters	3 Clusters	2 Clusters
dr_Pin	1	1	1	1
Pin	1	1	1	1
Pin_2	1	1	1	1
Pout	1	1	1	1
Pin_C	1	1	1	1
dr_C	1	1	1	1
D_PH2	1	1	1	1
D_PL	3	3	1	1
C	2	2	2	1
L_PRF	4	4	3	2
PRF_C	5	4	3	2

Fig. 5 shows a dendrogram constructed using the cluster method Average linkage (Between groups) for 10 significant variables and *Pout*. It is observed that the bigger gap between clusters is in the rescaled region [10, 19]. This gives the best cluster solution of four clusters, respectively:

Cluster 1: {*Pin*, *dr_Pin*, *Pin_2*, *Pout*, *dr_C*, *Pin_C*, *D_PH2*};
Cluster 2: {*L_PRF*, *PRF_C*};
Cluster 3: {*D_PL*};
Cluster 4: {*C*}.

This classification is the same as per factor analysis.

6.3 Discussion on classification results

The summary of equation (7) as well as the results from factor and cluster analyses indicate that the defining laser characteristic for the behavior of laser output power *Pout* is the supplied electric power *Pin*, on its own, in second degree form, as well as in second degree combinations with other independent laser characteristics. The second more important influence have the following characteristics: geometrical dimensions *D* and *dr* (inside diameters of the tube and of the diaphragms, respectively),

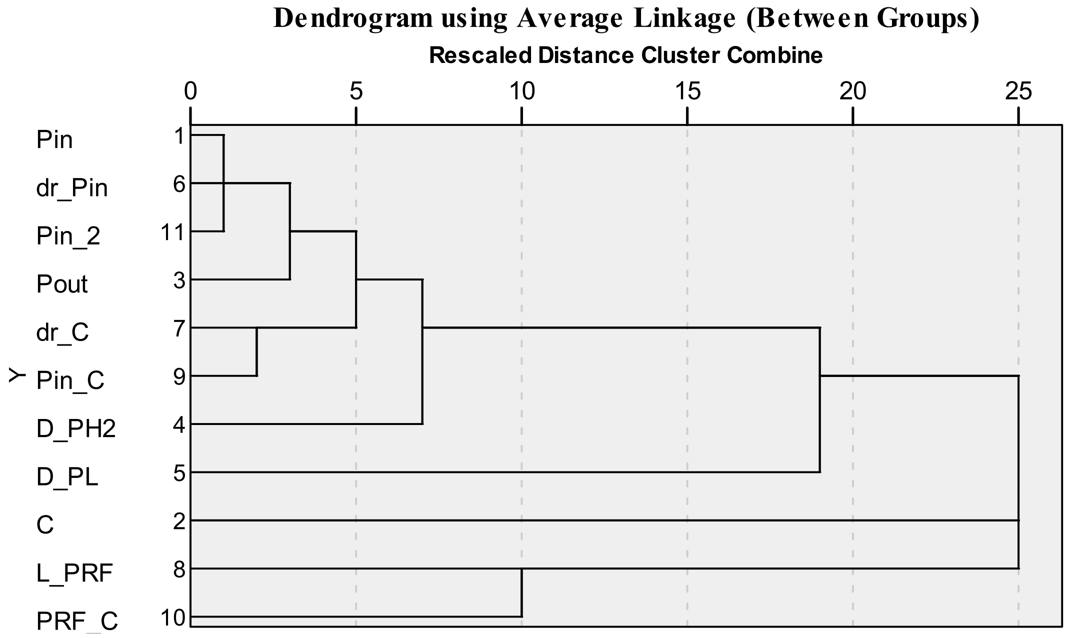

Fig. 5. Dendrogram from cluster analysis showing the influence if predictors in (7) on laser generation *Pout*.

the hydrogen pressure *PH*2 and the equivalent capacitance of the capacitor battery *C*.

These results have to be taken into account in planning and guiding experiments in order to increase the average laser generation *Pout* of existing and future copper bromide lasers of the investigated type.

7 Conclusion

It is the first time a parametric nonlinear polynomial model of second degree has been developed for the output power of a CuBr laser by using 10 input laser characteristics. By applying the stepwise procedure a regression model is derived for 65 initial predictors. The model includes 8 out of a total of 10 input independent variables. The adequacy of the developed regression model has been checked. It has been established that with regard to high-powered lasers there is very good fit between experiment results and the predictions obtained using the model with an average relative error of 7%. The influence of 8 significant variables on laser generation has been analyzed. It has been determined that the supplied electric power *Pin* has the strongest influence on laser generation. The developed regression model has been used to predict new, non-existent laser sources. This would reduce the time and costs needed for the development of laser devices with higher output generation.

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