Stability of Multidimensional Systems Using Genetic Algorithms

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***Abstract*—The study of the stability of m-dimensional systems is a difficult one especially when $m \geq 3$. There exist only a few results in the literature and unfortunately, there does not exist any practical criterion. In this brief, the stability of an m-dimensional system is dealt as a minimization problem of the absolute value of its characteristic polynomial over the boundaries of its variables (i.e., on the m unit circles). This minimization problem is solved by using genetic algorithms (GAs). Using GAs we obtain, in general, better results than other methods of minimization (numerical techniques, neural networks, etc.). Numerical examples are presented.**

I. Introduction

In the study of systems theory, stability plays an important role, since every designed system ought to be stable. A one-dimensional (1-D) discrete-time system is stable (in the bounded-input–bounded-output sense) if and only if its characteristic polynomial is devoid of any roots inside the unit disk and has no multiple roots on the unit circle. In the system theory literature, this kind of stability is also known as Schur stability. Also, for practical purposes and applications, there exist many tests such as Jury's and Hurwitz's tests that check the stability without finding the roots of the characteristic polynomial.

A multidimensional (m-D) linear, shift-invariant, discrete variable system described by the transfer function

$$G(z_1, \ldots, z_m) = \frac{A(z_1, \ldots, z_m)}{B(z_1, \ldots, z_m)} \tag{1}$$

and having no nonessential singularity of the second kind [1], [2] is stable (in the bounded-input–bounded-output sense) if and only if

$$\begin{aligned}
&B(0,\ldots,0,z_m)\neq 0, && \text{for } |z_m|\leq 1\\
&B(0,\ldots,0,z_{m-1},z_m)\neq 0, && \text{for } |z_{m-1}|\leq 1, |z_m|=1\\
&\quad\vdots\\
&B(0,z_2,\ldots,z_{m-1},z_m)\neq 0, && \text{for } |z_2|\leq 1, |z_3|=\ldots=|z_m|=1\\
&B(z_1,z_2,\ldots,z_m)\neq 0, && \text{for } |z_1|\leq 1, |z_2|=\ldots=|z_m|=1.
\end{aligned} \tag{2}$$

The above theorem is known as the theorem of *Anderson and Jury* [3], [4]. Unfortunately, for practical purposes (filtering, design of m-D filters, etc.,) we need some more practical tests than the above theorem. In two-dimensional (2-D) systems, a great variety of practical tests have been produced in the last three decades (Jury's 2-D test [1], [3], Schur–Cohn test [1], [3], Inners' test [5], Zeheb–Walach test [6], [7], Mastorakis–Barnett test [8], [9], Partial Energies' test [10], etc.). There are also a variety of special results and other considerations [11]–[13].

In m-D systems $(m > 2)$, unfortunately, we have a complete lack of such tests, though we must refer to the contributions of [4]–[7] and [14]–[22]. Thus, it is difficult to check if a given m-D polynomial $B(z_1, \ldots, z_m)$ corresponds to the characteristic polynomial of a stable m-D system when $m > 2$. In the sequel, by the term stable or unstable polynomial, we mean the characteristic polynomial of a stable or unstable m-D (linear, shift-invariant, discrete variables) system. An important result in the stability of m-D system is given by the following theorem, known as DeCarlo–Strintzis theorem [1], [3], [14].

DeCarlo–Strintzis Theorem: $B(z_1, \ldots\ldots, z_m)$ is a stable polynomial if and only if

$$B(z_1, 1, \ldots, 1) \neq 0, \quad \text{for} |z_1| \leq 1 \tag{3.1}$$

$$B(1, z_2, 1, \ldots, 1) \neq 0, \quad \text{for} |z_2| \leq 1 \tag{3.2}$$

$$\vdots$$

$$B(1, \ldots, 1, z_m) \neq 0, \quad \text{for} |z_m| \leq 1 \tag{3.m}$$

$$B(z_1, \ldots, z_m) \neq 0, \quad \text{for} |z_1| = \ldots = |z_m| = 1 \tag{3.m+1}$$

and the transfer function for which $B(z_1, \ldots\ldots, z_m)$ is the denominator has no nonessential singularity of the second kind.

In this brief, we will assume that the condition of the nonexistence of nonessential singularities of the second kind is fulfilled. The m first conditions of DeCarlo–Strintzis theorem actually consist of m 1-D conditions and are easy to be checked via any 1-D test (for example, the 1-D Jury test). In order to check the last equation of the DeCarlo–Strintzis' theorem, a methodology based on genetic algorithms (GAs) is proposed here. This methodology is presented in the next section.

II. GA for Checking the m-D Systems Stability

According to the DeCarlo–Strintzis Theorem, the first m conditions can be examined via any 1-D test (criterion). If some of these conditions are not satisfied, we easily conclude that the system is unstable, without examining the last condition (3.m+1). However, if these m conditions are fulfilled, then condition (3.m+1) is that which will "decide" the stability. If it is satisfied, then the system is stable; otherwise, it is unstable. In order to investigate the condition (3.m+1), we consider the minimum of the function f, where $f = f(w_1, w_2, \ldots, w_m) = |B(e^{jw_1}, e^{jw_2}, \ldots, e^{jw_n})|$. So, assuming that

$$\underline{M} = \min\ f \tag{4}$$

over w_i $0 \leq w_i \leq 2\pi$, the condition (3.m+1) is equivalent to

$$\underline{M} > 0. \tag{5}$$

If $\underline{M} = 0$, the polynomial $B(z_1, \ldots, z_m)$ is unstable. In general, the problem in question is how to find M avoiding any "trap" of local minima. Some of the existing methods of minimum search (numerical or neural networks' techniques) usually give only local minima. On the other hand, GAs can find the global minimum $\underline{M}$ in many cases, though such a convergence cannot always be guaranteed.

A brief overview of the theory of GAs is as follows. GAs are search algorithms, which initially were inspired by the process of natural genetics (reproduction of an original population, performance of crossover and mutation, selection of the best). The main idea for an optimization problem is to start our search not with one initial point, but with a population of initial points. The $2n$ numbers (parents) of this initial set (called population, quite analogously to the biological system) are converted to the binary system. In the sequel, they are considered as chromosomes (actually sequences of 0 and 1) "Parents" come to "reproduction" where they interchange parts of their "genetic material". This procedure is called crossover. Moreover a very small probability for a mutation exists. (Mutation is the phenomenon where quite randomly—with a very small probability though—a 0 becomes 1 or a 1 becomes 0). Assume that every pair of "parents" gives rise to

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k "children." By the process of reproduction, the population of the "parents" is enhanced by the "children" and we have an increase in the original population because new members have been added (parents always belong to the population considered). The new population has now $2n + kn$ members. Then, the process of *natural selection* is applied. According to the concept of natural selection, from the $2n + kn$ members, only $2n$ survive. These $2n$ members are selected as the members with the highest values of f, if we are attempting to *maximize* f (or with the lowest values of f, if we are attempting to *minimize* f). By repeated iterations of reproduction (under crossover and mutation) and natural selection, we can find the minimum (or maximum) of f as the point to which the best values of our population converge. The termination criterion is fulfilled if the mean value of f in the $2n$-members population is no longer improved (maximized or minimized), or if the number of iterations is greater than the maximum number of iterations $N_{\max}$, which is defined by us. A more detailed overview of GAs can be found in [23], [24].

In our problem of m-D systems stability, we wish to minimize f over $w_1, w_2, \ldots, w_m$ when $w_i \in [0, 2\pi]$, $i = 1, \ldots, \mathrm{m}$. To this end, $w_1, w_2, \ldots, w_m$ are converted to the binary system and are considered as part of a big chromosome. If we assume that every w_i is converted to a t-bits binary number, we need mt bits for the "chromosome" of $w_1, w_2, \ldots, w_m$. Our search starts with a randomly generated population of such $2n$ chromosomes. In quite a random manner, this population is split into pairs of parents that will be crossed, i.e., they will interchange their genetic material (with c crossovers) always under a small probability p for mutation (for example $p = 0.01$). By this reproduction, a new population of $2n + kn$ members will be formed, since each pair of parents give birth to k children. The new population is filtered and only the $2n$ better members (here "better" means the $2n$ lowest values of $f(w_1, w_2, \ldots, w_n)$) are retained in the population, and the others deleted. By repeated iterations of reproduction (under crossover and mutation) and natural selection, we can find the minimum of $f(w_1, w_2, \ldots, w_n)$, $0 \le w_i \le 2\pi$, $i = 1, \ldots, m$ as the point to which the best values of our population converge. The termination criterion is "the mean value of f in the population is no longer improved." The algorithm is summarized as follows.

STEP A: Find (randomly) the initial population of $2n$ members.

STEP B: Split the population (randomly) into n pairs.

STEP C: Make c crossovers and from each pair of parents take k children. Every bit of every child has a probability of p for a mutation.

STEP D: Find the new population $2n + 2k$ (parents + children).

STEP E: From the new population, select the $2n$ members with the lowest values of f.

STEP F: If the absolute value of the difference between the mean value of f in the population of this generation and the mean value of f in the population of the previous generation is $< \varepsilon$, or the number of iterations is greater than the maximum number of iterations, say, $N_{\max}$, (which is defined by us), then STOP; otherwise go to STEP C.

No technique or method in the area of global optimization can always guarantee the convergence to the global minimum. However, in most cases the evolutionary computation (GA) leads to convergence to the global minimum, and the strategy of evolutionary computation has proved to be better than the conventional numerical methods with regard to the convergence to the global minimum. However, the convergence is slower, but in most cases, we can find the global minimum.

The GA used here is the basic GA, and one can use more sophisticated schemata. In many cases, GAs find the global minimum of the minimization problem in question, in spite of the fact of its slow-convergence speed. While running the proposed GA, if we find that the minimum of

$$f = f(w_1, w_2, \ldots, w_m) = \left| B(e^{jw_1}, e^{jw_2}, \ldots\ldots, e^{jw_n}) \right|$$

is 0, then, the polynomial in question is unstable. However, if we find that the minimum of f is not zero, then we can guarantee that our polynomial is stable. The reason is that our function f is a function of $\cos(N_1{}^* w_1)$, $\sin(N_1{}^* w_1)$, $\cos(N_2{}^* w_2)$, and $\sin(N_2{}^* w_2)$, where N_1 and N_2 are the degrees of the polynomials with respect to z_1 and z_2 respectively, and $0 \le w_i \le 2\pi$, $i = 1, 2$.

Hence, taking appropriate equispaced initial population from 0 to 2π, for example, $2^* N_1$ points for w_1 and $2^* N_2$ for w_2, we avoid being trapped in a local minimum.

In our examples, we have used t from $t = 12$ up to $t = 20$. For examples up to ten variables $(m = 10)$, no convergence problem has been observed. The convergence is achieved after about 100–350 iterations. The time necessary to run the computer program on a PC Pentium 4 (2.4 GHz) is between 1 to 3 min, which is very satisfactory.

Example 1: Suppose that our 3-D system (without any nonessential singularity of the second kind), has the following characteristic polynomial:

$$B(z_1, z_2, z_3) = 0.8z_1 + 1.5z_1^2 z_2 + 1.8z_2^3 + 0.2z_3 + 1.3z_2 z_3^2 + 5.6.$$

Then, the first three conditions, i.e., (3.1)–(3.3) of the DeCarlo–Strintzis theorem are satisfied for $|z_i| \le 1$ with i =1, 2, and 3, respectively, since

$$\begin{aligned} B(z_1, 1, 1) &= 1.5z_1^2 + 0.8z_1 + 8.9 \neq 0 \\ B(1, z_2, 1) &= 1.8z_2^3 + 2.8z_2 + 6.6 \neq 0 \\ B(1, 1, z_3) &= 1.3z_3^2 + 0.2z_3 + 9.7 \neq 0. \end{aligned}$$

So, we have to examine the last equation in the DeCarlo–Strintzis theorem. To this end, let us consider $f = f(w_1, w_2, \ldots, w_m) = |B(e^{jw_1}, e^{jw_2}, \ldots\ldots, e^{jw_n})|$. We easily find that $f = \sqrt{Q_1^2 + Q_2^2}$, where

$$\begin{aligned} Q_1 = {} & 0.8\cos(w_1) + 1.5\cos(2w_1 + w_2) + 1.8\cos(3w_2) \\ & + 0.2\cos(w_3) + 1.3\cos(w_2 + 2w_3) + 5.6 \\ Q_2 = {} & 0.8\sin(w_1) + 1.5\sin(2w_1 + w_2) + 1.8\sin(3w_2) \\ & + 0.2\sin(w_3) + 1.3\sin(w_2 + 2w_3). \end{aligned}$$

Using now the previously presented GA with $n = 5$, $k = 4$, $t = 12$, $p = 0.01$, $c = 6$, and $\varepsilon = 10^{-4}$, we find that the optimum value of f as well as the mean value of f in each generation converges to zero (see Fig. 1). Therefore, for this example $\underline{M} = 0$ and the polynomial is (Schur) unstable.

Example 2: Suppose that our 3-D system (without any nonessential singularity of the second kind), has the following characteristic polynomial:

$$B(z_1, z_2, z_3) = z_1^2 + z_2^2 + z_3 - z_1 z_2 z_3 + 5.$$

The first three conditions, i.e., (3.1)–(3.3) of the DeCarlo–Strintzis theorem are satisfied for $|z_i| \le 1$ where $i = 1, 2, 3$ respectively, since

$$\begin{aligned} B(z_1, 1, 1) &= z_1^2 - z_1 + 7 \neq 0 \\ B(1, z_2, 1) &= z_2^2 - z_2 + 7 \neq 0 \\ B(1, 1, z_3) &= 7 \neq 0. \end{aligned}$$

Hence, we have to examine the last equation in the DeCarlo–Strintzis theorem. As in Example 1, one has $f = \sqrt{Q_1^2 + Q_2^2}$, where

$$\begin{aligned} Q_1 &= \cos(2w_1) + \cos(2w_2) + \cos(w_3) - \cos(w_1 + w_2 + w_3) + 5 \\ Q_2 &= \sin(2w_1) + \sin(2w_2) + \sin(w_3) - \sin(w_1 + w_2 + w_3). \end{aligned}$$

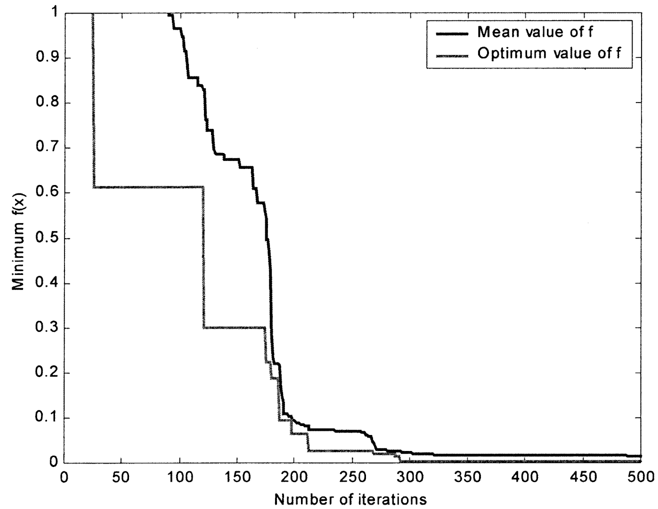

Fig. 1. Convergence of the optimum value of $\boldsymbol{f}$ as well as that of the mean value of $\boldsymbol{f}$ in every generation in Example 1.

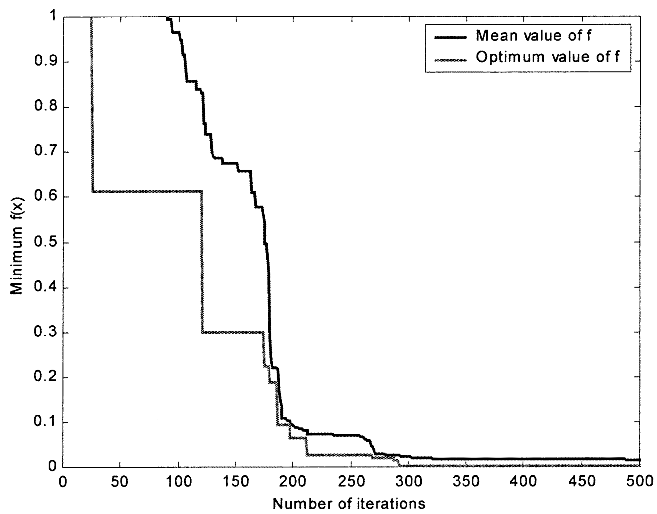

Fig. 2. Convergence of the optimum value of $\boldsymbol{f}$ as well as that of the mean value of $\boldsymbol{f}$ in every generation in Example 2.

Using the GA presented here with $n = 5, k = 4, t = 12, p = 0.01$, $c = 6$, and $\varepsilon = 10^{-4}$, we obtain that the optimum value of f as well as the mean value of f in each generation converges to unity (see Fig. 2). Therefore, for this example $\underline{M} > 0$, and the polynomial is (Schur) stable.

Example 3: Let our five-dimensional system (without any nonessential singularity of the second kind), has the following characteristic polynomial:

$$B(z_1, z_2, z_3, z_4, z_5) = z_1^2 z_3^3 + z_3^3 z_4^2 + z_1^3 z_2 z_5 + z_1 z_2 z_3 z_4 z_5 + 5.$$

The first five conditions, i.e., (3.1)–(3.5), of the DeCarlo–Strintzis theorem are satisfied for $|z_i| \leq 1$ where $i = 1, 2, 3, 4$, and 5, respectively, because

$$\begin{aligned}
B(z_1, 1, 1, 1, 1) &= z_1^3 + z_1^2 + z_1 + 7 \neq 0 \\
B(1, z_2, 1, 1, 1) &= 2z_2 + 8 \neq 0 \\
B(1, 1, z_3, 1, 1) &= 2z_3^3 + z_3 + 7 \neq 0 \\
B(1, 1, 1, z_4, 1) &= z_4^2 + z_4 + 8 \neq 0 \\
B(1, 1, 1, 1, z_5) &= z_5^3 + 2z_5 + 7 \neq 0.
\end{aligned}$$

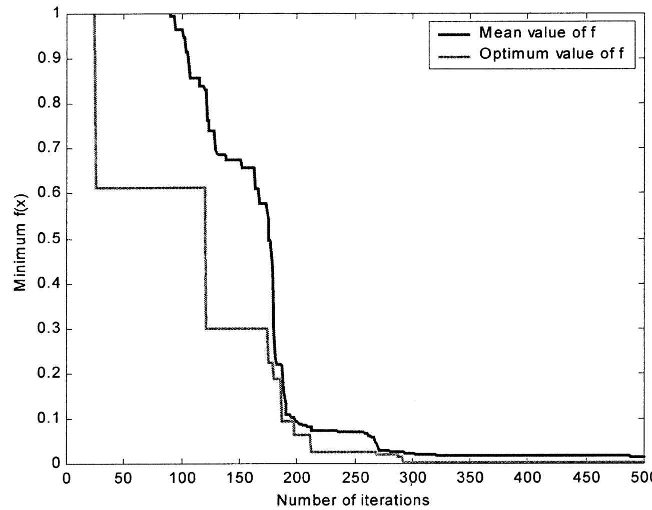

Fig. 3. Convergence of the optimum value of $\boldsymbol{f}$ as well as that of the mean value of $\boldsymbol{f}$ in every generation in Example 3.

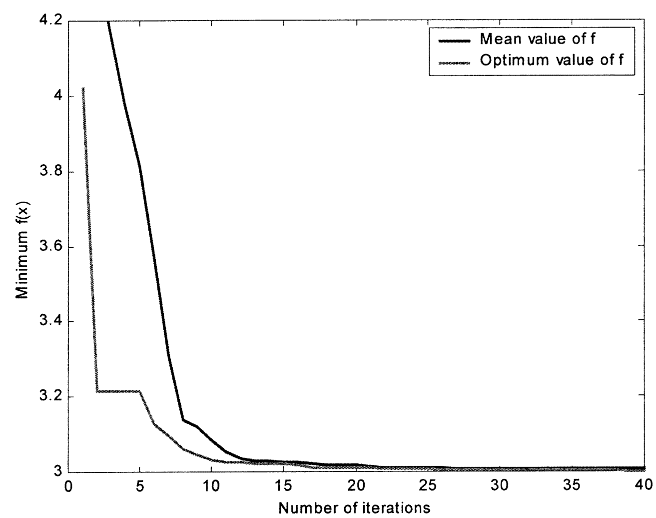

Fig. 4. Convergence of the optimum value of $\boldsymbol{f}$ as well as that of the mean value of $\boldsymbol{f}$ in every generation in Example 4.

So, we have to examine the last equation in the DeCarlo–Strintzis theorem. Here, $f = \sqrt{Q_1^2 + Q_2^2}$, where

$$\begin{aligned}
Q_1 &= \cos(2w_1 + 3w_3) + \cos(3w_3 + 2w_4) + \cos(3w_3 + w_2 + w_5) \\
&\quad + \cos(w_1 + w_2 + w_3 + w_4 + w_5) + 5 \\
Q_2 &= 2\cos(2w_1) + \sin(3w_2) + \sin(4w_3) \\
&\quad - \sin(5w_5) + 4\sin(w_4) + \sin(w_1 + w_2 + w_3).
\end{aligned}$$

Using now the GA with $n = 5, k = 4, t = 12, p = 0.01, c = 6$, and $\varepsilon = 10^{-4}$, we obtain that the optimum value of f as well as the mean value of f in each generation converges to zero (see Fig. 3). Therefore, for this example $\underline{M} = 0$ and the polynomial is (Schur) unstable.

Example 4: Let our 2-D system (without any nonessential singularity of the second kind), has the following characteristic polynomial [26]:

$$\begin{aligned}
B(z_1, z_2) = 6.5 + z_2 + 0.4z_2^2 + 0.4z_1 + 0.8z_1 z_2 \\
-0.5z_1 z_2^2 + 0.2z_1^2 - z_1^2 z_2 + z_1^2 z_2^2.
\end{aligned}$$

The first two conditions, i.e., (3.1), (3.2) of the DeCarlo–Strintzis theorem are satisfied for $|z_i| \leq 1$ where $i = 1, 2$ respectively, since

$$B(z_1, 1) = 0.2z_1^2 + 0.7z_1 + 7.9 \neq 0$$
$$B(1, z_2) = -0.1z_2^2 + 1.8z_2 + 7.1 \neq 0.$$

So, we have to examine the last equation in the DeCarlo–Strintzis theorem. As in Example 1, one has $f = \sqrt{Q_1^2 + Q_2^2}$, where

$$\begin{aligned} Q_1 = {} & \cos(w_2) + 0.4\cos(2w_2) + 0.4\cos(w_1) \\ & + 0.8\cos(w_1 + w_2) - 0.5\cos(w_1 + 2w_2) \\ & + 0.2\cos(2w_1) - \cos(2w_1 + w_2) \\ & + \cos(2w_1 + 2w_2) + 6.5 \\ Q_2 = {} & \sin(w_2) + 0.4\sin(2w_2) + 0.4\sin(w_1) \\ & + 0.8\sin(w_1 + w_2) - 0.5\sin(w_1 + 2w_2) \\ & + 0.2\sin(2w_1) - \sin(2w_1 + w_2) + \sin(2w_1 + 2w_2). \end{aligned}$$

Using the GA presented here with $n = 20$, $k = 4$, $t = 16$, $p = 0.1$, $c = 6$, and $\varepsilon = 10^{-4}$, we obtain that the optimum value of f as well as the mean value of f in each generation converges to 3 (see Fig. 4). Therefore, for this example, $\underline{M} > 0$, and the polynomial is (Schur) stable. The stability of this example has also been shown in [26].

III. Conclusion

It has been shown that GAs could prove to be a new useful tool for checking the stability of m-D $(m \geq 3)$ systems. First, the m-D stability problem is reduced to an appropriate minimization problem by using the last condition of the DeCarlo–Strintzis theorem. Then, this minimization problem of the absolute value of its characteristic polynomial over the boundaries of its variables (i.e., on the m unit circles) can be tackled using GAs. The GA presented in this brief is the basic one, and we can use far more sophisticated schemata. In most cases the GAs find the global minimum of the minimization problem in question, in spite of its slow speed of convergence. Investigation of the concepts such as the m-D stability threshold or stability margin is left for future research. Furthermore, the present method can be improved if we use some heuristic techniques or other sophisticated techniques like the method of *variant mutation*, *zooming*, etc [25].

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