The following question arises naturally while applying mathematical and computational methods in science and engineering: what errors we commit when we replace functions that satisfy some equations only approximately by the exact solutions to those equations. Quite effective tools to evaluate such errors are provided in the theory of Ulam stability (also often called Hyers-Ulam stability). It has been motivated by a question posed by S.M. Ulam in 1940 (see [2, 4, 14, 15]) and concerning approximate homomorphisms of metric groups. At present, it is a large field of research and concerns various types of equations (e.g., difference, differential, functional, integral). Roughly speaking, nowadays, we understand such stability of an equation in the following way:

When a function satisfying an equation approximately (in some sense) must be near an exact solution to the equation?

The talk contains some basic definitions and examples of (early and recent) results, concerning mainly difference, differential, and integral equations (see, e.g., [1]–[17]). For instance, the following definition describes the main ideas of the notions of stability and hyperstability (see [4]).

**Definition 1.** Let $A$ be a nonempty set, $(X, d)$ be a metric space, $E \subset C \subset \mathbb{R}_+^A$ be nonempty, $T$ be an operator mapping $C$ into $\mathbb{R}_+^A$ and $F_1, F_2$ be operators mapping a nonempty set $D \subset X^A$ into $X^A$.

We say that the equation

$$F_1\varphi(x_1, \ldots, x_n) = F_2\varphi(x_1, \ldots, x_n)$$  \hspace{1cm} (1)

is $(E, T)$-stable provided for any $\varepsilon \in E$ and $\varphi_0 \in D$ with

$$d(F_1\varphi_0(x_1, \ldots, x_n), F_2\varphi_0(x_1, \ldots, x_n)) \leq \varepsilon(x_1, \ldots, x_n), \quad x_1, \ldots, x_n \in A,$$  \hspace{1cm} (2)

there exists a solution $\varphi \in D$ of equation (1) such that

$$d(\varphi(x), \varphi_0(x)) \leq T\varepsilon(x), \quad x \in A.$$  \hspace{1cm} (3)

Next, given $\varepsilon \in \mathbb{R}_+^A$, we say that equation (1) is $\varepsilon$-hyperstable provided every $\varphi_0 \in D$, satisfying (2), fulfills equation (1).

The notion of non-stability is discussed, as well, on the examples of the difference equation of the form

$$x_{n+1} = F(x_n),$$

and its generalizations (also of higher orders).
References

Janusz Brzdęk

Present permanent employment: Department of Mathematics, Pedagogical University, Kraków, Poland; position of professor

Major research interests: functional equations and inequalities with their applications, Ulam’s type stability (e.g., of difference, differential, functional, integral and operator equations), real and functional analysis, fixed point theory

Author of more than 100 published papers

Chairman of the Scientific Committee of the series of conferences: *International Conference on Functional Equations and Inequalities* (ICFEI)


Chairman of the Scientific and Organizing Committees of *Conference on Ulam’s Type Stability* in Ustroń (Poland, June 2-6, 2014, http://cuts.up.krakow.pl/2014/) and Chairman of the Scientific Committee of Conference on Ulam’s Type Stability 2016 in Cluj-Napoca (Romania, July 4-9, 2016, http://cuts.up.krakow.pl/)

Member of the Program or Scientific Committees of numerous other international conferences

Editor (jointly with Th.M. Rassias) of the monograph *Functional Equations in Mathematical Analysis* (nearly 750 pages; collection of 47 papers of 67 authors), volume 52 (2013) of *Springer Optimization and Its Applications* series, dedicated to the 100th anniversary of S.M. Ulam

Lead Editor of Banach Center Publications volume 99 (2013) titled: *Recent Developments in Functional Equations and Inequalities. Selected Topics*

Lead Guest Editor of Abstract and Applied Analysis annual special issues: *Ulam’s Type Stability* (http://www.hindawi.com/journals/aaa/type.stability/) in the years 2012, 2013

Lead Guest Editor of Journal of Function Spaces (formerly: Journal of Function Spaces and Applications) special issue: *Ulam’s Type Stability and Fixed Points Methods* (http://www.hindawi.com/journals/jfs/si/329604/cfp/)

Lead Guest Editor of Discrete Dynamics in Nature and Society special issue: *Approximate and Iterative Methods* (http://www.hindawi.com/journals/ddns/si/473241/)

Supervisor of five promoted PhD students

Editor of several international journals

Plenary speaker of several international conferences