Editors
Valeri Mladenov
Nikos Mastorakis

Information Theoretic Models and Their Applications

by Prof. Om Parkash

ISBN: 978-1-61804-371-9

Information Theoretic Models and Their Applications

Editors
Prof. Valeri Mladenov
Prof. Nikos Mastorakis

Author

Om Parkash

Published by WSEAS Press
www.wseas.org

ISBN: 978-1-61804-371-9

Information Theoretic Models and their Applications

Published by WSEAS Press
www.wseas.org

Copyright © 2016, by WSEAS Press

All the copyright of the present book belongs to the World Scientific and Engineering Academy and Society Press. All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the Editor of World Scientific and Engineering Academy and Society Press.

All papers of the present volume were peer reviewed by two independent reviewers. Acceptance was granted when both reviewers' recommendations were positive.
See also: http://www.worldses.org/review/index.html

ISBN: 978-1-61804-371-9

World Scientific and Engineering Academy and Society

Preface

The objective of the present book entitled “Information Theoretic Models and Their Applications” is to acquaint the readers with the quantitative measure of information theoretic entropy discovered by well known American Mathematician C.E. Shannon. This discovery has played an increasingly significant role towards its applications in various disciplines of Science and Engineering. On the other hand, peculiar to information theory, fuzziness is a feature of imperfect information which gave birth to fuzzy entropy, loosely representing the information of uncertainty, and was introduced by an eminent American Electrical Engineer, Lofti Zadeh. The measures of entropy for probability and fuzzy distributions have a great deal in common and the knowledge of one may be used to enrich the literature on the other and vice-versa. The present manuscript provides the contribution of both types of entropy measures.

The two basic concepts, viz, entropy and coding are closely related to each other. In coding theory, we develop optimal and uniquely decipherable codes by using various measures of entropy, and these codes find tremendous applications in defense and banking industry. Another idea providing a holistic view of problems comes under the domain of Jaynes “Maximum Entropy Principle” which deals with the problems of obtaining the most unbiased probability distributions under a set of specified constraints. The contents of the book provide a study of uniquely decipherable codes and the maximum entropy principle.

It is worth mentioning here that engineers, scientists, and mathematicians want to experience the sheer joy of formulating and solving mathematical problems and thus have very practical reasons for doing mathematical modeling. The mathematical models find tremendous applications through their use in a number of decision-making contexts. This is to be emphasized that the use of mathematical models avoids intuition and, in certain cases, the risk involved, time consumed and the cost associated with the study of primary research. The book provides a variety of mathematical models dealing with discrete probability and fuzzy distributions.

I am thankful to Guru Nanak Dev University, Amritsar, India, for providing me sabbatical leave to write this book. I am also thankful to my wife Mrs. Asha, my daughter Miss Tanvi and my son Mr. Mayank for their continuous encouragements towards my

academic activities and also for providing the congenial atmosphere in the family for writing this book. I would like to express my gratitude to my research scholars, Mr. Mukesh and Ms. Priyanka Kakkar, Department of Mathematics, Guru Nanak Dev University, Amritsar, India, for their fruitful academic discussions and efforts made in meticulous proof reading for the completion of the book project. I shall be failing in my duty if I do not thank the WSEAS publishing team for their help and cooperation extended in publishing the present book.

I have every right to assume that the contents of this reference book will be useful to the scientists interested in information theoretic measures, and using entropy optimization problems in a variety of disciplines. I would like to express my gratitude for the services rendered by eminent reviewers for carrying out the reviewing process and their fruitful suggestions for revising the present volume. I sincerely hope that the book will be a source of inspiration to the budding researchers, teachers and scientists for the discovery of new principles, ideas and concepts underlying a variety of disciplines of Information Theory. Also, it will go a long way, I expect, in removing the cobwebs in the existing ones. I shall be highly obliged and gratefully accept from the readers any criticism and suggestions for the improvement of the present volume.

Om Parkash
Professor, Department of Mathematics
Guru Nanak Dev University, Amritsar, India

Forward

The book "Information Theoretic Models and their Applications" written by Dr. Om Parkash, Professor of Mathematics, Guru Nanak Dev University, Amritsar, India, is an advanced treatise in information theory. This volume will serve as a reference material to research scholars and students of mathematics, statistics and operations research. The scholarly aptitude of Dr. Om Parkash is evident from his high rated contributions in the field of information theory. He is a meticulous, methodical and mellowed worker, with an in depth knowledge on the subject.

Dr. R.K.Tuteja
Ex-Professor of Mathematics
Maharshi Dayanand University, Rohtak, India
President, Indian Society of Information
Theory and Applications

Table of Contents

Preface

Forward

1. Information Theoretic Measures Based Upon Discrete Probability Distributions 1

1.1 Introduction 1

1.2 A new generalized probabilistic measure of entropy 7

1.3 New generalized measures of weighted entropy 10

1.4 Probabilistic measures of directed divergence 17

2. Generalized Measures of Fuzzy Entropy-Their Properties and Contribution 28

2.1 Introduction 28

2.2 New measures of fuzzy entropy for discrete fuzzy distributions 31

2.3 Monotonic character of new measures of fuzzy entropy 35

2.4 Partial information about a fuzzy set-a measurement 38

2.5 Generating functions for measures of fuzzy entropy 42

2.6 Normalizing measures of fuzzy entropy 45

3. New Generalized Measures of Fuzzy Divergence and Their Detailed Properties 50

3.1 Introduction 50

3.2 New generalised measures of weighted fuzzy divergence 52

3.3 Generating measures of fuzzy entropy through fuzzy divergence measures 60

3.4 Some quantitative-qualitative measures of crispness 65

3.5 Generating functions for various weighted measures of fuzzy divergence 68

4. Optimum Values of Various Measures of Weighted Fuzzy Information 73

4.1 Introduction 73

4.2 Optimum values of generalized measures of weighted fuzzy entropy 76

4.3 Optimum values of generalized measures of weighted fuzzy divergence 82

5. Applications of Information Measures to Portfolio Analysis and Queueing Theory 94

5.1 Introduction 94

5.2 Development of new optimization principle 99

5.3 Measuring risk in portfolio analysis using parametric measures of divergence 102

5.4 Applications of information measures to the field of queueing theory 104

6. New Mean Codeword Lengths and Their Correspondence with Information Measures 114

6.1 Introduction 114

6.2 Development of exponentiated codeword lengths through measures of divergence 120

6.3 Derivations of well known existing codeword lengths 125

6.4 Development of information theoretic inequalities via coding theory and measures of divergence 130

6.5 Generating possible generalized measures of weighted entropy via coding theory 133

6.6 Measures of entropy as possible lower bounds 145

7. A Study of Maximum Entropy Principle for Discrete Probability Distributions 151

7.1 Introduction 151

7.2 Maximum entropy principle for discrete probability distributions 156

7.3 Maximum entropy principle for approximating a given probability distribution 171

Index 178

A

Abdel-Fadel, A. M. A.
Aczel, J. 5
Additivity property 9
Arimoto, S. 145
Arithmetic mean 16, 116,157, 167, 176
Arrival process 105
Autar, R. 145

B

Baciu, G. 156
Behara, M. 145
Belis, M. 10, 52, 68, 127, 133
Bera, A. K. 95
Bertsimas, D. 97
Best 1-1 code 118
Bezdek, J. C. 31, 52
Bhandari, D. 30, 51, 60, 70, 74
Bhattacharya, A. 97
Biology 6, 97
Birth-death process 105
Birth rate 106
Boltzmann-Shannon entropy 20
Booker, J.M. 29
Brissaud, J. B. 6
Bugár, G. 95
Burg, J. P. 157
Burgin, M. 6

C

Cai, H. 19
Campbell, L. L. 118
Carlson, B. A. 96
Characteristic function 29
Charvat, F. 5, 51, 64, 74, 77, 103, 173
Chawla, J. S. 145
Chemistry 6
Chen, X. 20
Chen, Y. 97
Clements, M. A. 96
Cluster analysis 96
Code alphabet 115
Code characters 115
Codes 114
Codeword 115
Coding theory 73, 114
Coefficient of determination 17
Computer science 114
Concave function 7, 12, 32, 61, 77, 164
Concavity 7, 32
Conditional entropy 29
Convex function 7, 23, 54
Convex optimization problem 98
Covariance 98
Covariance matrix 95
Cover, T.M. 118
Crispness 50
Crisp set 29, 42, 61
Cross entropy 23, 68, 95

D

Daroczy, Z. 5

Death rate 106
Decreasing function 31, 61, 77
Degenerate distributions 8, 11, 152
Dehmer, M. 6
De Luca, A. 30, 63, 74
Deterministic 153
Directed divergence 50, 65, 133, 155
Dispersion 16
Diversity 16
Dubois, D. 50, 75
Duckstein, L. 51

E

Ebanks, B. R. 31, 75
Ecology 97
Economics 97
Eguchi, S. 20
El-Sersy, K. F.
Encoding 114
Entropic approach 105
Entropy 104, 114, 151
Expected utility 94, 111
Exponentiated mean 116

F

Fan, J. L. 50, 75
Fan, J. X. 97
Ferreri, C. 20, 51, 76
Folger, T. A. 31
Fuzziness gap 39
Fuzzy cross entropy 82
Fuzzy distributions 43, 50, 71, 156, 176
Fuzzy divergence
Fuzzy entropy 50, 73
Fuzzy numbers 51
Fuzzy set 28, 73
Fuzzy uncertainty 29, 60, 65
Fuzzy vector 29, 66, 82

G

Gallager, R. 105
Garrido, A. 6
Generating function 28, 42, 50, 68
Geometric mean 16, 163, 169, 176
Gibbs entropy 2
Global minimum 18
Golomb, S. 42
Gray, R. M. 96
Guiasu, S. 10, 52, 68, 76, 104, 127, 156
Guo, X. Z. 31, 75
Gurdial 128
Gutter M. S. 97

H

Hanna, S. D. 97
Harmonic mean 16, 116
Havrada, J. H. 5, 52, 64, 74, 77, 103, 173
Herniter, J. D.156
Hessian matrix 22
Hillion, E. 6
Holder's inequality 125, 130
Hooda, D. S. 156

Hu, Q. 30

I

Ibraheem, N. M. A. 31
Impossible event 8
Inaccuracy 117
Increasing function 9, 31, 77, 114, 152
Information gain 17
Information improvement 97
Input-output analysis 151
Instantaneous code 117

J

Jaynes, E.T. 3, 151, 155
Jensen Burg measure 131
Jianshe, O. 95
Johnson, R. W. 96
Joint entropy 29
Joint probability distribution 9

K

Kakkar, P. 119
Kapur, J. N. 6, 29, 51, 74, 105, 120, 156
Kar, S. 20
Kato, S. 20
Kaufmann, A. 50
Kerridge, D. F. 117
Kesavan, H. K. 76, 156
Khan, A. B. 145
Klir, G. J. 31
Kosko, B. 50
Kraft, L. G. 116
Kraft's inequality 116
Kulkarni, S. 19
Kullback, S. 17, 50, 68, 75, 96, 117, 155
Kumar, R. 119
Kumar, S. 119

L

Lagrange's method 8, 73, 151
Laplace's principle 151
Lauprete, G. J. 97
Leibler, R. A. 50, 68, 75, 96, 117, 155
Leung-Yan-Cheong, S. K. 118
Li, X. 31
Linear programming 73, 151
Linguistics 6
Liu, B. 31
Local minimum 18
Log linear models 156
Longo, G. 127, 138
Lowen, R. 52

M

Mahajan, R. 156
Majumder, D. D. 52, 74
Marketing 156
Markowitz, H. M. 94, 98
Maximizing probabilities 11
Maximum cross entropy principle 76
Maximum entropy principle 31, 76

Ma, Y. L. 50, 75
Mean arrival rates 104
Mean codeword length 114
Mean number of customers 104
Mean return 95
Mean service rate 104
Mean–Variance efficient frontier 95, 98
Measures of risk 95
Medhi, J. M. 105
Medical diagnosis 30
Membership function 29
MEPD 151, 156, 165, 176
Metric spaces 18
Minimum cross entropy 155
M/M/1 system 105
Moments 171
Monotonicity 37
Most fuzzy set 65
Mowshowitz, A. 6
Mukesh 17, 97
Mutual entropy 29

N

Nocetti, D. 95
Noiseless coding theorem 115, 145
Non-degenerate 153
Nonlinear optimization program 100
Nonlinear programming 151
Non steady birth death process 94
Non steady state 108
Non-symmetric measure 17
Normalized measures 28, 46

O

Onicescu, O. 167
Operations research 73, 171
Optimization principle 76, 99, 155
Optimization problems 73
Osman, M. S. A. 31

P

Pal, N. R. 30, 51, 60, 70, 75
Parkash, O. 6, 10, 17, 30, 52, 61, 74, 76, 97, 119,156
Park, S. Y. 95
Partial information 28, 38, 156
Pattern classification 9
Pattern recognition 30, 151
Permutationally symmetric 7, 11, 77
Pessoa, F. 128
Physical sciences 97
Picard, C. F. 127
Piecewise convex 86
Portfolio analysis 73, 94, 156
Portfolio investment 151
Posterior distribution 155
Power mean 16
Prabhakar, B. 105
Prade, H. 50, 52, 75
Prefix code 127
Priori distribution 155
Priyanka 119

Probability distribution 17, 68, 104, 116, 151, 169
Probability generating function 105
Probability spaces 18

Q

Qualitative-quantitative measure 10
Quality control 156
Queueing theory 94, 104, 151

R

Ralescu, D. A. 20
Ramamoorthy, A. 119
Randomness 17
Regional planning 151
Reisher, C. 68
Relative entropy 17, 29
Relative information 68
Reliability theory 156
Renyi, A. 30, 51, 68, 74, 102, 117, 145
Risk-averse 102, 111
Risk prone 102, 111
Rosenfeld, A. 50, 75
Rudas, I. J. 30

S

Samarov, A. 97
Satellite communication 114
Shannon, C. E. 1, 28, 73, 118, 151, 156
Sharma, B. D. 5, 52, 76, 145
Sharma, P. K. 31, 51, 62, 70, 74, 156
Shenitzer, A. 76, 156
Shore, J. E. 96
Siegel, J. J. 98
Signal processing 96
Singh, R. P. 31
Singh, R. S. 10
Singh, Y. B. 6
Singpurwalla, N. D. 29
Social sciences 97
Sociology 6
Source coding theorem 128, 145
Spectral density functions 97
Statistica 171
Statistical inference 151
Statistical mechanics 151
Statistical physics 20
Steady state 105
Steady state queueing process 94
Stochastic 153
Suitable code 116
Sunita 6
Symmetric directed divergence 21

T

Taneja, H. C. 10
Taneja, I. J. 5, 52, 76, 145
Termini, S. 30, 63, 74
Thaler, R. H. 98
Theil, H. 97
Thermodynamic entropy 3
Thukral, A. K. 17

Tomar, V. P. 31
Topsoe, F. 156
Tran, L. 51
Transportation planning 151
Triangle inequality 18
Tuli, R. K. 52

U

Uncertainty 1, 8, 20, 28, 60, 94, 104, 151
Uniform distribution 8, 151, 157, 167
Uniquely decipherable code 115, 145
Uzsoki, M. 95

V

Variance 17, 31, 94, 98
Verdu, S. 19

W

Weighted averages 160
Weighted distribution 1, 127
Weighted divergence measure 59
Weighted entropy 10, 114, 129
Weighted fuzzy divergence 52
Weighted fuzzy information 68, 73
Weighted mean codeword length 127

X

Xie, W. X. 50, 75
Xin, X. L. 30, 75

Y

Yager, R. R. 30, 50
Yang, Y. W. 30
Yu, D. 30

Z

Zadeh, L. A. 2, 29, 73, 74
Zheng, C. 97
Zyczkowski, K. 5

Om Parkash is a Professor at the Guru Nanak Dev University, Department of Mathematics in Amritsar, India.

Having 32 years of teaching and research experience, Dr. Om Parkash is a Professor of Mathematics with Guru Nanak Dev University, Amritsar, India. He has edited 5 books and published 95 research papers in national and international journals of high repute. Under his supervision, 9 scholars have been awarded Ph.D. degrees, and many more are in the pipeline.

Besides, being a member of several mathematical societies and on the panel of experts and editorial boards of several national and international journals, he has presented his research work and delivered invited talks in several national and international conferences in India and in abroad including those held at Hamburg, Germany and Tskuba, Japan. He has accomplished many research project in the field of information theory, organized many conferences sponsored by various Indian agencies and chaired many sessions at several national and international conferences. He was bestowed award of excellence by the Indian Society of Information Theory and Applications for his contribution to the field of information theory.

The present book comprising of various mathematical models, basically deals with three main scientific contributions, viz, entropy, distance and coding which are closely related to each other and will be useful to all those interested in the development of information measures, and using entropy optimization problems in a variety of disciplines. It will be of interest to statisticians, engineers, life-scientists, economists and operational researchers interested in applying the powerful methodology based on maximum entropy principle in their respective disciplines. It will also be a source of inspiration and help to information theoreticians who have been using fuzzy information in their research work and will serve as a source of ready reference for the scientific community.

978-1-61804-371-9