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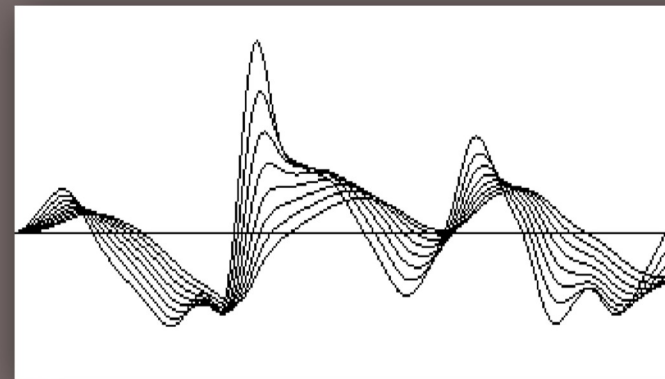
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# Advanced Topics on Applications of Fractional Calculus on Control Problems, System Stability and Modeling

*by Prof. Mihailo Lazarević*



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## Preface

In this monograph several aspects of fractional calculus will be presented ranging from its brief history over control applications and stability problems for time delay systems to applications in bio-engineering fields with illustrative examples.

The advantages of fractional calculus have been described and pointed out in the last few decades by many authors. Fractional calculus is based on derivatives and integrals of non integer arbitrary order, fractional differential equations and methods of their solution, approximations and implementation techniques. It has been shown that the fractional order models of real systems are regularly more adequate than usually used integer order models.

The monograph consists of seven chapters and an appendix where related a list of references include in the end of chapters.

The monograph begins in Chapter 1 with a brief historical review of the theory of fractional calculus and its applications. The theory of non-integer order differentiation and integration is almost as old as classical calculus itself, but nevertheless there seems to be an astonishing lack of knowledge of this field in most mathematicians. A look at the historical development can in parts explain the absence of this field in today's standard mathematics textbooks on calculus and in addition give the reader not familiar with this field a good access to the topics addressed in this monograph. In this chapter some well known definitions and properties of fractional order differ integrals are also stated.

Chapter 2 is devoted to the problem of discrete-time (digital) implementation of fractional order systems, i.e. fractional differ integrators, where two novel methods have been closely investigated: direct optimal and indirect. Both methods produce approximations of fractional differ integrators, which are then used to create approximations to more complex fractional order systems. It has been demonstrated by means of a number of numerical examples that both presented methods.

Some of stability problems for time delay systems have been discussed in the two following chapters (Chapters 3, 4). While Lyapunov methods have been developed for stability analysis and control law synthesis of integer linear systems and have been extended to stability of fractional systems, only few studies deal with non-Lyapunov stability of fractional systems. Here, finite-time stability of fractional order time-delay systems is considered in Chapter 3. Sufficient conditions for finite-time stability for (non) linear (non)homogeneous as well as perturbed fractional order time-delay systems are obtained and presented.

The problem of stability (simple stability and robust stability) of linear discrete-time fractional order systems is addressed in Chapter 4 where it is shown that some stability criteria for discrete time-delay systems could be applied with small changes to discrete fractional order state-space systems. The approach is based on the idea of constructing novel Lyapunov-Krasovskii functionals combined with free-weighting matrices or algebraic methods.

The next three Chapters (5, 6, 7) are related to applications of fractional calculus in bio-engineering fields. Chapter 5 is dedicated to the mathematical modeling of skin structure applying fractional calculus where it is proposed the skin structure as a more complex system consisting of several layers which describes series of structures via continuous generalizing (distributed order type) the Cole equation. According to this model and experimental data of the skin bioimpedance measurements, one may predict more complex equivalent electrical circuit and define new time parameters which correspond to each reduced Cole element.

In Chapter 6, a thermodynamically consistent rheological modified Zener model of viscoelastic body, i.e. standard fractional linear viscoelastic body is studied and presented. Proposed model comprises both fractional derivatives of stress and strain and the restrictions on the coefficients that follow from Clausius Duhem inequality. In that way, it should be included in both analytical and experimental projects ab initio, particularly in experiments in which newly developed materials are tested.

Finally, Chapter 7 concludes this monograph showing an useful modeling double DNA helix main chains of the free and forced fractional order vibrations applying fractional calculus. Different models are focusing on different aspects of the DNA molecule (biological, physical and chemical processes in which DNA is involved). The aim of this study was to model the DNA dynamics (vibrations of DNA chains) as a biological system in a specific boundary condition that are possible to occur in a life system during regular function of a DNA molecule.

I hope that this monograph will be value to Ph.D. students and fractional systems researchers as well as the other readers will find something in this monograph exciting.

Also, I want to thank very much Mrs. Ranki Gajic for the support in the preparation of the manuscript for English edition.

*Belgrade, August, 2012*

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Mihailo P. Lazarević is a Professor at the University of Belgrade, Faculty of Mechanical Engineering, Department of Mechanics in Serbia.

Mihailo P. Lazarević was born in Belgrade (Serbia) in 1964. He received four degrees from Belgrade University.

- *B.Sc in Mechanical Engineering (Aerospace),*
- *B.Sc in Electrical Engineering,*
- *M.Sc in Control and System Science Engineering and*
- *Ph. D. in Mechanical Engineering (Robotics).*

He worked at the University of Belgrade at the Faculty of Mechanical Engineering, Department of Mechanics as an Assistant from 1996-2001. He became an Assistant Professor in 2001 and Associate Professor in 2005 and since 2009 he is a full Professor at the same Faculty.

He is a member of the Serbian Society of Mechanics and the Secretary of the Serbian Society of Mechanics since July 2013. During the years of 2009 to 2012 he was the Chief of the Laboratory of Applied Mechanics of the Faculty of Mechanical Engineering in Belgrade University.

He published 4 National Monographs, 4 Chapters in International Monographs, 7 articles in Leading International Journals, 8 articles in International Journals, 12 articles in National Journals, 80 articles in the proceedings of International Meetings, Conferences and Symposiums, 1 Book and 2 Handbooks as collections of solutions and solved problems. Also, he participated in several research projects in Serbia and abroad (EUREKA) related to the field of (bio)robotics and applications of Fractional Calculus.

He is engaged as a reviewer by several International Journals and organizer of many International/National Conferences. According to Scopus, he has over 120 citations and his H index is 7.

His scientific research can be divided into several thematic areas among them the Mathematical Modeling and Control of Rigid Bodies by Electrical Devices (Robotics and Biomechatronics) which has been his first research interest. He has also been working on extension of ideas of Fractional Calculus to Modeling of Mechatronic Systems and Biomechanical Tissues.

In addition, a wide range of topics of his interest are in Different Classes of Time Delay Systems, in Theory of Electroviscoelasticity, in Applications of Fractional PID, in Fractional Iterative Learning Control and in Applications of Fractional Wavelet Transform in Signal and Image Processing.

